Lesson 55
NAME:

Probability of Independent Events

Start by navigating to the Online Lesson for instructions.

Objectives

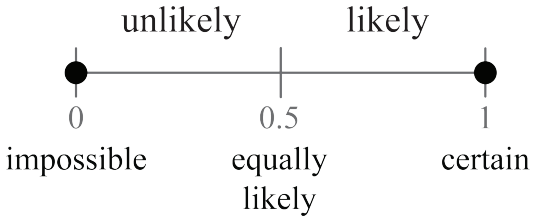
- Calculate the probability of an independent event.
- Calculate binomial probability.
- Determine if events are independent or dependent, and explain your thinking.

Why?

The likelihood of a favorable outcome changes when you have more than one event, and whether or not the events affect one another. Now you will be able to determine more complex probabilities and learn why a favorable outcome may be less likely than you think, such as guessing answers on a test.

Warm Up

1) Label the number line with percentages.



2) What are the symbols and words that represent union and intersection?

To continue, return to the Online Lesson.

Explore

Independent Events

Fill in the notes as you watch the video in the Online Lesson.

- When one event does not affect the probability of another event, the events are _____.
- The _____ states:

  When A and B are independent events, then $\mathrm{P}(A \text{ and } B) = \mathrm{P}(A) \cdot \mathrm{P}(B)$.

- The _____ is sometimes used in place of the word “and.”
- When completing two events in a row, the phrase _____ often represents independent probability because the total number of outcomes _____.
- When the occurrence of the first event changes the probability of subsequent events, the events are _____. This topic will be taught in the next lesson.

Example 1

Complete the example as you watch the video in the Online Lesson.

State whether the scenarios represent independent or dependent events.

A) One card is selected from a deck, placed to the side, and another card is drawn.

B) A coin is flipped ten times in a row.

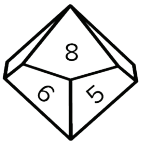
C) Three marbles are drawn from a bag. After each draw, the marble is replaced.

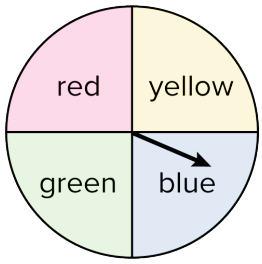
Example 2

Complete the example as you watch the video in the Online Lesson.

Determine the probability of rolling an eight-sided die and spinning the spinner arrow.

A) Rolling 7, and the arrow stopping on red





B) P(even and primary color)

C) Rolling anything but 2, and the arrow stopping on green

D) P(multiple of 4 ∩ red or blue)

Example 3

Complete the example as you watch the video in the Online Lesson.

What is the percent chance of flipping a coin and getting tails 3 times in a row?

Checkpoint: Independent Events

Marco has a standard deck of cards. Determine the probability of Marco drawing the cards in the order listed with replacement.

A) P(ace of spades ∩ 3 of spades)

B) P(♠ ∩ K)

To continue, return to the Online Lesson.

Binomial Probability

Fill in the notes as you watch the video in the Online Lesson.

- A _____ is a type of independent probability with n trials in which the trials are either a _____, p, or a _____, q.
- Because the outcome is either success or failure, the sum of p and q is one.

  $p + q = 1$, or $p = 1 - q$
- The formula for binomial probability is: $P(r) = nCr(n, r) \cdot p^r \cdot q^{n-r}$
  - n: _____
  - r: _____
  - p: _____
  - q: _____

Example 4

Complete the example as you watch the video in the Online Lesson.

Steve forgot to study for a ten-question, multiple-choice test. He needs a grade of 80%. What are the chances of correctly guessing exactly eight questions if each question has four choices and only one correct answer?

Example 5

Complete the example as you watch the video in the Online Lesson.

A coin is tossed three times. What is the likelihood that the coin lands tails up at least twice?

P(getting tails up two times or getting tails up three times)

Example 6

Complete the example as you watch the video in the Online Lesson.

At County High School, 3% of the students have earned a merit scholarship. If 40 students are selected at random, find the probability that four or five of the selected students have earned a merit scholarship.

Checkpoint: Binomial Probability

From Example 6, determine the probability that exactly one student has earned a merit scholarship.

To continue, return to the Online Lesson.

Practice 1

Complete problems on a separate sheet of paper.

State whether the scenario describes an independent or dependent event.

1) One thousand people were surveyed online and asked to select their favorite movie genre: action, comedy, animation, or fantasy. As the surveyors reviewed the results, they noticed five people in a row selected "action."

2) Sara has a laundry basket full of shirts that need to be hung up in her closet. She grabs a shirt, puts it on a hanger, and hangs it in the closet until the basket is empty.

3) Three students in Ainsley's probability class use a fair, four-section spinner to answer a 50-question multiple-choice test. The scores of each test are compared.

4) What is the percent chance of flipping a coin and getting tails 5 times in a row?

5) What is the probability of drawing an ace and then a king from a standard deck of cards when the card is replaced?

6) What is the percent chance of rolling the numbers 2, 3, 4, 5 in a row when rolling a six-sided die?

For problems 7–10, use the following scenario.

A bag of marbles contains four red, six blue, three green, and two yellow marbles. After each draw, the marble is replaced before a new one is selected. Determine the likelihood that the marbles are drawn in a specific order. Round to the nearest hundredth of a percent.

7) $P(R, R, Y)$

8) $P(B, R, Y)$

9) $P(B, G, B)$

10) $P(Y, \text{not } G)$

For problems 11–12, use the following scenario.

A basketball team successfully shoots 85% of its throws from the free-throw line. They attempt to shoot a total of 18 free throws in a game.

11) What is the probability that the team will successfully make exactly 16 free throws?

