

# Extension Lesson

## The 6 Trigonometric Functions

NAME:

### Objectives

- ☑ Identify each type of trigonometric function from a graph.
- ☑ Label key elements of trigonometric functions.
- ☑ Graph each type of trigonometric function on the coordinate plane.

### Extension of:

This lesson extends the learning from Unit 5.

## 🔍 Explore

### 🔍 The 6 Trigonometric Functions

- Trigonometry studies the relationship between angles and side lengths of triangles.
- Trigonometric functions cycle through the same values over and over in a repeating pattern (as compared to the other families of functions in this course: polynomials, radicals, logarithmic, etc.)
- There are six foundational trigonometry functions:
  - **sine (sin)**
  - **cosine (cos)**
  - **tangent (tan)**
  - **cotangent (cot)**
  - **secant (sec)**
  - **cosecant (csc)**
- Each trigonometric function is periodic, meaning each repeats its values at regular intervals across the  $x$ -axis.
- Sine, cosine, secant, and cosecant repeat values across an interval of  $2\pi$ .
- Tangent and cotangent repeat values across an interval of  $\pi$ .

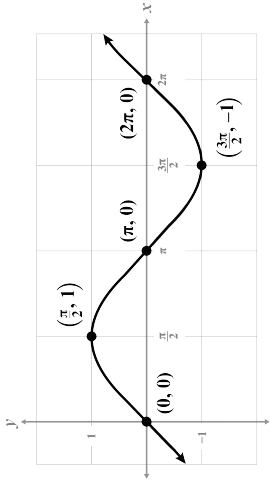
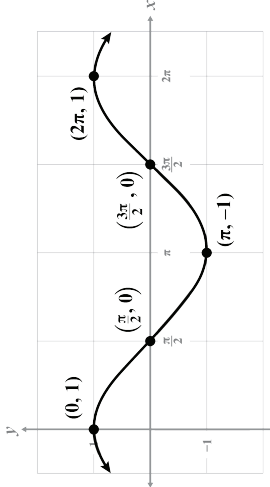
For this Extension Lesson, the scale of the  $x$ -axis will be in radians.

- For the parent graphs of the trigonometric functions, the scale of the  $x$ -axis is adjusted.
- For sine, cosine, secant, and cosecant, the scale of the  $x$ -axis increases or decreases by  $\frac{\pi}{2}$ .
- For tangent and cotangent, the scale of the  $x$ -axis increases or decreases by  $\frac{\pi}{4}$ .

- The scale of the  $y$ -axis increases or decreases by one.
- Each trigonometric function includes five key elements on the graph, which consist of intercepts and asymptotes.

To see the periodic nature of the trigonometric functions, go to the Online Lesson, under *Get Started*, and click the **Trigonometric Functions** link.

**🔗 Functions: Sine and Cosine**

	<b>Sine (sin)</b>	<b>Cosine (cos)</b>																								
<b>Equation</b>	$y = \sin x$	$y = \cos x$																								
<b>Graph and Key Elements</b>	 <table border="1" style="margin-left: auto; margin-right: auto;"> <thead> <tr> <th>x</th> <th>y</th> </tr> </thead> <tbody> <tr> <td>0</td> <td>0</td> </tr> <tr> <td><math>\frac{\pi}{2}</math></td> <td>1</td> </tr> <tr> <td><math>\pi</math></td> <td>0</td> </tr> <tr> <td><math>\frac{3\pi}{2}</math></td> <td>-1</td> </tr> <tr> <td><math>2\pi</math></td> <td>0</td> </tr> </tbody> </table>	x	y	0	0	$\frac{\pi}{2}$	1	$\pi$	0	$\frac{3\pi}{2}$	-1	$2\pi$	0	 <table border="1" style="margin-left: auto; margin-right: auto;"> <thead> <tr> <th>x</th> <th>y</th> </tr> </thead> <tbody> <tr> <td>0</td> <td>1</td> </tr> <tr> <td><math>\frac{\pi}{2}</math></td> <td>0</td> </tr> <tr> <td><math>\pi</math></td> <td>-1</td> </tr> <tr> <td><math>\frac{3\pi}{2}</math></td> <td>0</td> </tr> <tr> <td><math>2\pi</math></td> <td>1</td> </tr> </tbody> </table>	x	y	0	1	$\frac{\pi}{2}$	0	$\pi$	-1	$\frac{3\pi}{2}$	0	$2\pi$	1
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<b>Range</b>	$\{y   y \in \mathbb{R}, -1 \leq y \leq 1\}$	$\{y   y \in \mathbb{R}, -1 \leq y \leq 1\}$																								
<b>Asymptotes</b>	none	none																								
<b>Period</b>	$2\pi$ or $0 \leq x \leq 2\pi$	$2\pi$ or $0 \leq x \leq 2\pi$																								

