Lesson 46

NAME:

Standard Normal Distributions

Start by navigating to the Online Lesson for instructions.

Objectives

- Calculate z-scores.
- Estimate area with a z-table.
- Estimate population percentages.
- Evaluate reports based on data.

Why?

When developing new products, for example, an experimental drug to help with the symptoms of dementia, many different clinical studies are performed. While each study is performed differently, the scientists hope that the results all point to a similar conclusion: the effectiveness of the drug.

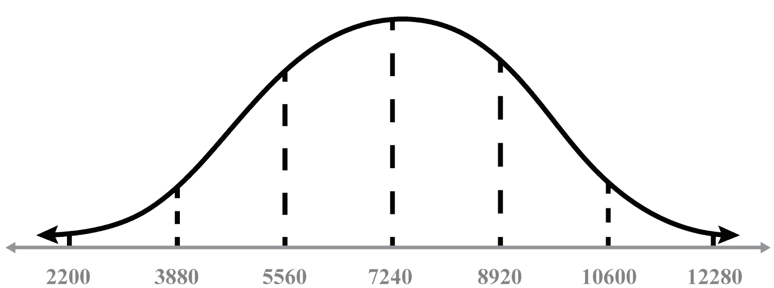
To compare the results from varied studies, z-scores are used. Understanding the standard normal distribution allows you to estimate the likelihood (probability) of any number occurring within a data set. By standardizing data with z-scores, you can use one universal tool (the z-table) to compare completely different data sets and make powerful predictions about populations.

Essentially, you CAN compare apples to oranges.

Warm Up

For problems 1–2, use the graph below.

Daily steps



1) Name the mean and standard deviation.

2) Name the ranges in standard deviation from the Empirical Rule.

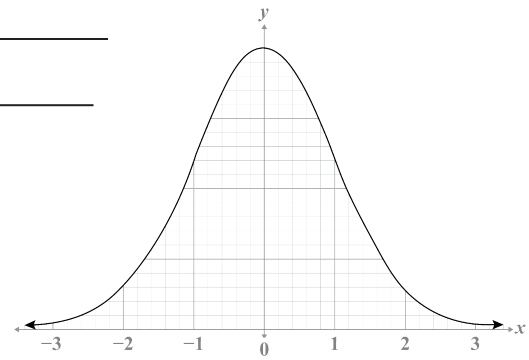
To continue, return to the Online Lesson.

Explore

z-Scores

Fill in the notes as you watch the video in the Online Lesson.

- The standard normal distribution is always ________________ on the x-axis with intervals that ________________________ __________ on each side of zero.



- Therefore:
  - _____________
  - _____________

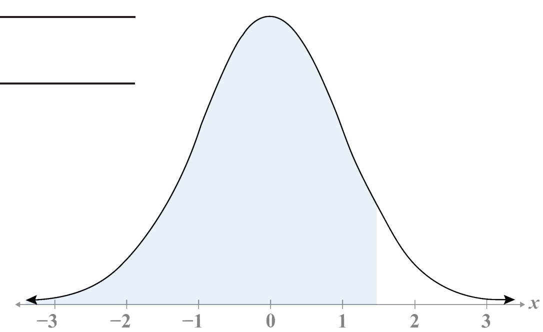
- This type of distribution is a _______________________ with an area _________________ ___________________________, or in terms of a proportion, _________.

- Because the mean is zero, ____ (and 50% is above the mean).

- Any normally distributed data set can be standardized using the ______________________ ____________________: $z = \dfrac{X - \mu}{\sigma}$

- A z-score is a measure of how many standard deviations, σ, ___________________________ ____________________________, μ.

- When a data set is standardized, it can be _________________ ____________________________________ using z-scores.



