Lesson 56
NAME:

Probability of Dependent Events

Start by navigating to the Online Lesson for instructions.

Objectives

- Calculate the probability of a dependent event.
- Calculate conditional probabilities.
- Explain if events are independent or dependent.

Why?

The outcome of an event can change depending on the order in which it occurs, for example, selecting your seat in a movie theater after other people are already seated, or your travel time based on weather and traffic conditions. The conditions of the problem affect the outcome and the likelihood it will occur.

Warm Up

1) What is the formula for independent events?

2) Kim needs a dress for the spring dance. If she purchases a dress she likes at the first store, she will not go to another store. Explain if this is an independent or dependent event.

To continue, return to the Online Lesson.

Explore

Dependent Events

Fill in the notes as you watch the video in the Online Lesson.

- When the occurrence of the first event changes the probability of the subsequent events, the events are _____.
- The _____ states:

  When A and B are dependent events, then $\mathrm{P(A \text{ and } B)} = \mathrm{P(A)} \cdot \mathrm{P(B \mid A)}$.

- The phrase _____ often represents dependent probability.
- Not replacing an element changes the _____ _____.

Multiplication Rule for:	
Independent Probability	**Dependent Probability**
$P(A \text{ and } B) = P(A) \cdot P(B)$	$P(A \text{ and } B) = P(A) \cdot P(B \mid A)$

Example 1

Complete the example as you watch the video in the Online Lesson.

A jar contains 11 red, 27 yellow, 16 blue, and 6 green marbles. Beau selects three marbles, one after the other, without replacement. If the marbles were selected randomly, what is the probability that Beau selects a yellow, then another yellow, and then a blue marble? Round to the nearest tenth of a percent.

$$P(Y, Y, B) = P(Y) \cdot P(Y \mid Y) \cdot P(B \mid Y, Y)$$

Example 2

Complete the example as you watch the video in the Online Lesson.

Meredith checked out four graphic novels, three mysteries, and three biographies from the library. What is the probability that the first four books Meredith randomly selects are a mystery, a graphic novel, a graphic novel, and a biography?

Checkpoint: Dependent Events

Marco has a standard deck of cards. Determine the probability of Marco randomly drawing the cards in the order listed without replacement. Round to the ten-thousandth place.

A) P(ace of spades, 3 of spades)

B) P(♠, ♠)

To continue, return to the Online Lesson.

Conditional Probability

Fill in the notes as you watch the video in the Online Lesson.

- Conditional probability, $P(B|A)$, is the probability _____ _____.
- The formula for conditional probability of dependent events is: $P(B|A) = \frac{P(A \cap B)}{P(A)}$
- Because the order in which events occur matters, _____.
- Conditional probability is useful when the conditions being compared _____ _____.
- This allows you to compare _____ rather than only to the total number of events.

- The words _____ often indicate there is a specific condition that must be considered.

Example Conditional Probability Phrases:
The probability of getting a ticket, *given that* you were speeding.
The probability of a student voting "yes" *if* they are in 8th grade.
The probability of seeing a dolphin *when* you are at the beach.
The probability of buying a new phone case *after* purchasing a new phone.

- When solving problems with conditional events:
  - Substitute what you know into the formula:

    $\mathrm{P}(A \text{ and } B) = \mathrm{P}(A) \cdot \mathrm{P}(B \mid A)$ or $\mathrm{P}(A \text{ and } B) = \mathrm{P}(B) \cdot \mathrm{P}(A \mid B)$

  - Then _____ algebraically.

Example 3

Complete the example as you watch the video in the Online Lesson.

A fruit bowl contains ten apples and eight nectarines. You decide to grab two pieces of fruit without looking. Determine the probability as a simplified fraction that:

A) Both pieces of fruit are apples.

B) You grab a nectarine after you already have an apple.

Example 4

Complete the example as you watch the video in the Online Lesson.

The employees of Buzz Coffee tracked orders on a busy morning. One of the employees created a Venn diagram to organize the orders and calculated some of the probabilities.

$P(\text{Latte}) = 0.4967$

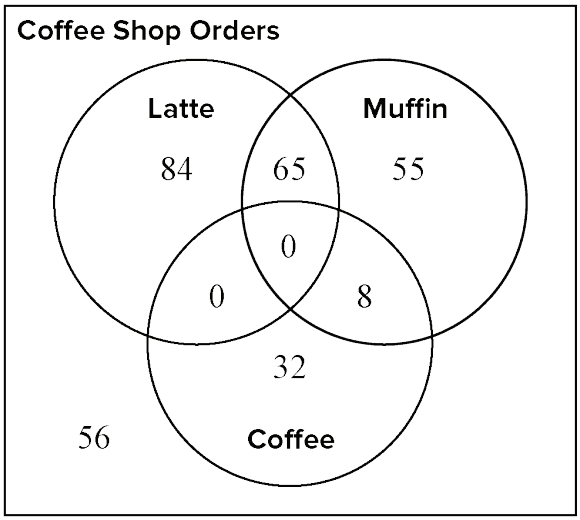
$P(\text{Muffin}) = 0.4$

$P(\text{Coffee}) = 0.1333$

$P(\text{Latte and Muffin}) = 0.2167$

$P(\text{Coffee and Muffin}) = 0.0267$

$P(\text{None}) = 0.1867$



A) Determine if the orders are independent events.

