Lesson 49
Error and Confidence

NAME:

Start by navigating to the Online Lesson for instructions.

Objectives

- Infer a population mean from confidence intervals.
- Calculate the margin of error of a sample.
- Evaluate reports.

Why?

When analyzing samples collected from a population, error is expected because a sample will not exactly represent every member of a population. To be confident in the sample's findings, the maximum error of the estimate is used to calculate the confidence interval. The interval gives the expected range where the population parameter will fall.

Warm Up

In the context of probabilities and statistics, define the vocabulary in your own words.

1) population

2) sample

3) parameter

To continue, return to the Online Lesson.

Explore

Error and Confidence

Fill in the notes as you watch the video in the Online Lesson.

- In inferential statistics, a sample is used because _____

  _____ (a census) is impractical.

- However, using a sample results in a _____ (or uncertainty) in the estimates.
- The variability between a random sample and a population produces a _____ _____.
- A key principle in statistics is that _____ _____, because a larger sample more accurately represents the population.

Maximum Error of the Estimate (Margin of Error)

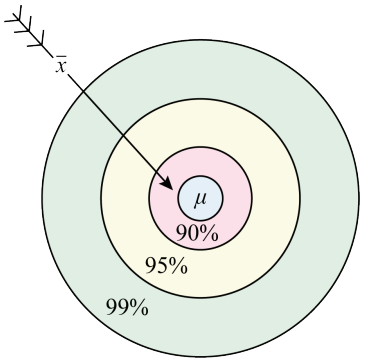
- The margin of error (or maximum error of the estimate) is the estimated difference between a _____ (sample mean or sample proportion) and the _____.
- A point estimate is a specific numerical value estimate of a _____ _____.
- The formula for the maximum error of the estimate, E, is: $E = z \cdot \frac{s}{\sqrt{n}}$
  - z: _____
  - s: _____
  - n: _____
- The maximum error of the estimate (or margin of error) is a single value that is used alongside a sample statistic to _____.
- The confidence in the reliability of the sample to estimate the population parameter can be expressed in two ways:
  - _____
  - _____

Confidence Level

- The confidence level is the probability that the estimate _____ _____.
- Confidence levels are written as percentages:

Confidence Level	z-score
	1.645
	1.960
	2.576

- As the confidence level _____, the precision of estimating the true population mean _____.



Confidence Interval

- A confidence interval, CI, is the _____ that represent the _____.
- It accounts for sampling error, because a CI is written as a _____ instead of a single point estimate.
- Therefore, population mean, μ, _____ from the confidence interval.
- It is calculated with a sample mean, $\bar{x}$, and the _____ _____.

- Once the maximum error of the estimate is calculated, the confidence interval is written as a _____: $\bar{x} - E \le \mu \le \bar{x} + E$ or $\bar{x} \pm E$
  - $\bar{x}$: _____
  - μ: _____
  - E: _____
- Remember, the _____ the interval, the more confident you can be that the population mean will be found within it.

Example 1

Complete the example as you watch the video in the Online Lesson.

In 2023, a random sample of 60 retail employees in a large city found the sample mean wage was \$17.25 per hour with a standard deviation of \$3.50 per hour. Calculate the confidence intervals (CI) with 90% and 99% certainty for the population mean. Explain.

$s = 3.50, \quad n = 60$

90% CI	**99% CI**
$z = 1.645$	$z = 2.576$
$E = 0.75$	$E = 1.16$

90% CI

$$\bar{x} - E \le \mu \le \bar{x} + E$$
$$17.25 - 0.75 \le \mu \le 17.25 + 0.75$$
$$\$16.50 \le \mu \le \$18.00$$

You can be _____ that the hourly wage of retail employees in the city will be _____ _____.

You can be _____ that the hourly wage of retail employees in the city will be _____ _____.

Example 2

Complete the example as you watch the video in the Online Lesson.

At the Zander Zoo, a random survey of 350 visitors found that 83.7% are satisfied with the maps for navigating the zoo. Find the interval that most likely contains the population parameter for visitors dissatisfied with the zoo's maps for a margin of error of $\pm 5.34\%$.

It is likely that _____ of the zoo's visitors are dissatisfied with the maps.

Example 3

Complete the example as you watch the video in the Online Lesson.

A national movie theater chain wants to estimate the average number of ounces of popcorn purchased per movie ticket sale. From years of data, they know the population standard deviation for popcorn purchases per ticket sale is 4.5 ounces. The chain wants to be 95% confident in their estimate and decides to take a random sample of 100 movie ticket sales.

The theater chain can be _____ confident that their estimate will be _____ away from the population mean.

Example 4

Complete the example as you watch the video in the Online Lesson.

A Bright Idea manufactures light bulbs and needs to estimate the average lifespan of a new type of bulb. They know from pilot studies that the standard deviation (s) of a light bulb's lifespan is 120 hours. They want to be 99% confident that their estimate is within 25 hours of the true average lifespan. What is the minimum sample size that can be tested to reach a 99% confidence level for A Bright Idea?

