Extension Lesson 3: Linear Quadratic Systems

Objectives

In this lesson, you will learn about linear quadratic systems.

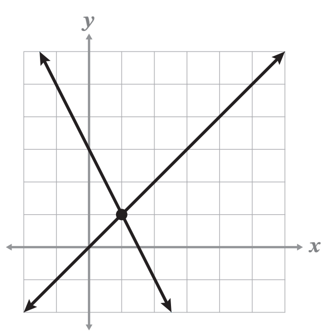
By the end of this lesson you will be able to do the following:

- Graph a system consisting of a linear and quadratic equation.
- Determine the solutions of a linear quadratic system of equations.
- Graph a system consisting of a linear and quadratic inequality.

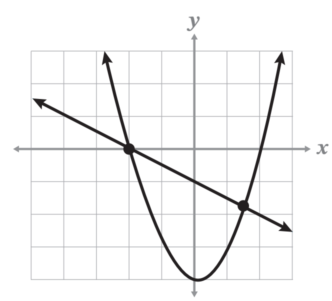
Graph a System of Linear and Quadratic Equations

- Systems of equations can contain more than one type of graph. For example, they can have a linear equation and a quadratic equation.
- Solutions to systems of equations:

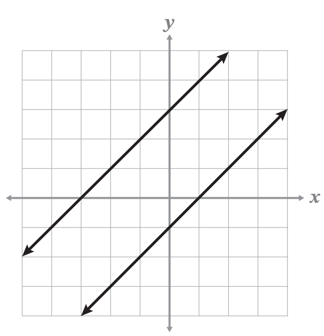
The solution to a system of linear equations is the point where the lines intersect.



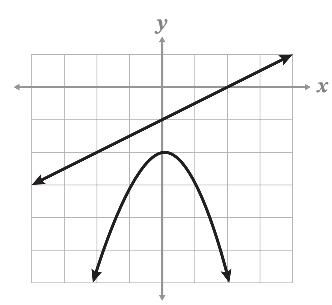
The solution to a system that includes a linear and a quadratic equation is where the line and the parabola intersect at either one or two points.



A system of linear equations that do not intersect has no solution.



A system that includes a linear and a quadratic equation that do not intersect has no solution.



- For systems of linear and quadratic equations, using technology to find the solution(s) is recommended because
    - the coordinates of the parabola can increase or decrease quickly, and
    - the intersection points will likely not be rational values.

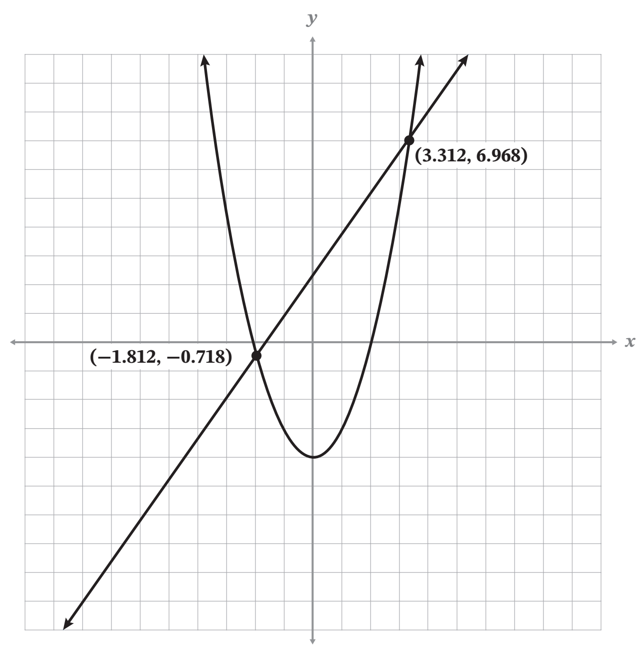
Example 1

Graph each of the equations on the coordinate plane. Extend the equation across the plane until the intersection points are found. Mark the solutions on the graph and list them as ordered pairs beside the graph.

$$y = x^2 - 4$$
$$y = \frac{3}{2}x + 2$$

solutions: (–1.812, –0.718), (3.312, 6.968)

You can use technology such as Desmos® to graph this system of equations. Technology will approximate irrational values.



Graph a System of Linear and Quadratic Inequalities

- Systems of inequalities using more than one type of graph follow similar rules as systems of linear inequalities.
    - The graph of each equation should be determined as solid or dashed and then placed on the same coordinate plane.
    - The shading will be added to show all possible solutions for all equations in the given system.
- Remember that a test point can be used to algebraically check where the shading belongs to make all equations true.

Example 2

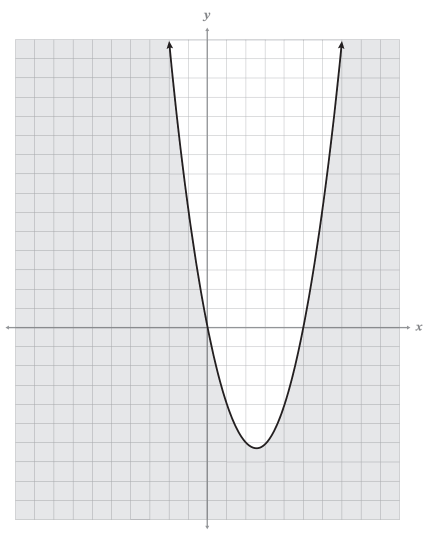
Graph each of the inequalities on the coordinate plane.

