Lesson 51
NAME:

Permutations and Combinations

Start by navigating to the Online Lesson for instructions.

Objectives

- Apply the Fundamental Counting Principle.
- Solve problems involving permutations and/or combinations.

Why?

How many ways can your group of six be seated at a table at a restaurant? Will we ever run out of phone numbers? With the Fundamental Counting Principle, you can determine the answers to these questions and more.

Warm Up

List all the possible combinations for the given scenario.

1) List all the ways to arrange the letters X, Y, Z in groups of three.

2) Jodi has three shirts: red, blue, and green, two pairs of pants: tan and black, and two pairs of shoes: sneakers and boots. If Jodi must wear a red shirt, how many outfits can she make?

A calculator is essential for this lesson because most answers are very large numbers.

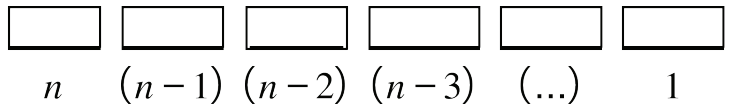
To continue, return to the Online Lesson.

Explore

The Fundamental Counting Principle

Fill in the notes as you watch the video in the Online Lesson.

- The Fundamental Counting Principle (FCP) is used to determine the _____ _____ (or arrangements) when there are _____ _____.
- If one choice has n options and another has m options, then nm (the product) is the _____.
- There are many different types of scenarios that _____ the principle.
- With decreasing options, if you have a set group of items and select them one at a time, you will _____.
- Each selection depends on _____.



- A factorial (!) is a mathematical shorthand used to represent _____ _____.
- It can be used to determine the _____ _____ of n-elements.
- Because you can arrange nothing into one group of nothing _____.

Example 1

Complete the example as you watch the video in the Online Lesson.

Mina only buys clothing that pairs well together. She has 7 tops, 4 bottoms, 3 pairs of shoes, and 3 bags. How many different outfits is Mina able to create?

_____ _____ _____ _____

tops bottoms shoes bags

Mina can create _____ outfits with her current options.

Example 2

Complete the example as you watch the video in the Online Lesson.

A) Determine the number of 5-digit PINs you can create using the numbers zero through nine.

_____ _____ _____ _____ _____

digit 1 digit 2 digit 3 digit 4 digit 5

B) Determine the number of 5-digit PINs you can create using the numbers zero through nine with no repeating digits.

_____ _____ _____ _____ _____

digit 1 digit 2 digit 3 digit 4 digit 5

Example 3

Complete the example as you watch the video in the Online Lesson.

At the quick checkout lane, Marion removed the eight different items from the basket one at a time and scanned them before paying. How many arrangements are there when scanning the items?

☑ Checkpoint: The Fundamental Counting Principle

How many ways can you arrange *The Limit Does Not Exist*, *The Integral of Intrigue*, and *The Derivative of Deception* on a bookshelf?

To continue, return to the Online Lesson.

Permutations and Combinations

Fill in the notes as you watch the video in the Online Lesson.

- Counting problems that are more complex can be categorized into _____ and _____.
- The formulas use the variables n and r.
  - n = _____
  - r = _____
- A permutation is an ordered arrangement in which no element is used more than once, and _____.
- The formula for permutation is: $nPr(n, r) = \frac{n!}{(n-r)!}$
- A combination is an arrangement in which no element is used more than once, and _____.
- The formula for combination is: $nCr(n, r) = \frac{n!}{r!(n-r)!}$

Example 4

Complete the example as you watch the video in the Online Lesson.

A teacher has a class of 24 students and plans to divide them into groups of 4. One option is to assign each member of a group a specific role, and the other is to have them work in committees. How many different groups can be formed, with and without specific roles?

Groups with specific roles	**Committees**
$nPr(n, r) = \frac{n!}{(n-r)!}$	$nCr(n, r) = \frac{n!}{r!(n-r)!}$
When students are assigned specific roles, there are _____ possible ways to arrange the 24 students into groups of 4.	There are _____ different groups of 4 when 24 students are arranged into committees.

Example 5

Complete the example as you watch the video in the Online Lesson.

An investment group was picking stocks from the Fortune 100 list. They plan to have 25 different stocks in their portfolio. How many different options are possible?

Example 6

Complete the example as you watch the video in the Online Lesson.

Javier is creating a playlist of his top ten songs. Determine the number of ways Javier's songs can be arranged.

Checkpoint: Permutations and Combinations

Roma Pizza's most popular pizza has three toppings of your choice. Possible topping choices include green peppers, extra cheese, pepperoni, olives, sausage, onions, anchovies, and pineapple. How many pizzas can be created if any three toppings create a mouthwatering pizza?

To continue, return to the Online Lesson.

Practice 1

Complete problems on a separate sheet of paper.

Evaluate. Do not use a calculator.

1) $4!$

2) $nPr(4, 4)$

3) $nCr(8, 6)$

4) $nCr(4, 4)$

5) $6!$

6) $nPr(6, 2)$

Evaluate with a calculator.

7) $7!$

8) $12!$

9) In the 1800s, maritime vessels could display flags to send messages to other ships at sea. A ship would usually have 40 flags and might display five at a time. How many messages could a ship display?

10) How many unique 4-letter codes can you create using the 26 letters of the English alphabet?

