Extension Lesson 1: Completing the Square

Objectives

In this lesson, you will learn about completing the square.

By the end of this lesson you will be able to do the following:

- Determine the value of c to make a perfect square trinomial for $x^2 + bx + c$.
- Solve quadratic equations by completing the square.

Vocabulary

- completing the square

Determining the Value of *c* to Make a Perfect Square Trinomial

- Some quadratic trinomials are perfect square trinomials: $a^2 + 2ab + b^2$
- With a perfect square trinomial in standard form ($ax^2 + bx + c$), the terms a and c are perfect squares and $b = 2\sqrt{ac}$
  - Because a and c are perfect square terms, finding *twice* the product of their square roots is likely something you have already done without even realizing it.
- You can prove that this formula is true by using a perfect square trinomial where the value of c is known.
  - For example,
    $x^2 + 8x + 16 = (x + 4)^2$

Plan	Implement
Determine a, b, and c using standard form.	$a = 1, b = 8, c = 16$
Find the value of b and c using the formula $b = 2\sqrt{ac}$	$b = 2\sqrt{1 \cdot 16} = 2 \cdot 4 = 8$
	$c = \left(\frac{b}{2}\right)^2 = \left(\frac{8}{2}\right)^2$

- When you know the values of a and b, you can use this formula to find the value of c:
  - If $a = 1$, then $b = 2\sqrt{c}$, which means $c = \left(\frac{b}{2}\right)^2$ by inverse operations.
  - Notice that a is simplified from the original formula since it is equal to one.
- For Algebra 1, the value of a will equal one ($a = 1$), and the missing value will be c.

Example 1

Find the value of c that will make the expression a perfect square trinomial.

$x^2 + 6x + c$

Implement	Explain
$a = 1, b = 6$	◀ Enter the values into the formula $c = \left(\frac{b}{2}\right)^2$ and solve for c.
$c = \left(\frac{6}{2}\right)^2 = \frac{(6)^2}{(2)^2} = \frac{36}{4} = 9$	◀ Remember to square the numerator and denominator.
$x^2 + 6x + 9$	◀ Write your trinomial with the value of c.

Check

$(x + 3)(x + 3) = (x + 3)^2$	◀ To check, write the trinomial as its binomial factors. Since this can be written as a binomial-squared, the answer is correct.

Example 2

Find the value of c that will make the expression a perfect square trinomial.

$x^2 - 15x + c$

Implement	Explain
$a = 1, b = -15$	◀ Enter the values into the formula $c = \left(\frac{b}{2}\right)^2$ and solve for c.
$c = \left(\frac{-15}{2}\right)^2 = \frac{(-15)^2}{(2)^2} = \frac{225}{4}$	◀ Make sure to square the numerator and denominator and leave the answer as an improper fraction.
$x^2 - 15x + \frac{225}{4}$	◀ Write your trinomial with the value of c.

Check

$\left(x - \frac{15}{2}\right)\left(x - \frac{15}{2}\right) = \left(x - \frac{15}{2}\right)^2$	◀ Factor the expression to check your work.

Solving Quadratic Equations with Perfect Squares

- **Completing the square** is used to solve a quadratic equation, particularly one that cannot be factored.
- There are several steps to follow to complete a square, including
    - calculating the term that will make the expression a perfect square trinomial and
    - taking the square root of both sides of the equation.
- Make sure to follow these steps exactly. This will help you to see that quadratic equations, factoring, and graphing are all different representations of the same thing.

- Here are some important tips to remember when completing a square:
  - When you take a square root, write the plus/minus symbol ($\pm$) in front of the square root symbol. When a number is squared you no longer know if the original value is positive or negative. Using this symbol ensures that all solutions to the quadratic equation are represented.
  - Perform the *same* operation on *both sides* of the equation so that equality is maintained. (You can see this especially in steps 2, 3, 4, and 7 in Example 3.)

Example 3

Solve the equation by completing the square.