**🔗 Functions: Tangent and Cotangent**

Equation	$y = \tan x$	$y = \cot x$																								
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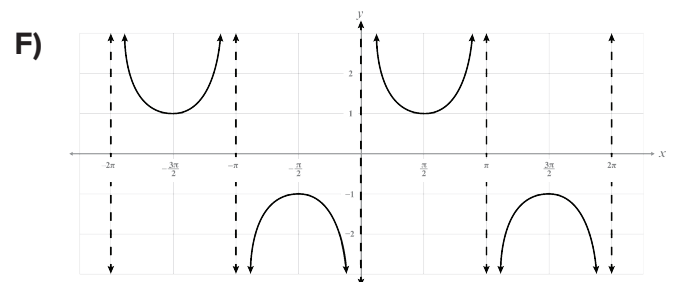
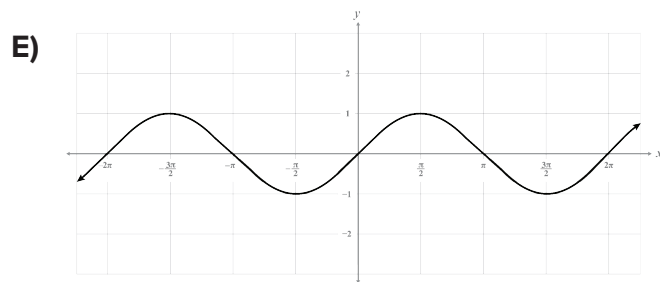
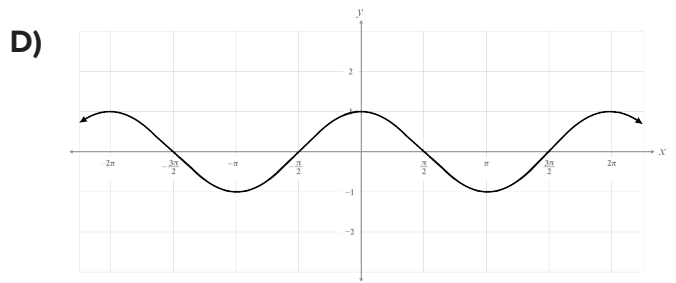
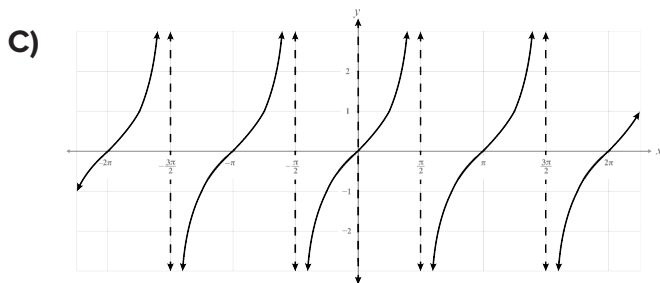
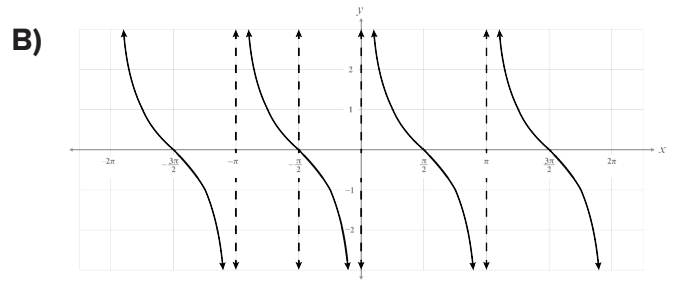
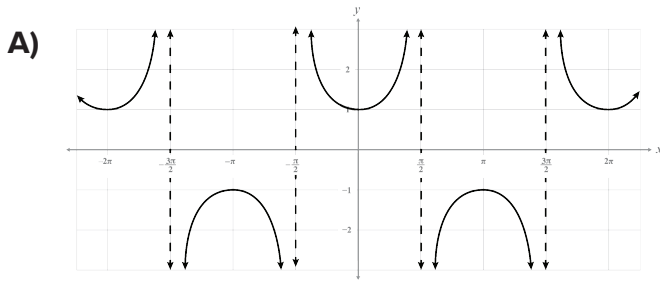
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 Practice

For this Practice, complete your work directly on these pages, as trigonometry-scaled graphs are not provided.

