Extension Lesson 2: The Quadratic Formula

Objectives

In this lesson, you will learn about the quadratic formula.

By the end of this lesson, you will be able to do the following:

- Use the quadratic formula to find solutions to a quadratic equation.
- Determine the number of real solutions to a quadratic equation using the discriminant.

Vocabulary

- discriminant
- quadratic formula

Deriving the Quadratic Formula by Completing the Square

- You already know how to solve a quadratic equation with these methods:
  - Factoring
  - Graphing
  - Completing the square
- The last method to solve quadratic equations is the quadratic formula.
- The quadratic formula in standard form is written as:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

- The quadratic formula is derived by taking the standard form of a quadratic equation and completing the square to solve for x.
  - This will give you the formula which you can use to solve any quadratic equation in standard form.

Example 1

Use the steps for completing a square and a, b, and c to practice solving to represent a quadratic equation.

Implement	Explain (steps for completing a square)
$ax^2 + bx + c = 0$	**1)** Write the equation in standard form.
$\frac{a}{a}x^2 + \frac{b}{a}x + \frac{c}{a} = \frac{0}{a}$	**2)** Divide all terms by the leading coefficient, a. If $a = 1$, this step can be skipped.
$x^2 + \frac{b}{a}x + \frac{c}{a} - \frac{c}{a} = -\frac{c}{a}$	**3)** Add $-\frac{c}{a}$ to both sides of the equation
$x^2 + \frac{b}{a}x + \_\_\_\_ = -\frac{c}{a} + \_\_\_\_$	**4)** Simplify and write + _____ after the terms on both sides.
$+ \_\_\_\_ = \left(\frac{b}{2a}\right)^2$	**5)** Calculate the value that will make the LEFT side of the equation a perfect square trinomial.
$x^2 + \frac{b}{a}x + \left(\frac{b}{2a}\right)^2 = -\frac{c}{a} + \left(\frac{b}{2a}\right)^2$	Add this value to the blanks on BOTH SIDES of the equation.
$\left(x + \frac{b}{2a}\right)^2 = -\frac{c}{a} + \frac{b^2}{4a^2}$	**6)** Write the left side of the equation as the product of a binomial squared. Simplify the right side of the equation.
LCD: $4a^2$	**7)** Find the LCD for the expression on the right-side of the equation.
$-\left(\frac{c}{a}\right)\left(\frac{4a}{4a}\right) + \frac{b^2}{4a^2} = \frac{-4ac + b^2}{4a^2}$	
$\sqrt{\left(x + \frac{b}{2a}\right)^2} = \sqrt{\frac{b^2 - 4ac}{4a^2}}$	**8)** Solve for x by taking the square root of the entire equation.
$x + \frac{b}{2a} = \pm\frac{\sqrt{b^2 - 4ac}}{2a}$	**9)** Simplify the equation. (Include $\pm$ to represent both solutions.)
$x = -\frac{b}{2a} \pm \frac{\sqrt{b^2 - 4ac}}{2a}$	**10)** Add $-\frac{b}{2a}$ to both sides of the equation to find the values of x.
$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$	When the problem is written with one denominator, the quadratic formula is revealed.

Using the Quadratic Formula to Find Solutions to a Quadratic Equation

- Now that you have seen how the quadratic formula is derived, you can use it to solve quadratic equations.
  - quadratic formula: $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
- The quadratic formula will work for any quadratic equation in standard form.
- Because there are so many variables and + and − signs in the quadratic formula, it is important that you carefully input values when you use it.

> In Algebra 1, you will only solve for real number solutions. But in later levels, you can solve for imaginary solutions as well.

Example 2

Use the quadratic formula to solve. Write your answers in radical form unless they simplify to rational numbers.

$-x^2 - 3x + 7 = 0$

Implement	Explain
$a = -1, b = -3, c = 7$	◀ Identify a, b, and c
$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-(-3) \pm \sqrt{(-3)^2 - 4(-1)(7)}}{2(-1)}$	◀ Substitute values into the quadratic formula
$x = \frac{3 \pm \sqrt{9 - (-28)}}{-2}$	◀ Simplify
$x = \frac{3 \pm \sqrt{37}}{-2}$	
or	
$x = -\frac{3 \pm \sqrt{37}}{2}$	◀ Write the final answer in the form that makes the most sense for the question being asked.
or	
$x = \frac{3 + \sqrt{37}}{-2}, \frac{3 - \sqrt{37}}{-2}$	

Your solution can be written using the ± symbol or written as two separate solutions.

Example 3

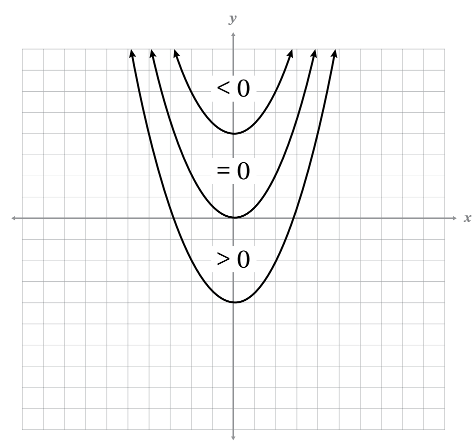
Use the quadratic formula to solve. Write your answers in radical form unless they simplify to rational numbers.