$y < 2x - 1$

$y \le x^2 - 5x$

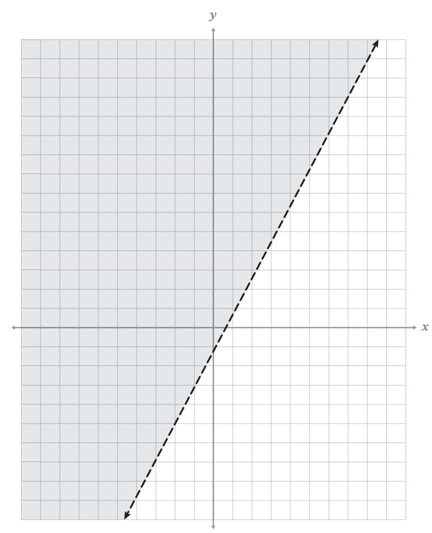
Looking at the inequalities separately, you can see that

- The shading *surrounds* the *solid* parabola $y \le x^2 - 5x$



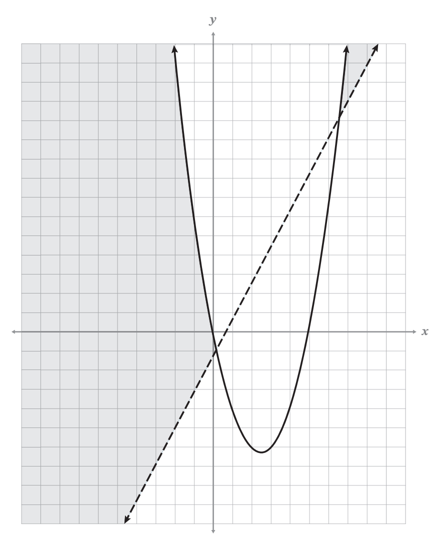
and

- The shading is *above* the *dashed* line. $y < 2x - 1$



When the inequalities are graphed on the *same* coordinate plane:

- The shaded region is on *both the left and right* sides of the parabola because the line extends through the parabola.
- The region where the solutions for each inequality overlaps is the solution to the system.



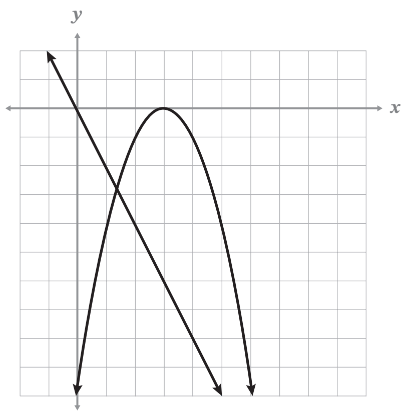
With technology, you can zoom in or out to see the best possible view of what is happening on the coordinate plane before graphing it on grid paper.

Practice

Complete the problems on a separate sheet of paper.

Determine if the given systems of equations have zero, one, or two solutions.

1)



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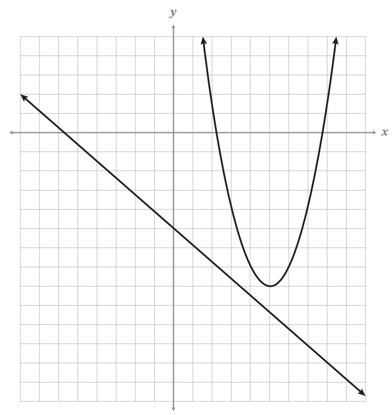

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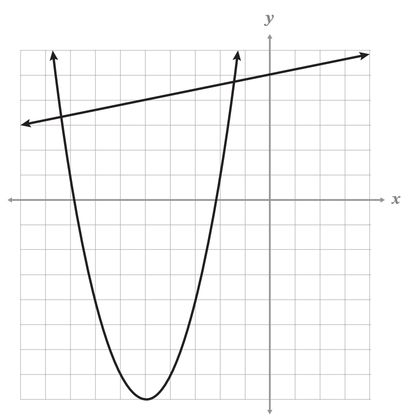

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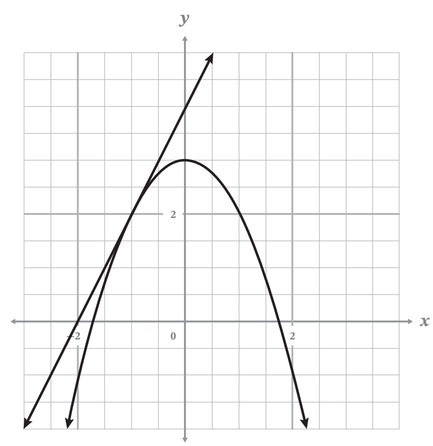

Graph the system of equations. Mark your solution(s) on the graph and list them as ordered pairs beside the coordinate plane.

5) $y = -2x + 3$
$y = x^2$

6) $y = 3x + 6$
$y = -x^2$

7) $y = (x - 2)^2 + 6$
$y = 6$

8) $y = \frac{1}{3}x - 4$
$y = -x^2 + 1$

9) $y = x^2 + 2x + 2$
$y = -\frac{4}{3}x + 1$

Graph the system of inequalities.

10) $y > x^2 + 3x + 2$
$y \le \frac{3}{5}x + 7$

11) $y < -\frac{1}{2}x + 3$
$y \ge -x^2$

12) $y \le -\frac{4}{5}x$
$y \le x^2 + 5x + 6$

13) $y \le \frac{1}{4}x^2 - 9$
$y > \frac{2}{5}x + 7$

14) $y < -(x - 1)^2 + 5$
$y > 2x - 4$

15) $y < -x^2 + x + 6$
$y \le x$