- Once the z-score is calculated, a standard normal table, or ________________, is used as a reference to find where the value ______________________________.
- The z-table determines the __________________ __________________ of a specific z-score.
- The area can be written as a ____________________ ____________________ from the z-table.
- The __________________ and __________________ correspond with the z-score.

z	.00	.01	.02	.03
0.0	.0000	.0040	.0080	.0120
0.1	.0398	.0832	.0478	.0517
0.2	.0793	.0832	.0871	.0910
0.3	.1179	.1217	.1255	.1293
0.4	.1554	.1591	.1628	.1664
0.5	.1915	.1950	.1985	.2019
0.1	.2257	.2291	.2324	.2357
0.2	.2580	.2611	.2642	.2673
0.3	.2881	.2910	.2939	.2967
0.4	.3159	.3186	.3212	.3238
1.0	.3412	.3438	.3461	.3485
1.1	.3643	.3665	.3686	8708
1.2	.3849	.3869	.3888	.3907
1.3	.4032	.4049	.4066	.4082
1.4	.4192	.4207	.4222	.4236

- When analyzing z-values using the z-table:
  - Ask yourself: What proportion of the data __________________?
  - Write answers using the math shorthand: __________________.

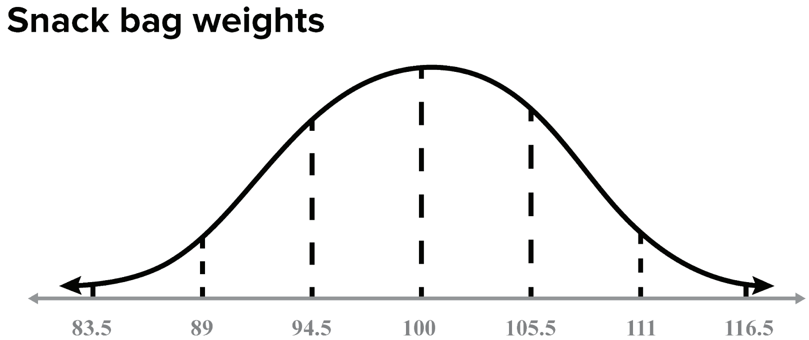
You will need the z-table for this lesson.

The z-table is located in the Statistics and Probability Formula Sheet.

Example 1

Complete the example as you watch the video in the Online Lesson.

Standardize the normal distribution.



$$z = \frac{X - \mu}{\sigma}$$

$$z = \frac{100 - 100}{5.5} = 0 \qquad z = \frac{83.5 - 100}{5.5} = -3$$

Example 2

Complete the example as you watch the video in the Online Lesson.

A high school track coach calculated the post-season mile times to be $\mu = 6.417$, $\sigma = 0.333$. What proportion of runners on the track team ran a mile in less than 6:30 minutes?

z	.00	.01	.02	.03	.04	.05
0.0	.5000	.5040	.5080	.5120	.5160	.5199
0.1	.5398	.5438	.5478	.5517	.5557	.5596
0.2	.5793	.5832	.5871	.5910	.5948	.5987
0.3	.6179	.6217	.6255	.6293	.6331	.6368

Example 3

Complete the example as you watch the video in the Online Lesson.

The area for a standard normal distribution is 0.0495 with a mean of 7.5 and a standard deviation of 1.5. Determine the raw data value. Explain.

Checkpoint: z-Scores

The technology supervisor determined that $\mu = 11$, $\sigma = 2$, for the number of emails received each day by teachers at Fair View High School.

A) What proportion of teachers received fewer than eight emails in one day?

B) How many emails will a teacher receive in the 85th percentile?

To continue, return to the Online Lesson.

Application of z-Scores

Fill in the notes as you watch the video in the Online Lesson.

- The z-table provides the area under the curve ______________________________ (to the left of) the z-score.
- The total area under a standard normal curve is ______________.
- A negative z-score represents ______________________________, and a positive z-score represents ______________________________.
- If two z-scores are given and the area between those points needs to be determined, ______________________________.

