# Lesson 44

NAME:

# Applications of Exponents and Logarithms

Start by navigating to the Online Lesson for instructions.

## Objectives

- Apply the properties of exponents and logarithms.
- Apply exponential and logarithmic formulas to real-life scenarios.

## Why?

Exponential and logarithmic problems are part of everyday life, including compound interest rates, weather reporting, and decibel levels. Applying the properties of exponents and logs to real life will help you strengthen your knowledge of this unit.

## Warm Up

**1)** Solve. Round to the nearest thousandth.

$$12^{2x-3} = 15$$

**2)** Determine if $f(x)$ and $g(x)$ are inverses.

$$f(x) = \left(\frac{1}{2}\right)^{x+5}$$

$$g(x) = -\log_2 x - 5$$

**To continue, return to the Online Lesson.**

# Explore

## Applications of Exponents and Logs

*Fill in the notes as you watch the video in the Online Lesson.*

- Each application of exponents and logarithms has its own ____________________ and ____________________.
- To start, remember to identify the ________________________________________ based on the formula provided.
- After solving, be sure to ____________________ your answer.
- ______________________ notation is often used to write the final answer because exponents and logarithms increase or decrease rapidly.

**Half-Life Application**

- Half-life is the time it takes for a substance to ______________________________ of its original value, which can be true of radioactive materials, bacteria, medicine, etc.
- The general formula for a half-life is: $A(t) = A_0\left(\frac{1}{2}\right)^{\frac{t}{h}}$
    - $A_0$: _______________ quantity
    - $t$: ______________, in years
    - $h$: ___________________
    - $A(t)$: ______________ remaining at time $t$

**Continuous Compounding Application**

- The formula for continuous compounding events is: $y = Pe^{rt}$

________________________ → $y$

$$y = Pe^{rt}$$

________________________ → $rt$

________________________ → $P$

- It can be used for ____________________________ when the rate is continuously compounding.

## Example 1

*Complete the example as you watch the video in the Online Lesson.*

A new type of battery has a half-life of $5$ years. To determine total life of the battery, lab technicians set up an experiment that starts with a fully charged battery $(100\%)$ and runs until the power remaining is $10\%$. How many years is the battery expected to last?

$$P(t) = P_0\left(\frac{1}{2}\right)^{\frac{t}{h}}$$

**Plan**

Define known variables
Substitute values into the formula
Take the log of both sides
Simplify logs
Solve

**Implement**

$P(t)$: power remaining at time $t$
$P_0$: initial power
$t$: time (in years)
$h$: half-life

**Example 2**

*Complete the example as you watch the video in the Online Lesson.*

A population ecologist is studying the introduction of an invasive species of bugs into an environment with no predators. The initial number of bugs was estimated to be 400. If the ecologist estimates a population of 50,000 after 46 weeks, what is the continuous growth rate? Round to the nearest tenth of a percent.

$y = Pe^{rt}$

**Example 3**

*Complete the example as you watch the video in the Online Lesson.*

The pH of a solution measures its acidity or alkalinity. The pH is defined using the equation $\text{pH} = -\log[\text{H}+]$ and is measured in moles per liter.

The pH level for a section of the Schuylkill River near the city of Philadelphia, PA is 6.5. Determine the hydrogen ion concentration of the river water in moles per liter.

### Checkpoint: Applications of Exponents and Logs

A biologist is studying a strain of bacteria that triples $(r = 3)$ every hour. If she starts with a culture containing $100$ bacteria, how long will it take for $24{,}000$ bacteria to be present? Round to the nearest quarter of an hour, $t$, using the formula: $Q(t) = Q_0(r)^t$

**To continue, return to the Online Lesson.**

## Modeling Applications

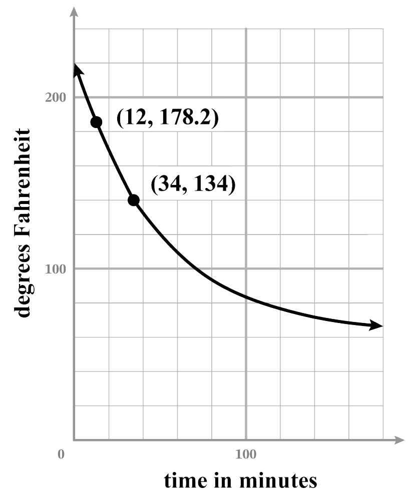
*Fill in the notes as you watch the video in the Online Lesson.*

- Mathematical ______________ of real-life scenarios can be created from tables, graphs, and/or equations.
- The models do not always match the scenario exactly, but they are close ___________________________.
- They can be used to analyze and predict using _______________________ and _______________________.