A Bright Idea needs to test a minimum

of _____

to be _____

about the lifespan of their light bulbs.

When looking for the minimum sample, *round up* to the nearest whole number.

Checkpoint: Error and Confidence

A Bright Idea decides to test a sample of 153 light bulbs at a 90% confidence level. The standard deviation remains 120 hours. What is the confidence interval for a sample mean of $7{,}250$ hours when the confidence level is changed? (Round to the nearest hour.)

To continue, return to the Online Lesson.

Practice 1

Complete problems on a separate sheet of paper.

Use the following scenario for problems 1–4.

The Department of Transportation is studying the average commute time for residents in a large metropolitan area. They take a random sample of 250 commuters. Based on prior census data, they assume the population standard deviation of commute times is 6.2 minutes.

1) Calculate the maximum error of the estimate if they aim for a 90% confidence level in their study. Explain.

2) Calculate the maximum error of the estimate if the random sample increased to 500 commuters. Explain.

3) Why is the error reduced when the sample increases?

4) If the sample mean for 500 commuters is 38 minutes, determine the confidence interval (CI).

Use the following scenario for problems 5–7.

Lakeforest School District wants to estimate the mean reading test score for all third-grade students. It takes a random sample of 64 students. Previous data suggests the standard deviation for the test is 15 points. Lakeforest is using a 99% confidence level to ensure a high degree of certainty for its public report.

5) Calculate the maximum error of the estimate.

6) Why would the school use a 99% confidence level instead of a 90% confidence level?

7) Lakeforest School District surveyed 136 third-grade parents about the reading program their children use at school. Approximately 11% of parents were dissatisfied with the program. Using a $\pm 8.4\%$ margin of error, find the interval that most likely contains the population parameter for satisfied parents.

Use the following scenario for problems 8–9.

An automobile parts supplier needs to estimate the average diameter of a new piston part. They need to be 95% confident that their estimate is within 0.01 millimeters (mm) of the true mean diameter. Historical data suggests the population standard deviation, σ, is 0.05 mm.

8) What is the minimum number of parts to be tested to match the confidence level?

9) Determine the population mean from the random sample of pistons when the sample mean is 95.5 mm.

Use the following scenario for problems 10–14.

A hospital administrator wants to estimate the average length of stay (in days) for general surgery patients. They sampled 120 patient records. The population standard deviation, σ, for general surgery stays is historically 2.4 days. They chose a 90% confidence level for their audit.

10) Determine the maximum error of the estimate in days and hours.

11) If the average stay of a randomly selected group of patients is 4.5 days, what is the population mean?

12) The hospital administrator decided to adjust the confidence level. Calculate the maximum error of the estimate for a 95% confidence level.

13) If hospital employees are scheduled in 8-hour shifts, explain which confidence level should be used.

14) The same sample of patients was also given a discharge survey asking them to rate their hospital stay as positive, neutral, or negative. Of the 120 patients surveyed, 17% rated their stay as neutral. Find the interval when the margin of error is $\pm 9.13\%$.

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Mastery Check

Show What You Know

Danell's Delivery Service claims its average delivery time is 3.0 days. A competitor challenges this claim and states, "It is simply not possible in this region."

The Consumer Advocacy Group (CAG) opted to conduct an investigation using a random sample of 200 customer deliveries. For the standard deviation, CAG referenced national delivery service data, which indicated a value of 1.5 days.

A) What is the maximum error of the estimate for a 90% confidence level?

B) CAG found that the sample mean delivery time is 3.25 days. Construct the confidence interval (CI) for 90% confidence level.

C) Based on the CI, is Danell's Delivery Service claim of 3.0 days plausible? Explain what this means for CAG.

D) Danell's Delivery Service noted that, if a 99% confidence level had been used, 3.0 days would have been within the CI. CAG said 90% is more helpful for customers. Explain why this was CAG's response.

E) Rather than using average delivery time, Danell's Delivery Service decided to switch to customer satisfaction ratings. In a random survey of 750 customers, 87% reported being very satisfied. Determine the interval when the margin of error is $\pm 3.65\%$.

Say What You Know

In your own words, talk about what you have learned using the objectives for this lesson and your work on this page.

To continue, return to the Online Lesson.

Practice 2

Complete problems on a separate sheet of paper.

Use the following scenario for problems 1–3.

A farming co-op is testing a new fertilizer to estimate the average yield per acre of corn. They randomly select and measure the yield from 81 acres. Based on decades of records for this region, the population standard deviation for corn yield is known to be 8.5 bushels/acre.

1) Estimate the co-op's yield for a 95% confidence level.

2) Determine the confidence interval when the sample mean yield is 150 bushels/acre.

3) The co-op surveyed 162 community members to determine if the farming co-op had a positive, neutral, or negative perception. The results showed that 84% of the community had a positive perception of the farming co-op. Using a margin of error of $\pm 7.86\%$, find the interval.

Use the following scenario for problems 4–7.

An environmental group is studying the average pH level of a large lake. Using historical data, they know the standard deviation is a pH of 0.5 parts per unit and are using a 95% confidence level.