Example 4

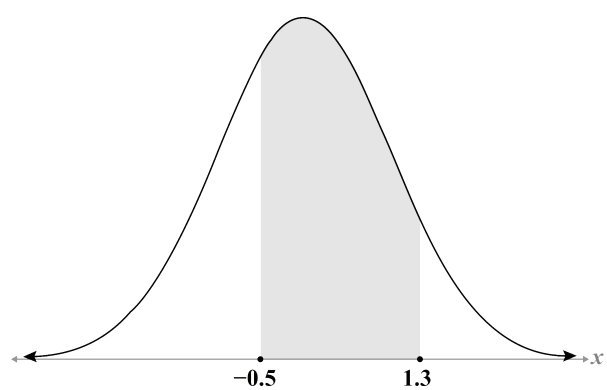
Complete the example as you watch the video in the Online Lesson.

Determine the area of the shaded and unshaded regions. Then find the area above $z = 1.3$.

Plan

Find areas on the z-table
Calculate the shaded region
Calculate the unshaded region

Shaded region: $P(-0.5 < z < 1.3)$



Example 5

Complete the example as you watch the video in the Online Lesson.

The heights of college basketball players were measured. The average height was 195 cm, with a standard deviation of 6.5 cm. If sixteen players were randomly selected from all the college players, does the data appear to be normally distributed? Explain.

Plan

List the normal distribution to $\pm 2\sigma$
Tally the data points $< \pm 2\sigma$
Explain

Player Height (cm)			
198	194	199	205
193	193	205	200
192	199	192	192
197	183	184	191

Basketball Player Height Distribution					
z	–2	–1	0	1	2
Area $< z$	0.02	0.16	0.5	0.84	0.98
Predicted: $16z$	0.32	2.26	8	13.44	15.68
$X < \mu \pm 2\sigma$					
Actual Count					

Example 6

Complete the example as you watch the video in the Online Lesson.

According to the United States Mint, the average diameter of a nickel is 21.21 millimeters. The likelihood of a nickel having a diameter less than 21.288 mm is 94.7%.

A) Determine the standard deviation for the diameter of a nickel.

B) Determine $P(21.21 < X < 21.288)$.

Plan

Write a proportionality statement
Determine the z-score
Solve

Checkpoint: Application of z-Scores

In the 20th century, the United States Mint began manufacturing pennies with an average diameter of 19.05 millimeters. To ensure consistent dimensions for all pennies, the standard deviation was set to 0.015 millimeters. What is the likelihood that a randomly selected penny has a diameter between 19.01 and 19.044 mm?

There is a ______________ chance that a randomly selected penny will be __________________

______________________________.

To continue, return to the Online Lesson.

Practice 1

Complete problems on a separate sheet of paper.

Name the area for the standard normal curve as a proportion and a percent.

1) $z = -2.36$

2) $z = 0.48$

Determine the z-score given the proportion.

3) 0.9642

4) 0.1207

For problems 5–6, determine the missing value in the formula $z = \dfrac{X - \mu}{\sigma}$. Round to the hundredth.

5) $z = -1.35,\ \mu = 13.45,\ X = 5.1$

6) $\text{P} = 0.624,\ \mu = 24.74,\ \sigma = 4.32$

For problems 7–10, calculate the z-score. Then determine the proportion or raw data score.

After measuring the length of the fish population to stock a lake, a fisheries biologist calculated the average length to be 10.12 centimeters and the standard deviation to be 1.57 centimeters.

7) What proportion of fish have a length smaller than 10 centimeters?

8) What proportion of fish were between 7.5 centimeters and 11 centimeters in length?

9) What proportion of fish were longer than 11.5 centimeters?

10) What length would represent 93% of the fish in the lake?

For problems 11–14, use the scenario.

The media corporation, Time Inc., collected data on employee travel time from home to work in minutes. The average commute time was 20 minutes, and the standard deviation was 5 minutes.

11) How long is the commute for 90% of Time Inc. employees?

12) What proportion of employees have a commute of 10 minutes or less?