## Example 4

*Complete the example as you watch the video in the Online Lesson.*

After making a cup of tea at 8 a.m., Rheema decided to measure how quickly it was cooling. She noted two points on an exponential decay graph.




**A)** Write an equation to represent the graph of $y = ab^x + 55$.

**B)** At what approximate time will the temperature of the tea reach $66°\text{F}$?

**C)** Find the rate of change between the given points.

**A)** $(12, 178.2), (34, 134)$

$$y = ab^x + 55$$

$$134 = ab^{34} + 55$$

$$79 = ab^{34}$$

$$a = \frac{79}{b^{34}}$$

**B)**

**C)**

## Checkpoint: Modeling Applications

**Use Example 4.**

What was the temperature of the tea after one hour?

**To continue, return to the Online Lesson.**

## Practice 1

*Complete problems on a separate sheet of paper.*

**Use the information on decibel levels for problems 1–3.**

The loudness of a sound L is measured in decibels (dB).

The formula for the loudness of a sound is: $L = 10 \cdot \log\left(\frac{I}{I_0}\right)$

Where:

- I: The intensity of the sound
- $I_0$: The reference intensity of the softest sound a human can hear: $10^{-12}\ W/m^2$

**1)** A normal conversation has a loudness of about 60 dB. What is the intensity level?

**2)** A rock concert has a loudness of about 120 dB. How many times more intense is sound at a rock concert than in a normal conversation?

**3)** The sound of a vacuum cleaner has a registered intensity of $10^{-4.5}\ W/m^2$. What is the decibel level?

**Use the information on the continuous compounding formula for problems 4–5.**

$y = Pe^{rt}$

$P$: Initial amount
$r$: Rate
$t$: Time
$y$: Final amount

**4)** The intensity of a certain earthquake's seismic waves diminishes continuously at a rate of 9% ($r = -0.09$) per kilometer as they travel through the Earth's crust. If the initial intensity of the waves is measured at 7.5 on the Richter scale, at what distance will the earthquake wave intensity reach 4.0 on the Richter scale? Round to the nearest hundredth.

**5)** Kristin invested $6000 in a savings account with a 6% annual interest rate, compounded continuously. How much money will she have in the account after 10 years?

**Use the information on the compound interest formula for problems 6–7.**

$A = P\left(1 + \frac{r}{n}\right)^{nt}$

$P$: Initial amount
$r$: Annual interest rate
$n$: Compounding number
$t$: Time in years
$A$: Final amount with interest

**6)** Kristin invested $6000 in a savings account that offers a 6% annual interest rate, compounded annually. How much money will she have in the account after 10 years?

**7)** Natalee invests $5000 in a savings account with a 6.5% interest rate that is compounded twice a year. How many years will it take for her to have $15,000 in her account?

**Use the information on the half-life formula for problems 8–9.**

$$A(t) = A_0\left(\frac{1}{2}\right)^{\frac{t}{h}}$$

$A_0$: Initial quantity
$t$: Time in years
$h$: Half-life
$A(t)$: Quantity remaining at time $t$

**8)** A scientist discovers a new radioactive element, Element X. After observing a $100$ gram sample for $10$ days, they find that only $25$ grams remain. What is the half-life of Element X?

**9)** A biologist is studying a bacterial culture with a very short half-life of $30$ minutes. If they start with a sample of $1$ million bacteria, how many bacteria will be left after two hours?

**Use the information below for problems 10–12.**

A baker takes a fresh apple pie out of a $350°\text{F}$ oven and places it on a cooling rack in the kitchen, where the room temperature is a constant $72°\text{F}$. After $15$ minutes, the pie has cooled to $250°\text{F}$.

$$T(t) = T_a + \left(T_0 - T_a\right)e^{(-kt)}$$

$T(t)$: Temperature of the object at time $t$
$T_k$: Ambient temperature
$T_0$: Initial temperature of the object
$k$: Cooling constant

**10)** Determine the cooling constant, $k$. Round to the nearest hundredth.

**11)** If the baker took the pie out of the oven at $6{:}00$ p.m., when will it reach a temperature of $100°\text{F}$? Round to the nearest whole minute.