For problems 1–6, match the graph to its parent function.



1)  $y = \tan x$

2)  $y = \sec x$

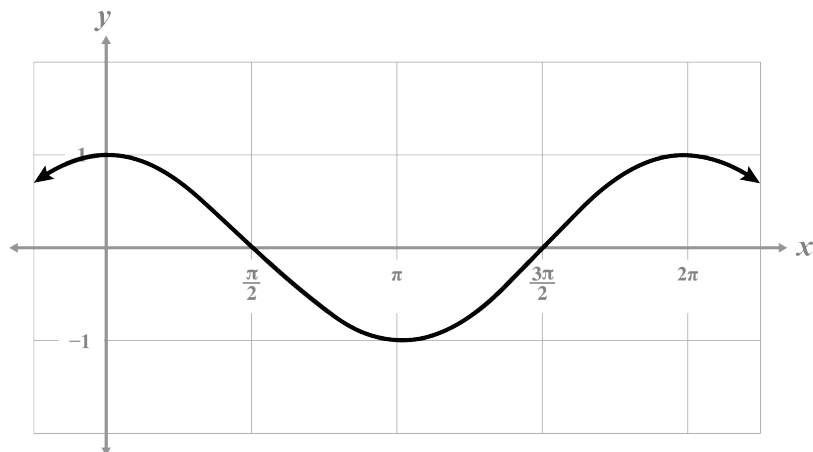
3)  $y = \sin x$

4)  $y = \cot x$

5)  $y = \csc x$

6)  $y = \cos x$

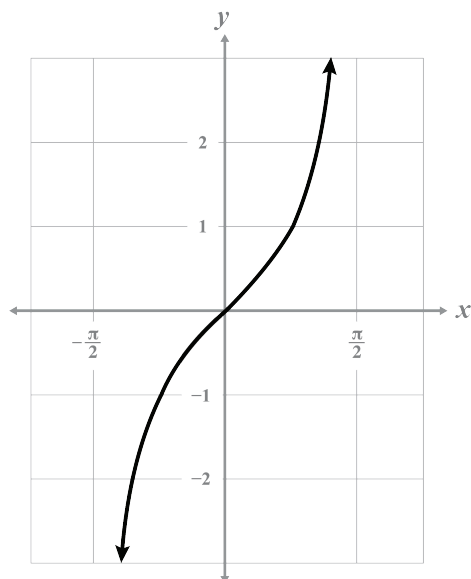
- 7) Name the function and label the key elements on the sketch over the interval  $0 \leq x \leq 2\pi$ .



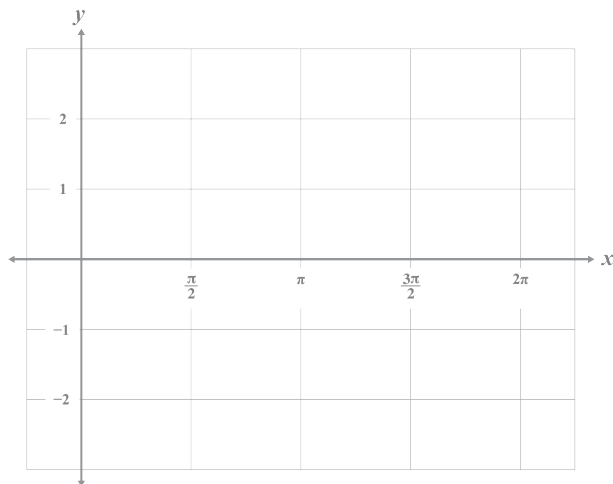
- 8) Name the asymptotes for  $y = \sec x$  across the interval  $0 \leq x \leq 2\pi$ .

- 9) Name the trigonometric functions that have a period of  $\pi$ .