12) What is the probability that the team successfully makes *at least* 16 free throws?

For problems 13–16, use the following scenario.

A manufacturer produces items with a 3% defect rate. If a random sample of 15 items is selected, to the nearest hundredth of a percent, what is the chance that:

13) Exactly two items are defective

14) All items are without defects

15) At most, two items are defective

16) If there are only 13 items, determine the percent chance that exactly 4 items are defective.

To continue, return to the Online Lesson.

Mastery Check

Show What You Know

Keanu was rushing through a six-question assignment and decided to guess the answers.

A) If all the problems were true/false and Keanu decided to flip a coin to fill in the answers, what is the likelihood that he answers all the problems correctly?

B) Explain why the events are independent when guessing.

C) If all the problems were multiple choice with four answer options each, how likely is it that he would guess the correct answer for all six problems?

D) If all the problems were multiple choice with four answer options each, how likely is Keanu to guess at least four of the six answers correctly?

E) Explain to Keanu why guessing is not a good strategy.

Say What You Know

In your own words, talk about what you have learned using the objectives for this lesson and your work on this page.

To continue, return to the Online Lesson.

Practice 2

Complete problems on a separate sheet of paper.

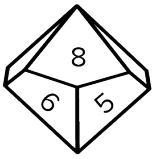
State whether the scenario describes an independent or dependent event.

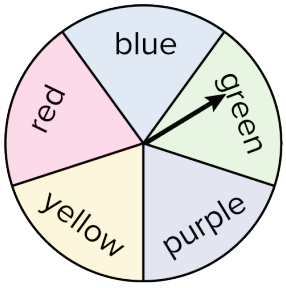
1) A teacher is assigning the order in which students will present their projects this month. Once a student presents, their turn is complete for that month.

2) Three experiments were conducted simultaneously: flipping a coin, rolling one die, and using a spinner.

3) A baseball team calculated its batting average after calculating the individual batting averages of its top nine players.

For problems 4–7, use the spinner and the eight-sided die with numbers 1 through 8 to find the probability.





4) Rolling an even number and landing on yellow

5) Rolling a multiple of 3 and landing on a primary color

6) Rolling a 5 and not landing on purple

7) Rolling any number except 7 and landing on purple or green

For problems 8–11, use the following scenario.

Cards are drawn from a standard deck with replacement. Determine the percent chance to the nearest hundredth of a percent.

8) Heart, then a five

9) $\mathrm{P}(10, \mathrm{A})$

10) Three face cards in a row

11) $\mathrm{P}(\mathrm{K}, \mathrm{Q}, \mathrm{J})$

For problems 12–16, use the following scenario.

A recent survey found that 75% of people left a positive review of a product. If eight customers are randomly selected, determine to the nearest hundredth of a percent, the chance of:

12) Exactly six people leaving a positive review

13) At least six people leaving a positive review

14) Fewer than two people leaving a *negative* review

15) All eight people leaving a positive review

16) If *ten* people were randomly selected, determine, to the nearest hundredth of a percent, the chance of exactly five people leaving a positive review.

To continue, return to the Online Lesson.

Targeted Review

Complete items on a separate sheet of paper.

For problems 1–4, use the table.

An observational study was conducted with 152 people. Observers recorded each person's dominant hand and teeth-brushing hand.

		Teeth-Brushing Hand	
		Right	**Left**
Dominant Hand	**Right**	85%	4%
	Left	2%	9%

1) Determine the number of people who are left-handed and brush their teeth with their right hand.

2) Determine the percentage of people who are left-handed and also brush their teeth left-handed.

3) If a person is selected randomly, how likely is it that they will use their right hand to brush their teeth?

4) Determine the number of people who use their dominant hand to brush their teeth.

5) Expand: $(2x-1)^5$

6) Charlotte has 5 shirts, 6 pairs of jeans, and 4 pairs of shoes. How many different outfits can she put together?

7) A television network wants to estimate the average number of minutes a household watches a particular prime-time news program. The network requires the estimate to be within 5 minutes of the true average viewing time. If a 99% confidence interval is used, what is the *minimum* sample size that can be taken using a standard deviation of 22.5 minutes?

8) What is the z-score of the observed difference?

Multiple Choice

____ **9)** Emil has a bag containing two red dice, ten blue dice, and four white dice. If Emil selects a single die from the bag without looking, what is the chance of selecting white?

A) 1 out of 8

B) 1 out of 4

C) 5 out of 8

D) 3 out of 4

Multiple Choice

_____ **10)** The probability of a baby being born with blue eyes when both parents have brown eyes is around 25%. What is the probability that the baby will *not* have blue eyes?

A) 0%

B) 75%

C) 25%

D) 100%

_____ **11)** Select the question that is unbiased.

A) Would your ride to work be more enjoyable if the traffic were less horrendous?

B) Does it really take that long to drive to work?

C) How long was your drive to work today?

D) If your job actually makes you drive to the office, how long does it take?

_____ **12)** New equipment for physical therapy was introduced at one of eight office locations. The therapist saw that patients recovered 3.2 weeks faster ($z = 2.32$), but the cost of the equipment was $\$17{,}500$. What should be considered when deciding if all eight offices will update their equipment?

A) Only the cost should be considered.

B) Only the statistical significance should be considered.

C) Both the statistical significance and the cost should be considered.

D) No considerations are needed when the results are statistically significant.

Problem	1	2	3	4	5	6	7	8	9	10	11	12
Origin	L54	L54	L54	L54	L52	L51	L49	L50	L53	L53	L48	L50

L = Lesson in this level, A1 = Algebra 1: Principles of Secondary Mathematics

To continue, return to the Online Lesson.