**12)** Sketch the graph. Include labels.

**To continue, return to the Online Lesson.**

# Mastery Check

## Show What You Know

Sara invests $1500 into a savings account earning 7% compound interest every quarter and a second account that compounds continuously. Determine how long it would take for each account to reach $3000.

$$A = P\left(1 + \frac{r}{n}\right)^{nt}$$

$P$: Initial amount
$r$: Annual interest rate
$n$: Compounding number
$t$: Time in years
$A$: Final amount with interest

$$y = Pe^{rt}$$

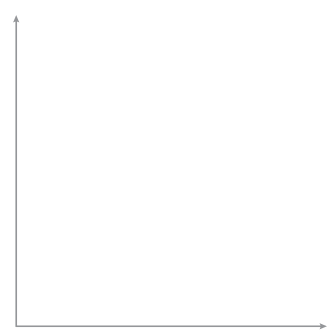
$P$: Initial amount
$r$: Rate
$t$: Time
$y$: Final amount

Determine the time it will take to earn $3000 for each scenario.

**A)** Quarterly

**B)** Continuously

**C)** Sketch a graph to approximate the savings account for continuous growth. Include labels on the graph.



## Say What You Know

In your own words, talk about what you have learned using the objectives for this lesson and your work on this page.

**To continue, return to the Online Lesson.**

# Practice 2

*Complete problems on a separate sheet of paper.*

**Use the information on earthquakes for problems 1–3.**

The Richter scale is used to measure the magnitude of an earthquake.

The magnitude of an earthquake is given by the formula: $\mathrm{M} = \log\left(\frac{I}{I_0}\right)$

- $I$ is the intensity of the earthquake.
- $I_0 = 1$ is the intensity of a standard earthquake.

**1)** In 1906, San Francisco was hit by a major earthquake with a magnitude of 7.8 on the Richter scale. What was the intensity of the earthquake?

**2)** In 1989, another earthquake hit San Francisco, this time with a magnitude of 6.9. How many times more intense was the 1906 earthquake than the 1989 earthquake?

**Use the information on the compound interest formula for problems 3–4.**

$$A = P\left(1 + \frac{r}{n}\right)^{nt}$$

$P$: Initial amount
$r$: Annual interest rate
$n$: Compounding number
$t$: Time in years
$A$: Final amount with interest

**3)** Edmond invested \$2500 in a compound interest account at his local bank. The bank offered an interest rate of 5% compounded monthly. When will Edmond's account reach \$7500? Round to the nearest whole year.

**4)** How much more would Edmond have in his account in the same amount of time if the bank offered 8% interest? Round to the nearest cent.

**Use the information on the pH formula for problems 5–6.**

$$pH = -\log_{10}[H^+]$$

**5)** A solution has a pH of 4. What is the concentration of hydrogen ions in the solution?

**6)** A substance had a concentration of hydrogen ions of $2.67 \times 10^{-8}$. Determine the pH to the nearest hundredth.

**Use the information on the continuous compounding formula for problems 7–8.**

$y = Pe^{rt}$

$P$: Initial amount
$r$: Rate
$t$: Time
$y$: Final amount

**7)** A biologist is studying a new strain of bacteria that exhibits continuous exponential growth. They begin with a small culture of 500 bacteria. After just 4 hours, the number of bacteria has skyrocketed to 27,000. What is the continuous hourly growth rate of this bacteria strain?

**8)** Another bacteria grows continuously at a rate of 52.3%. The initial culture started with 1200 bacteria. What is the bacteria population after 8 hours?

**Use the information on the half-life formula for problems 9–10.**

$$A(t) = A_0\left(\frac{1}{2}\right)^{\frac{t}{h}}$$

$A_0$: Initial quantity
$t$: Time in years
$h$: Half-life
$A(t)$: Quantity remaining at time $t$

**9)** A sample of a newly discovered radioactive substance has a mass of 300 grams. Scientists observe that after 5 years, only 50 grams of the new substance remain. What is the half-life of this mysterious substance?

**10)** A specific type of medication has a half-life of 4 hours in the human body. If a patient is given an initial dose of 400 milligrams, how long will it take for the amount of medication in their system to decrease to 25 milligrams?

**Use the information below for problems 11–12.**

The population of a small town grows exponentially. In the year 2000, the population was 4000. By 2010, the population had grown to 7000.

**11)** Write the equation in the form $y = ab^x$.

**12)** If this trend continues, what will the population be in the year 2030?

**To continue, return to the Online Lesson.**