

Lesson 52

Binomial Counting Principles

NAME:

 Start by navigating to the Online Lesson for instructions.

Objectives

- ✓ Apply Pascal's Triangle to problems.
- ✓ Know and apply the Binomial Theorem to binomial expressions.

Why?

The Binomial Theorem is shorthand for expanding binomial expressions raised to the n^{th} power because they can be quite long and prone to error. You will be able to combine your knowledge of the repeated distributive property, combinations, and Pascal's Triangle to deepen your understanding of the relationship between algebraic concepts and probability.

Warm Up

Solve without a calculator. Show your work.


1) $nCr(12, 2)$

2) $nCr(12, 3)$

 To continue, return to the Online Lesson.

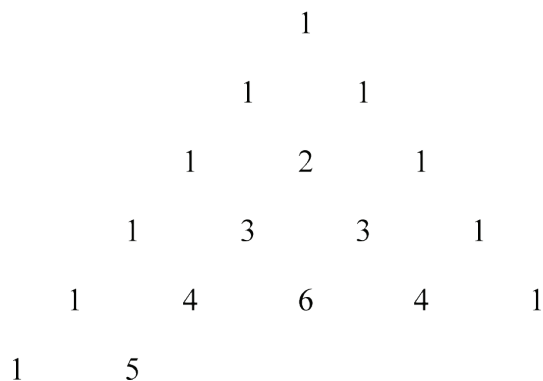
Explore

Pascal's Triangle

 Fill in the notes as you watch the video in the Online Lesson.

- _____ is a triangular arrangement of numbers in which each number is the sum of the two numbers directly above it.
- Mathematicians have found many applications for Pascal's Triangle, including:
 - Finding the number _____
 - Determining the coefficients _____
 - Finding the sum _____

- Pascal's Triangle is constructed by _____ of the two numbers in the row directly above it.
 - Each row starts and ends with _____.
 - The top row of the triangle is "_____."
 - In each row that follows, the _____ represents which row it is (row 1, row 2, etc.).



- To determine a combination, nCr , using Pascal's Triangle:
 - _____
 - _____ (the first term will always be the left-most "1" in the row)
 - _____

For this lesson, a Pascal's Triangle handout has been provided for you. You should be able to create additional rows on your own using the established pattern.

Example 1

 Complete the example as you watch the video in the Online Lesson.

Use Pascal's Triangle to evaluate.

A) $nCr(13, 7)$

Explain

▶ The $(r + 1)^{th}$ term in Pascal's Triangle has the answer to the combination $(7 + 1 = 8^{th}$ term)

▶ Find the 8^{th} term in the 13^{th} row

B) $nCr(15, 11)$

Example 2

 Complete the example as you watch the video in the Online Lesson.

Use Pascal's Triangle to determine the sum of the row.

A) Row 6

B) Row 17

Checkpoint: Pascal's Triangle

Use Pascal's Triangle to solve.

A) $nCr(7, 5)$

B) Determine the sum of row 7.



To continue, return to the Online Lesson.

The Binomial Theorem

 Fill in the notes as you watch the video in the Online Lesson.

- When a binomial expression is raised to the n^{th} power where n is a whole number, _____ in the coefficients and the powers of the terms.

$$(x + y)^1 =$$

$$(x + y)^2 =$$

$$(x + y)^3 =$$

$$(x + y)^4 = 1x^4y^0 + 4x^3y^1 + 6x^2y^2 + 4x^1y^3 + 1x^0y^4$$

$$(x + y)^5 = 1x^5y^0 + 5x^4y^1 + 10x^3y^2 + 10x^2y^3 + 5x^1y^4 + 1x^0y^5$$

- The Binomial Theorem determines the coefficients of an n^{th} power binomial _____.

$$(x + y)^n = \binom{n}{0}x^n y^0 + \binom{n}{1}x^{n-1}y^1 + \binom{n}{2}x^{n-2}y^2 + \dots + \binom{n}{n}x^0 y^n$$

- In this formula, the “stacked” terms inside the parentheses represent _____.
- You can use Pascal’s Triangle in place of the Binomial Theorem, but you would need to _____.
- The two terms of the binomial expression are _____.
- If x or y has a _____, use _____ to correctly expand the binomial.
- When the binomial contains a numerical term:
 - _____
 - _____

- This means the _____
once the terms are completely simplified.
- When the terms of a binomial expression are subtracted, $(x - y)^n$, the signs of the terms will _____, starting with a positive value.
- Finding the n^{th} term:
 - The Binomial Theorem can be used to find a specific term in a binomial expression _____.
 - The value of r will be _____ than the term you are looking for (i.e., if you need the fourth term, $r = 3$, if you need the eighth term, $r = 7$).

$$\binom{n}{r} x^{n-r} y^r$$

Example 3

▶ Complete the example as you watch the video in the Online Lesson.