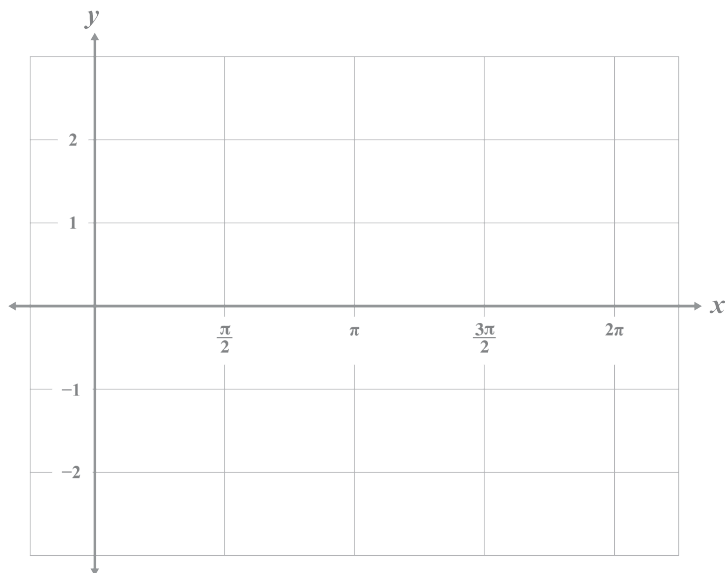
- 10) Name the function and label the key elements on the sketch over the interval  $-\frac{\pi}{2} < x < \frac{\pi}{2}$ .



- 11) Sketch  $y = \sin x$  over the interval  $0 \leq x \leq 2\pi$ . Label the five key elements.



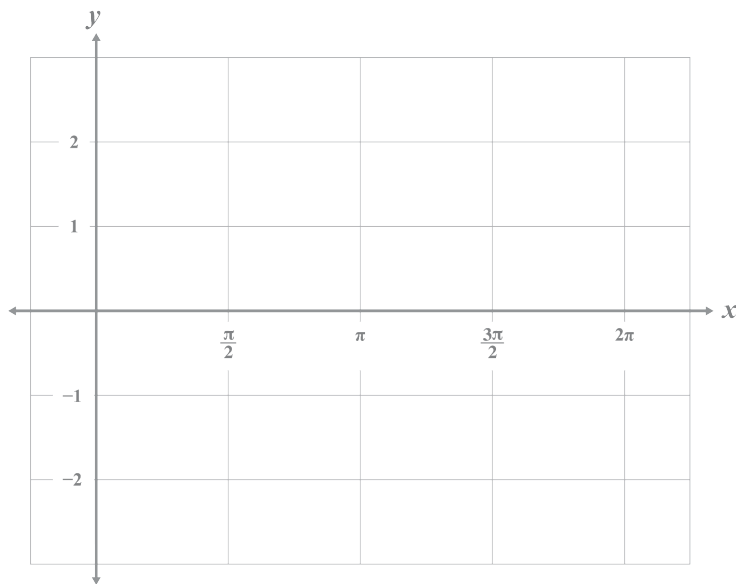
- 12) Sketch  $y = \csc x$  over the interval  $0 \leq x \leq 2\pi$ . Label the key elements.



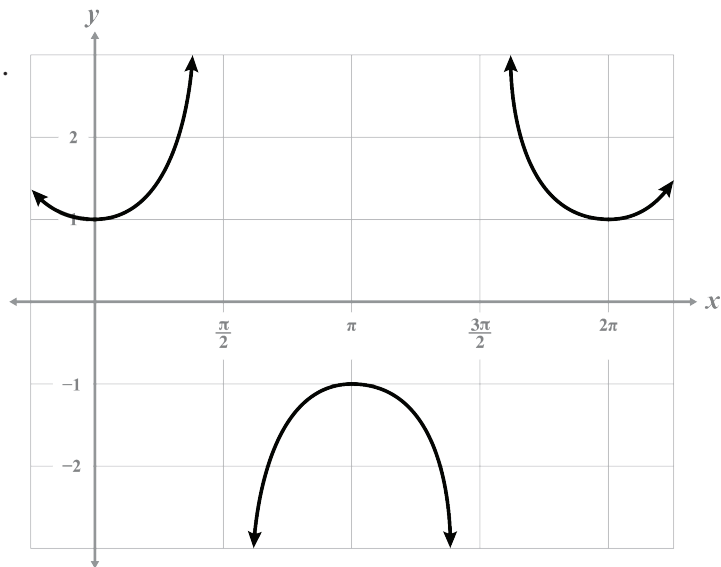
- 13) Name the asymptotes for  $y = \cot x$  across the interval  $0 \leq x \leq 2\pi$ .

- 14) Name the trigonometric functions that have a period of  $2\pi$ .

- 15) Sketch  $y = \cot x$  over the interval  $0 < x < \pi$ . Label the key elements.



- 16) Name the function and label the key elements on the sketch over the interval  $0 \leq x \leq 2\pi$



To continue, return to the Online Lesson.