Independent: $P(A \text{ and } B) = P(A) \cdot P(B)$

Latte and Muffin

Coffee and Muffin

Find the probability that:

B) A customer ordered a muffin *after* they ordered a latte.

$P(M \mid L)$

C) A customer ordered a latte *when* they ordered a muffin.

$P(L \mid M)$

D) Why are the results of the probabilities in parts B and C different?

Example 5

Complete the example as you watch the video in the Online Lesson.

On a given day, the probability that Adrian will wear sneakers is 60%. The probability that Adrian wears a baseball hat is 40%. The likelihood that she wears sneakers *if* she is already wearing a hat is 30%.

Determine the probability of Adrian wearing a hat *and* sneakers.

Determine the probability of Adrian wearing a hat *if* she is already wearing sneakers.

Checkpoint: Conditional Probability

Find the probability that:

$P(\text{Muffin}) = 0.4$

$P(\text{Coffee}) = 0.1333$

$P(\text{Coffee and Muffin}) = 0.0267$

A) A customer ordered a muffin if they ordered a coffee.

B) A customer ordered a coffee when they ordered a muffin.

To continue, return to the Online Lesson.

Practice 1

Complete problems on a separate sheet of paper.

For problems 1–3, use the scenario.

For a standard deck of 52 cards, a student draws cards randomly, without replacement. Determine the probability. Write your answer as a fraction.

1) $P(K, Q, 2)$

2) $P(\text{spade}, \text{spade}, \text{heart})$

3) $P(7, 7, 7)$

For problems 4–6, use the scenario.

A room consists of three 9th graders, five 10th graders, eight 11th graders, and four 12th graders. Determine the percent chance of students being randomly selected into groups of:

4) Two ninth graders, one tenth grader, and one eleventh grader

5) One twelfth grader and one eleventh grader

6) One ninth grader, one tenth grader, one twelfth grader

For problems 7–11, use the scenario.

In the Midwest, there is a 0.7 probability of a sunny day during the 100 days of summer. If it is sunny, there is a 0.2 probability of a thunderstorm in the afternoon. If it is raining, the probability of a thunderstorm in the afternoon is 0.6.

- S: Sunny day
- R: Rain
- T: Thunderstorm

$$P(S) = 0.7$$
$$P(R) = 1 - P(S) = 0.3$$
$$P(T|S) = 0.2$$
$$P(T|R) = 0.6$$

7) Determine the number of summer days that are sunny, AND there is a thunderstorm.

8) Determine the total number of summer days a thunderstorm can be expected.

9) Determine the number of days in the summer that are sunny, AND there is NO thunderstorm.

10) On how many summer days will there likely be rain and NO thunderstorm?

11) How many summer days will likely have no thunderstorms?

For problems 12–14, use the scenario.

At a local gym, 60% of the members use the cardio machines while 45% of the members lift weights. The probability that a member uses the cardio machines, given they lift weights, is 80%.

12) Explain why the events are not independent.

13) What is the percent chance that a randomly selected member uses the cardio machines and lifts weights?

14) $P(W|C)$

To continue, return to the Online Lesson.

Mastery Check

Show What You Know

The school newspaper, *The Daily Scoop*, recently conducted a survey to find out student participation in extracurricular activities. They surveyed 200 students. Here is what they found:

- 80 students participate in sports (S)
- 60 students participate in music (M)
- 30 students participate in both sports and music
- The rest of the students don't participate in either of these activities.

A) Are the events "participating in sports" and "participating in music" independent? Explain your reasoning.

B) If a student participates in sports, what is the probability that they also participate in music, $P(M|S)$?

C) What is the probability that a randomly selected student participates in sports AND does *not* participate in music?

D) If a student does NOT participate in sports, what is the probability that they participate in music?

Say What You Know

In your own words, talk about what you have learned using the objectives for this lesson and your work on this page.

To continue, return to the Online Lesson.

Practice 2

Complete problems on a separate sheet of paper.

For problems 1–3, use the scenario.

A bag contains eight green marbles, five blue marbles, and seven red marbles. Determine the probability if marbles are drawn without replacement. Write your answer as a fraction.

1) $P(G, G, R)$

2) $P(R, B, B)$

3) $P(R, B, R)$

For problems 4–6, use the scenario.

Your school is having a raffle, and there are 50 tickets sold. You (Y) and your friend (F) each buy three tickets. Only one ticket will be drawn for a $\$100$ gift card, and a second ticket will be drawn for a $\$25$ gift card. The first ticket drawn is not replaced. Determine the probability and write your answer to the tenth of a percent.

4) The probability that you win both prizes

5) The probability that you each win a prize

6) If you each bought 5 tickets, by how much would the probability of you both winning a prize increase?

For problems 7–9, use the scenario.

In a particular town, the probability of rain on any day is 25%. The probability of a traffic jam is 40%. The chance of a traffic jam given that it is raining is 70%.

7) What is the probability that it is raining and there is a traffic jam?

8) What is the probability of rain given a traffic jam?

9) Explain why the events are dependent.

For problems 10–12, use the scenario.

Kristin wrapped up books on her summer reading list in identical wrapping paper. She is required to read three books. There are eight fiction (F) and five non-fiction (N) books. She randomly selects one wrapped book at a time to read. Determine the probability of selecting the books in the given order. Write the solution as a fraction.

10) $P(N, N, F)$

11) Two fiction books and one non-fiction book

12) $P(F, F, F)$

To continue, return to the Online Lesson.