

# Extension Lesson

## Degrees, Radians, and Arc Length

NAME:

### Objectives

- ✓ Convert between degree measure and radians.
- ✓ Calculate the central angle in radians.
- ✓ Find the arc length.

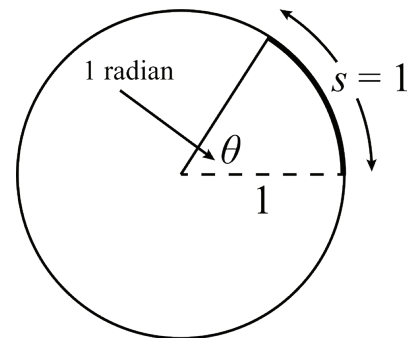
### Extension of:

This lesson extends the learning from Unit 5.

## 🔍 Explore

### 🔍 Degrees, Radians, and Arc Length

- Circles have angles and arcs that can be measured.
- An arc is a portion of the circumference of a circle.
- The central angle of a circle is an angle whose vertex is the center of the circle.
- The angles of a circle can be measured in degrees or in radians,  $\theta$ .
- A radian is a unit of angle measure for a circle.
- One radian is the measure of the central angle of a circle that intersects an arc equal in length to the radius of the circle.
- The arc length of a circle,  $s$ , is directly proportional to both the central angle and the radius.
- The formula to determine the arc length of a circle is:  $s = r\theta$ , where  $\theta$  must be in radians.
- The formula to convert from degrees ( $deg$ ) to radians ( $rad$ ) is:  $rad = deg \cdot \frac{\pi}{180}$
- The formula to convert from radians ( $rad$ ) to degrees ( $deg$ ) is:  $deg = rad \cdot \frac{180}{\pi}$



**Example 1**

Convert  $150^\circ$  to radians using  $\pi$ .

$$rad = deg \cdot \frac{\pi}{180}$$

$$rad = 150 \cdot \frac{\pi}{180} = \frac{150\cancel{\pi}}{180}$$

$$rad = \frac{5\pi}{6}$$

**Example 2**

Convert the radian measure,  $\frac{\pi}{2}$ , to a degree measure.

$$deg = rad \cdot \frac{180}{\pi}$$

$$deg = \frac{\pi}{2} \cdot \frac{180}{\pi} = \frac{180\cancel{\pi}}{2\cancel{\pi}}$$

$$deg = 90^\circ$$

**Example 3**

Determine the arc length of a circle with a  $225^\circ$  central angle and a radius of 7.

$$r = 7$$

$$rad = 225 \cdot \frac{\pi}{180} = \frac{225\pi}{180} = \frac{5\pi}{4}$$

$$s = r\theta$$

$$s = 7 \cdot \frac{5\pi}{4}$$

$$s = \frac{35\pi}{4}$$

 **Practice**

*Complete problems on a separate sheet of paper.*

**For problems 1–10, if the problem is in degrees, convert it to radians using  $\pi$ .  
If the problem is in radians, convert it to degrees.**

1)  $\frac{7\pi}{6}$

2)  $\frac{5\pi}{12}$

3)  $30^\circ$

4)  $\frac{3\pi}{2}$

5)  $360^\circ$

6)  $310^\circ$

7)  $\frac{13\pi}{20}$

8)  $115^\circ$

9)  $120^\circ$

10)  $\frac{\pi}{4}$

**Determine the arc length of a circle with the given central angle and radius.**

11)  $210^\circ, r = 3$

12)  $90^\circ, r = 8$

13)  $300^\circ, r = 1$

14)  $135^\circ, r = 10$

**Find the missing value.**

15)  $deg = 75^\circ, s = \frac{5\pi}{6}, r = \underline{\hspace{1cm}}$

16)  $r = 5, s = \frac{25\pi}{8}, deg = \underline{\hspace{1cm}}^\circ$