Expand the binomial with Pascal's Triangle.

$$(x + y)^5$$

Row 5:

$$1x^5y^0 + 5x^4y^1 + 10x^3y^2 + 10x^2y^3 + 5x^1y^4 + 1x^0y^5$$

Example 4

▶ Complete the example as you watch the video in the Online Lesson.

Determine the middle term of the binomial.

A) $(a - b)^{10}$

B) $(a - 3)^{10}$

Example 5

▶ Complete the example as you watch the video in the Online Lesson.

Determine the third term of the binomial expression.

$$(3x + y)^7$$

Plan

Find r
Determine nCr
Expand the specific binomial term

Implement

Example 6

▶ Complete the example as you watch the video in the Online Lesson.

Expand the binomial.

$$\begin{aligned} & \left(2x - \frac{1}{3}y\right)^4 \\ &= \binom{4}{0}(2x)^4\left(-\frac{1}{3}y\right)^0 + \binom{4}{1}(2x)^3\left(-\frac{1}{3}y\right)^1 + \binom{4}{2}(2x)^2\left(-\frac{1}{3}y\right)^2 + \binom{4}{3}(2x)^1\left(-\frac{1}{3}y\right)^3 + \binom{4}{4}(2x)^0\left(-\frac{1}{3}y\right)^4 \\ &= (1)(2^4)\left(-\frac{1}{3}\right)^0 x^4 y^0 + (4)(2^3)\left(-\frac{1}{3}\right)^1 x^3 y^1 + (6)(2^2)\left(-\frac{1}{3}\right)^2 x^2 y^2 + \end{aligned}$$

Checkpoint: The Binomial Theorem

Determine the fifth term of the binomial.

$$(x - 2)^8$$



To continue, return to the Online Lesson.

 **Practice 1**

Complete problems on a separate sheet of paper.

- 1) Write rows 0 through 6 of Pascal's Triangle.
- 2) Explain how to find the next row of Pascal's Triangle.

Determine the combination using Pascal's Triangle.

- 3) $nCr(12, 7)$
- 4) $nCr(9, 8)$
- 5) Determine the sum of row 8.
- 6) Write the sum of the 39th row as a base raised to a power. Do not evaluate.

Expand the binomial with Pascal's Triangle.

- 7) $(a + h)^6$
- 8) $(2 - n)^3$
- 9) Determine the third term of the binomial: $(x - y)^4$
- 10) Determine the sixth term of the binomial: $(a + 1)^{16}$

Expand the binomial.

- 11) $(3x + 2)^4$
- 12) $\left(x - \frac{1}{2}y\right)^5$



To continue, return to the Online Lesson.

 **Mastery Check** **Show What You Know**

- A)** How can Pascal's Triangle be used with the Binomial Theorem?
- B)** Explain what happens to the exponents of each term as you work through the Binomial Theorem.
- C)** Determine the thirteenth term of the binomial: $(0.5x + y)^{14}$
- D)** Determine the middle term of the binomial: $(0.5x + y)^{14}$

 **Say What You Know**

In your own words, talk about what you have learned using the objectives for this lesson and your work on this page.



To continue, return to the Online Lesson.

 **Practice 2**

Complete problems on a separate sheet of paper.

Determine the answers using Pascal's Triangle.

- 1) $nCr(15, 7)$
- 2) $nCr(8, 5)$
- 3) $nCr(14, 10)$
- 4) $nCr(10, 3)$
- 5) Determine the sum of row 9.
- 6) Write the sum of the 85th row as a base raised to a power. Do not evaluate.

Expand the binomial with Pascal's Triangle.

- 7) $(x-y)^7$
- 8) $(m+2)^4$
- 9) Determine the seventh term of the binomial: $(b-c)^{11}$
- 10) Determine the middle term of the binomial: $(a+1)^{16}$

Expand the binomial.

- 11) $(2x-1)^5$
- 12) $(a+2b)^4$



To continue, return to the Online Lesson.