Combinations of Functions

Sum

$(f+g)(x)=f(x)+g(x)$

Difference

$(f-g)(x)=f(x)-g(x)$

Product

$(fg)(x)=f(x)\cdot g(x)$

Quotient

$\left(\frac{f}{g}\right)(x)=\frac{f(x)}{g(x)},\ g(x)\neq 0$

Composition

$[f\circ g](x)=f[g(x)]$

Properties of Logs

Where $\{a, b, c, n \in \mathbb{R}\}$ and $a>0$, $a\neq 1$, $b>0$, $c>0$

Foundational Properties

If $a^0=1$, then $\log_a 1=0$ or $\ln 1=0$

If $a^n=a^n$, then $\log_a\left(a^n\right)=n$ or $\ln e^x=x$

If $a^1=a$, then $\log_a(a)=1$ or $\ln e=1$

Product: $\log_a bc=\log_a b+\log_a c$

Quotient: $\log_a \frac{b}{c}=\log_a b-\log_a c$

Power: $\log_a b^n=n\cdot\log_a b$

Change of Base

The variables a, b, and X are positive real numbers, and $a\neq 1$, $b\neq 1$.

$\log_b X=\frac{\log_a X}{\log_a b}$

Properties of Equality

Where $\{a, b, c\in\mathbb{R}\}$, $b>0$, $b\neq 1$

Exponential Equation

If $b^a=b^c$, then $a=c$.

Logarithmic Equation

When a is a positive number not equal to one, $\log_a x=\log_a y$ if and only if $x=y$.

Rational Root Theorem

Potential rational roots, $x=\frac{\text{Factors of the constant}}{\text{Factors of the leading coefficient}}$

Inverse of Functions

If $[f\circ g](x)=x$ and $[g\circ f](x)=x$, then the functions are inverses.

Polynomial Function

$p(x)=a_n x^n+a_{n-1}x^{n-1}+\ldots+a_2x^2+a_1x+a_0$ where $a_n\neq 0$, n is a whole number and the coefficients are real numbers.

More Functions

Exponential

$f(x) = ab^{x-h} + k,\ b > 0,\ b \neq 1$

Natural Exponential

$f(x) = e^x$

Logarithmic

$f(x) = a\log_b(x-h) + k,\ x > 0$

Variation

Where k is the constant of variation:

Direct

$y = kx^n,\ n > 0$

Inverse

$y = \dfrac{k}{x^n},\ n > 0$

Joint

$y = kxz$

Converting Between Exponents and Logarithms

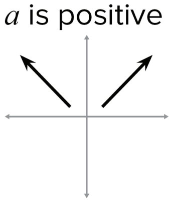
$b^x = y \quad \log_b y = x$

End Behavior of Polynomial Functions

End Behavior

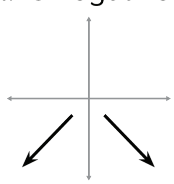
Even Degree

a is positive



$x \to \pm\infty, f(x) \to +\infty$

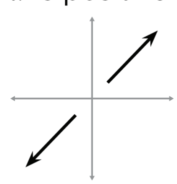
a is negative



$x \to \pm\infty, f(x) \to -\infty$

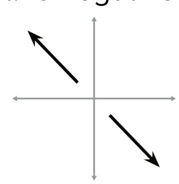
Odd Degree

a is positive



$x \to -\infty, f(x) \to -\infty$
and $x \to +\infty, f(x) \to +\infty$

a is negative



$x \to -\infty, f(x) \to +\infty$
and $x \to +\infty, f(x) \to -\infty$

Plan, Implement, Explain Method for Problem-Solving

Plan how you will approach the problem.

Implement your plan to complete the problem, and then **check** your work.

Explain why your answer makes sense for the given problem.