$x^2 + 35 = 12x$

Step	Implement	Explain
1)	$x^2 - 12x + 35 = 0$	◀ Write the equation in standard form.
2)	$a = 1$, move to step 3	◀ Divide all terms by the leading coefficient, a. (Since $a = 1$, this step can be skipped.)
3)	$x^2 - 12x + 35 = 0$ $-35 \quad -35$	◀ Add $-\frac{c}{a}$ to both sides of the equation. In this case, since $a = 1$, you add the additive inverse of c: $-\frac{c}{a} = -\frac{(35)}{1} = -35$
4)	$x^2 - 12x + \_\_\_ = -35 + \_\_\_$	◀ Simplify and write + ____ after the terms on *both sides.*
5)	$+\_\_\_ = \left(\frac{-12}{2}\right)^2 = (-6)^2 = 36$ $x^2 - 12x + (-6)^2 = -35 + (-6)^2$	◀ Calculate the value that will make the LEFT side of the equation a perfect square trinomial. Add this value, $\left(\frac{b}{2}\right)^2$, to the blanks on *both sides* of the equation. You may find it helpful not to simplify this value completely, especially if it is a fraction. In this case, $(-6)^2$ is added instead of 36 so that $\left(\frac{b}{2}\right)$ can be identified in the next step.
6)	$(x - 6)^2 = -35 + 36$ $(x - 6)^2 = 1$	◀ Write the left side of the equation as the product of a binomial squared: $\left(x - \frac{b}{2}\right)^2$
7)	$\sqrt{(x - 6)^2} = \sqrt{1}$	◀ Solve for x by taking the square root of the entire equation.
8)	$x - 6 = \pm 1$	◀ Simplify the equation.
9)	$x = 6 \pm 1$ $x = 5, 7$	◀ Add $-\frac{b}{2a}$ to *both sides* of the equation to find the values of x. In this case, $-\frac{b}{2a} = -\frac{(-12)}{2(1)} = 6$

Example 4

Solve the equation by completing the square.

$x^2 - 8x - 31 = 0$

Step	Implement	Explain
1)	$x^2 - 8x - 31 = 0$	Write the equation in standard form.
2)	$a = 1$, move to step 3	Divide all terms by the leading coefficient, a. (Since $a = 1$, this step can be skipped.)
3)	$x^2 - 8x - 31 = 0$ $+31 \quad +31$	Add $-\frac{c}{a}$ to both sides of the equation. In this case, since $a = 1$, you add the additive inverse of c: $-\frac{c}{a} = -\frac{(-31)}{1} = 31$
4)	$x^2 - 8x + \_\_\_ = 31 + \_\_\_$	Simplify and write $+\_\_\_\_$ after the terms on *both sides.*
5)	$+\_\_\_ = \left(\frac{-8}{2}\right)^2 = (-4)^2 = 16$ $x^2 - 8x + (-4)^2 = 31 + (-4)^2$	Calculate the value that will make the LEFT side of the equation a perfect square trinomial. Add this value, $\left(\frac{b}{2}\right)^2$, to the blanks on *both sides* of the equation. Note that $(-4)^2$ or 16 can be used since the values are equivalent.
6)	$(x - 4)^2 = 31 + 16$ $(x - 4)^2 = 47$	Write the left side of the equation as the product of a binomial squared: $\left(x - \frac{b}{2}\right)^2$ Then, simplify the right side of the equation.
7)	$\sqrt{(x - 4)^2 = 47}$	Solve for x by taking the square root of the entire equation.
8)	$x - 4 = \pm\sqrt{47}$	Simplify the equation.
9)	$x = 4 \pm \sqrt{47}$	Add $-\frac{b}{2a}$ to *both sides* of the equation to find the values of x. In this case, $-\frac{b}{2a} = -\frac{(-8)}{2(1)} = 4$

The answer $x = 4 \pm \sqrt{47}$ means there are two solutions to the quadratic equation:

- $x = 4 + \sqrt{47}$
- $x = 4 - \sqrt{47}$

You can use the "±" symbol as shorthand rather than writing the solutions separately.

Practice

Complete the problems on a separate sheet of paper.

Determine the value that will make the expression a perfect square trinomial. Show your work.

1) $x^2 + 16x + c$

2) $x^2 - 24x + c$

3) $x^2 + 6x + c$

4) $x^2 + 7x + c$

5) $x^2 - 11x + c$

6) $x^2 + \frac{5}{2}x + c$

Solve the quadratic equations by completing the square.

7) $x^2 - 6x + 8 = 0$

8) $x^2 - 10x + 14 = 0$

9) $x^2 + 6x - 22 = 0$

10) $x^2 + 20x + 94 = 0$

11) $x^2 - 2x - 38 = 0$

12) $x^2 + 5x - 9 = 0$