13) What is the maximum expected commute time for 6% of employees?

14) Determine the proportion of employees who have a commute between 15 and 20 minutes at Time Inc.

For problems 15–16, use the scenario.

The average typing speed for fifteen-year-olds at Barnesville High is 36 words per minute (wpm), and a standard deviation of three words per minute.

Typing Speed (wpm)			
41	30	35	38
33	39	37	34
38	36	32	35

15) Complete the table to determine if the sample group of students represents a normal data set.

z	-2	-1	0	1	2
Area $< z$	0.02	0.16	0.5	0.84	0.98
Predicted: $12z$	0.24	1.92	6	10.08	11.76
$X < \mu \pm 2\sigma$					
Actual Count					

16) Is the sample normally distributed? Explain.

To continue, return to the Online Lesson.

Mastery Check

Show What You Know

Guthrie Girls Autobody tracked data over a few years and found that, on average, customers brought in their Model G car for an oil change every 8,255 miles.

A) If 70.2% of customers bring in their Model G every 8,435 miles, what is the standard deviation for oil changes to the nearest whole number?

B) What is the likelihood that a car is serviced at the manufacturer's recommendation of every 7,500 miles?

C) The same Model G cars need an alignment between 6,000 and 12,000 miles. On average, a customer has their car aligned every 10,500 miles with a standard deviation of 1,982 miles. What percentage of cars are in the given range?

Say What You Know

In your own words, talk about what you have learned using the objectives for this lesson and your work on this page.

To continue, return to the Online Lesson.

Practice 2

Complete problems on a separate sheet of paper.

Name the area for the standard normal curve as a proportion and a percent.

1) $z = 3.02$

2) $z = -2.34$

Determine the z-score given the proportion.

3) 0.3970

4) 0.3781

For problems 5–8, calculate the z-score. Then determine the percentage or raw data score.

Mr. Bartz's class recorded their nightly reading times in minutes over a two-week period. He calculated the class average to be 22 minutes with a standard deviation of 3 minutes.

5) What percentage of the class read less than 15 minutes per night?

6) What percentage of Mr. Bartz's students read more than 25 minutes per night?

7) What percentage of students read between 17 and 20 minutes per night?

8) How long, to the nearest minute, did 66% of Mr. Bartz's class read?

For problems 9–12, use the scenario.

Ms. Troglin's botany class studied the growth of Flower B plants in centimeters (cm) over thirty days. On day zero, students planted seeds. Then on day thirty, students measured the height of each Flower B plant. Using their measurements, Ms. Troglin's class calculated that the average height of the plants was 26.4 cm with a 1.2 cm standard deviation.

9) What height does Ms. Troglin's class expect 40% of all Flower B plants to be?

10) Determine the height of the top 5.05% of the plants.

11) What percentage of plants were less than 23 centimeters?

12) What percentage of plants were between 25 and 27 centimeters?

For problems 13–14, determine the missing value in the formula $z = \frac{X - \mu}{\sigma}$.

13) $\mathrm{P} = 0.1093,\ X = 13.2,\ \sigma = 7.51$

14) $z = 0.72,\ \mu = 18,\ \sigma = 4.35$

For problems 15–16, use the scenario.

A fisheries biologist calculated the average length of a lake fish population to be 10.12 centimeters and the standard deviation to be 1.57 centimeters. Then, one year later, a sample of the population was measured.

Fish Length (cm)		
8.0	8.4	9.5
9.4	7.3	7.4
6.2	8.5	
10.1	10.0	

15) Complete the table to determine if the sample group of fish represents a normal data set.

z	**–2**	**–1**	**0**	**1**	**2**
Area $< z$	0.02	0.16	0.5	0.84	0.98
Predicted: 10z	0.2	1.6	6	8.4	9.8
$X < \mu \pm 2\sigma$					
Actual Count					

16) Is the sample normally distributed? Explain.

To continue, return to the Online Lesson.