4) If the group takes an initial random sample of 50 water measurements, calculate the maximum error of the estimate.

5) A new member of the environmental group suggested taking 200 samples from the lake. Without making calculations, why would a new member make this suggestion?

6) How many times larger is the new sample size?

7) The value of E is 0.0692 for 200 lake samples. How many times smaller is the new value of E?

Use the following scenario for problems 8–10.

A+ Service Company decides to estimate the average hold time for customers calling their support line. They want to be 95% confident that their estimate is within 0.5 minutes (30 seconds) of the true average. From prior data, the population standard deviation for hold time is 1.8 minutes.

8) What is the minimum sample size of customer calls A+ Service Company needs to measure to meet this requirement?

9) After the customers were surveyed, the sample mean for hold time was calculated to be 2.3 minutes. Determine the confidence interval.

10) At the end of every customer interaction, clients are asked to rate their experience with A+ Service Company on a scale of 1 (terrible) to 5 (wonderful). A random sample of 340 responses was collected, revealing that 76.5% of customers rated their experience as 4 or higher. Using a margin of error of $\pm 5.42\%$, calculate the interval.

Use the following scenario for problems 11–12.

Civil engineers are measuring the length of a bridge span and must have a high degree of certainty in their measurements. They take 196 independent measurements using instruments with a known population standard deviation of 0.8 inches. They must be 99% confident in their estimation of the true length.

11) Determine the maximum error of the estimate if the engineers must be 99% confident in their estimate of the true length.

12) The engineers found the sample mean bridge length to be 89.9 feet ($1{,}078.8$ inches) from the sample measurements. Determine the population mean with 99% confidence.

Use the following scenario for problems 13–14.

A food safety inspector is estimating the average sodium content (in mg) of a new frozen meal. They need to be very precise, requiring an estimate that is within 5 mg of the true mean. The known population standard deviation for sodium content in this type of meal is 18 mg.

13) What is the minimum sample size of meals they need to test to guarantee a 99% confidence level?

14) What should the food inspector do if the value of E needs to be more precise (smaller)?

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◎ Targeted Review

Complete items on a separate sheet of paper.

Use the following scenario for problems 1–2.

A consumer advocacy group investigated whether cereal box color (red, blue, or yellow) affects calories per serving. They sampled 200 US cereal boxes, noting color and calorie count from labels.

1) Name the type of study and the population.

2) Identify the variables and classify them as quantitative or categorical.

Use the following scenario for problems 3–4.

The fuel efficiency (in miles per gallon, mpg) of a specific make and model of car is collected from all owners who purchased the car in the first six months of sales. The average fuel efficiency is 28 mpg with a standard deviation of 3 mpg.

3) Sketch a normal curve showing the Empirical rule.

4) Determine the fuel efficiency for cars in the 97.5th percentile.

5) Determine the value of x if $\log_4 x = -3$.

6) Solve: $\left(\frac{5}{3}\right)^x = \left(\frac{27}{125}\right)^{3x-2}$

7) Sketch a graph with technology. Label the intercept(s) and turning points.
$f(x) = x^4 + 3x^3 + x^2 - 3x - 2$

8) Sketch a graph with technology. Label the intercept(s).
$y = \frac{1}{2}(3)^x - 1$

Multiple Choice

_____ **9)** Describe the transformation from $f(x)$ to $g(x)$.
$f(x) = (2)^x$, $g(x) = \frac{1}{4}(2)^{x+4}$

A) $g(x)$ is vertically stretched and translated 4 units up

B) $g(x)$ is vertically stretched and translated 4 units left

C) $g(x)$ is vertically compressed and translated 4 units up

D) $g(x)$ is vertically compressed and translated 4 units left

Multiple Choice

_____ **10)** Identify the sampling method.

A teacher wants to select a student at random to present their project. They write each student's name on a separate slip of paper, put all the slips into a jar, and draw one name without looking.

A) Simple random sample

B) Systematic

C) Stratified

D) Cluster

_____ **11)** Identify the type of study conducted.

A beverage company wants to test whether its new energy drink improves mental performance. They recruit 100 college students and randomly divide them into two groups. Group A receives the new drink, and Group B is given a placebo.

A) Observation

B) Experiment

C) Survey

D) Cannot be determined

_____ **12)** Boyle's Law states that pressure, p, of a compressed gas is inversely proportional to the volume, v, or $p_1 v_1 = p_2 v_2$. If the volume of gas is 200 cubic inches when a pressure of 25 pounds per square inch exists, what is the pressure for a volume of 125 cubic inches?

A) $25 \text{ lb}/\text{in}^2$

B) $40 \text{ lb}/\text{in}^2$

C) $125 \text{ lb}/\text{in}^2$

D) $1000 \text{ lb}/\text{in}^2$

Problem	1	2	3	4	5	6	7	8	9	10	11	12
Origin	L47	L47	L45	L46	L39	L38	L34, L35	L37	L37	L48	L47	L36

L = Lesson in this level, A1 = Algebra 1: Principles of Secondary Mathematics, FD = Foundational Knowledge

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