$2x^2 + 3x - 5 = 0$

Implement	Explain
$a = 2, b = 3, c = -5$	◀ Identify a, b, and c
$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-(3) \pm \sqrt{(3)^2 - 4(2)(-5)}}{2(2)}$	◀ Substitute values into the quadratic formula
$x = \frac{-3 \pm \sqrt{9 - (-40)}}{4}$	◀ Simplify
$x = \frac{-3 \pm \sqrt{49}}{4}$	
$x = \frac{-3 \pm 7}{4} = \frac{-10}{4}, \frac{4}{4}$	◀ Simplify the radical and write answer in simplest form.
$x = -2.5, 1$	

Determining the Number of Real Solutions to Quadratic Equations Using the Discriminant

- The discriminant is the part of the quadratic formula under the square root symbol: $b^2 - 4ac$
- Remember, that a quadratic equation can have 0, 1, or 2 real solutions.
- The discriminant can be used as its own formula to determine the number of solutions a quadratic equation will have.
    - The discriminant does not tell you the exact values for the solutions; it only determines whether you will have 0, 1, or 2 real solutions.
    - It can be helpful to use the discriminant before using the quadratic formula to determine the number of solutions.
        - If the discriminant shows there are zero real solutions, you do not need to continue solving the equation because the answer has already been found.
        - If the discriminant shows there are one or two solutions, you can use it to make sure you get the correct number of solutions when using the quadratic formula.
- When the quadratic equation is written in standard form, $ax^2 + bx + c = 0$, the values of a, b, and c are used in the discriminant.
    - When $b^2 - 4ac > 0$, there will be 2 real solutions, or two x-intercepts.
    - When $b^2 - 4ac = 0$, there will be 1 real solution, or one x-intercept.
    - When $b^2 - 4ac < 0$, there will be 0 real solutions, or zero x-intercepts.



- You can see why these rules work by placing the discriminant back into the quadratic formula under the radical sign.
    - If the value under the radical is greater than zero, there are two solutions:
    $x = \frac{-3 \pm \sqrt{49}}{4} = \frac{-3 \pm 7}{4} = \frac{-10}{4}, \frac{4}{4}$
    - If the value under the radical is equal to zero, there is only one solution because adding or subtracting zero does not change the original value:
    $x = \frac{-3 \pm \sqrt{0}}{4} = -\frac{3}{4}$
    - If the value under the radical is less than zero, there are no real solutions because a negative value under the radical sign is not a real number:
    $x = \frac{-3 \pm \sqrt{-49}}{4} =$ no real solution

Example 4

Determine the number of solutions to the quadratic equation using the discriminant.

A) $2x^2 + 8x - 3 = 0$

Implement	Explain
$a = 2, b = 8, c = -3$	◀ Identify a, b, and c
$b^2 - 4ac$	◀ Write the discriminant formula
$(8)^2 - 4(2)(-3)$	◀ Substitute values into the formula
$64 - (-24) = 64 + 24 = 88$	◀ Simplify

Because the discriminant is positive, there are 2 real solutions.

B) $x^2 - 5x + 8 = 0$

Implement	Explain
$a = 1, b = -5, c = 8$	◀ Identify a, b, and c
$b^2 - 4ac$	◀ Write the discriminant formula
$(-5)^2 - 4(1)(8)$	◀ Substitute values into the formula
$25 - 32 = -7$	◀ Simplify

Because the discriminant is less than 0, there are no real solutions.

C) $2x^2 + 2 = 4x$

Implement	Explain
$2x^2 - 4x + 2 = 0$	◀ Equation in standard form
$a = 2, b = -4, c = 2$	◀ Identify a, b, and c
$b^2 - 4ac$	◀ Write the discriminant formula
$(-4)^2 - 4(2)(2)$	◀ Substitute values into the formula
$16 - 16 = 0$	◀ Simplify

Because the discriminant is 0, there is 1 real solution.

Practice

Complete the problems on a separate sheet of paper.

Use the quadratic formula to solve. Write answers in simplest radical form.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

1) $5x^2 - 11x + 6 = 0$

2) $8x^2 - 5x = 0$

3) $x^2 + 4x + 4 = 0$

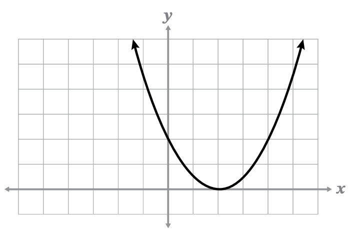
4) $3x^2 + 10x - 2 = 0$

5) $x^2 + 6x - 2 = 0$

6) $-2x^2 + 11x - 12 = 0$

Determine the number of solutions from the graph. Write the inequality or equation that matches the number of solutions.

7)



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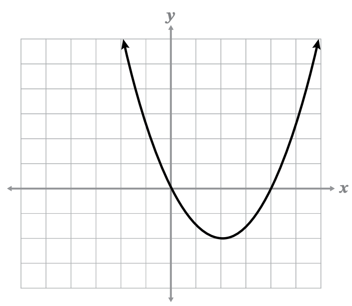

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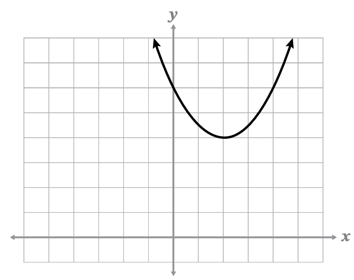

Use the discriminant to determine the number of real solutions for the quadratic equations.

Discriminant: $b^2 - 4ac$

10) $5x^2 - 11x + 6 = 0$

11) $8x^2 - 5x = 0$

12) $x^2 + 6x + 12 = 0$

13) $x^2 + 4x + 4 = 0$

14) $3x^2 + 10x = 2$

Use this Desmos® extension to check your work and explore the quadratic formula further.