11) A jury of 12 people will be selected from a group of 50 for a trial. Each member of the jury has the same roles and responsibilities. How many combinations of juries can be selected?

12) Gerry is making granola for breakfast. She has two types of oats, five types of dried fruit, and three types of seeds. How many different granola recipes can she create?

13) Something About Salad offers thirteen ingredients to customize a salad. Customers can select five ingredients to create their salads. How many different salads can be created?

14) The finals of the 100-meter dash consist of eight runners. Gold, silver, and bronze medals are awarded for first, second, and third place. How many ways can the runners place?

Given the scenario, explain why it represents a permutation or combination.

15) Playing notes of a song on an instrument

16) Selecting coffee beans for your own signature blend

To continue, return to the Online Lesson.

Mastery Check

Show What You Know

Chef Julianne is opening a small restaurant and wants to serve a three-course meal that includes this selection: five hors d'oeuvres, six entrees, and two desserts. Each dinner guest will be able to choose one option for each course.

A) How many different dinner arrangements can be served?

B) The staff reports to Chef Julianne that dinner guests want 3 more dessert options and a soup or salad with dinner. How many more options would dinner guests have?

C) Eight staff members are available to do the restaurant's prep work before opening, but Chef Julianne determines that only six people are needed. If each person is assigned a specific job when they arrive, how many possible ways can Chef Julianne assign her staff?

Say What You Know

In your own words, talk about what you have learned using the objectives for this lesson and your work on this page.

To continue, return to the Online Lesson.

Practice 2

Complete problems on a separate sheet of paper.

Evaluate. Show your work. Do not use a calculator.

1) $5!$

2) $nPr(5, 3)$

3) $nCr(20, 18)$

4) $3!$

5) $nPr(7, 2)$

6) $nCr(10, 8)$

Evaluate with a calculator.

7) Determine how many ways you can arrange the letters MATH. None of the arrangements needs to form a new word.

8) The Billie Book Shoppe has a "Buy 3, Get 1 Free" deal from a selection of 20. How many different combinations of four books can you choose?

9) The Department of Motor Vehicles requires license plates on all vehicles. How many unique license plates can be made if a three-letter, three-number combination is used?

10) A team of scientists programmed a boogie robot. The dancing robot knows 17 dance moves. To submit the robot for a competition, a routine with six dance moves must be prepared. How many ways can the boogie robot (get down) be programmed?

11) Pettigrew's Petals keeps fifteen different types of flowers in stock for their arrangements. If they guarantee every arrangement will have eight types of flowers, how many flower arrangements can be created?

12) There are 12 players on a basketball team, but only 5 will be on the court at the same time, each in a unique position. How many ways can the coach set the lineup for the specific positions?

13) The tenth-grade student council at Stars Hollow High has ten members. They need a committee of four students to plan the upcoming Spring Fling dance. How many different committees are possible?

14) Luke's Diner offers 5 types of bread, 7 types of meat, 4 cheeses, and 8 toppings. How many different sandwiches can you create if you choose one type of bread, one meat, one cheese, and two toppings?

Given the scenario, explain why it represents a permutation or combination.

15) Picking a color palette to decorate a room

16) Creating a physical therapy plan after surgery with numbered steps

To continue, return to the Online Lesson.

Targeted Review

Complete items on a separate sheet of paper.

1) Name the type of data distribution (left-/right-skewed, symmetric) when the mean is 6.78, the median is 5.5, and the mode is 1.

2) The area for a standard normal distribution is 0.59 with a mean of 87 and a standard deviation of 6.2.

For problems 3–4:

A t-shirt distributor is trying to figure out which college's merchandise is most popular with its customers. The distributor divided stores into regions. Then, stores in three regions were selected to record the college logo on every t-shirt sold for one week.

3) Name the population and the sample.

4) Name the type of sampling method used.

5) Determine the roots, their multiplicities, and the degree.
$h(x) = x(x-3)^2(x+1)(x-5)$

For problems 6–8, use the functions: $f(x) = -\frac{1}{2}(x-6)+10, \quad g(x) = x^2 - 9$

6) $f(x) + g(x)$

7) $(f+g)(6)$

8) $(f \circ g)(x)$

Multiple Choice

____ 9) A research team is studying the effectiveness of a new teaching method in elementary schools. They identify all elementary schools in a large school district, randomly select 8 schools (clusters). Then they implement the new teaching method and collect data from all students in those selected schools.

A) simple random **B)** systematic

C) stratified **D)** cluster

____ 10) Solve: $-3\log_2(3x+1) = -12$

A) $\frac{7}{3}$ **B)** $\frac{17}{3}$

C) 5 **D)** no solution

Multiple Choice

_____ **11)** Select the expression equivalent to: $\log\left(\frac{ab}{h^c}\right)$

A) $\log a + \log b - c\log h$

B) $\log a + \log b - h\log c$

C) $\log(a+b) - c\log h$

D) $b\log a - c\log h$

12) Select all measures of center.

- ☐ mean
- ☐ median
- ☐ mode
- ☐ range

Problem	1	2	3	4	5	6	7	8	9	10	11	12
Origin	L45	L46	L47	L48	L34	L31	L31	L32	L48	L40	L42	L45

L = Lesson in this level, A1 = Algebra 1: Principles of Secondary Mathematics

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