# Lesson 39
# Introduction to Logarithms

NAME:

Start by navigating to the Online Lesson for instructions.

## Objectives

- Write an exponential expression or equation in logarithmic form.
- Write a logarithmic expression or equation in exponential form.
- Evaluate a logarithmic expression.

## Why?

Logarithms and exponents are inverses of one another. Knowing how to write expressions in either form allows you to solve more complex problems in Algebra 2.

## Warm Up

**Simplify. Do not use a calculator.**

**1)** $81^{\frac{3}{4}}$

**2)** $32^{-\frac{2}{5}}$

**To continue, return to the Online Lesson.**

## Explore

### Convert between Exponential and Logarithmic Equations

*Fill in the notes as you watch the video in the Online Lesson.*

- Exponential equations and logarithmic equations are _____ of one another.
- To define exponential and logarithmic equations: $\log_b y = x$ $\quad b^x = y$
  - $b$ is called the _____ and $b > 0, b \neq 1$.
  - $x$ is called the _____ or exponent.
  - $y$ is called the _____ or the answer and $y > 0$.

- _____ an exponential equation to a logarithmic equation, and vice versa, allows you to work with both exponential and logarithmic equations.
- When working with logs, think: _____ multiplied by itself _____ times results in _____ .
- In words, "base $b$ raised to the $x$ power _____ $y$" and "log base $b$ of $y$ _____ $x$"

| Convert equation from exponential to logarithmic | Convert equation from logarithmic to exponential |
|---|---|
| $b^x = y \qquad \log_b y = x$ | $\log_b y = x \qquad b^x = y$ |

## Example 1

*Complete the example as you watch the video in the Online Lesson.*

**Convert each equation to the equivalent logarithmic form.**

**A)** $64^{\frac{1}{3}} = 4$

**B)** $2^5 = 32$

## Example 2

*Complete the example as you watch the video in the Online Lesson.*

**Convert each equation to the equivalent exponential form.**

**A)** $\log_8 32 = \frac{5}{3}$

**B)** $\log_3 81 = 4$

## Checkpoint: Convert between Exponential and Logarithmic Equations

**Convert the equation to its inverse (exponential to logarithmic or logarithmic to exponential).**

**A)** $4^3 = 64$

**B)** $\log_{64} 32 = \frac{5}{6}$

**To continue, return to the Online Lesson.**

## Evaluating Logarithmic Expressions

*Fill in the notes as you watch the video in the Online Lesson.*

- Foundational properties of logs when _____:
  - If $a^0 = 1$, then $\log_a 1 = 0$
  - If $a^n = a^n$, then $\log_a \left(a^n\right) = n$
- Foundational log properties and _____ rules are used to evaluate logs without using technology.
- A common _____ must exist to find the value of the variable.

**To evaluate logarithms (by hand):**

**1)** Set the expression equal to a _____. (In this lesson, use $x$.)

**2)** _____ the logarithmic equation to an exponential equation.

**3)** Write both sides of the equation with the _____.

**4)** _____.

Note: When no base is given, the base, $b$, is $10$.

## Example 3

*Complete the example as you watch the video in the Online Lesson.*

**Evaluate.**

**A)** $\log_{16} 2$

**B)** $\log_2\left(\frac{1}{16}\right)$

## Example 4

*Complete the example as you watch the video in the Online Lesson.*

**Evaluate.**

**A)** $\log_{0.25} 64$

**B)** $\log_3\left(\sqrt[5]{3}\right)$

Most logs are irrational numbers, and therefore, technology is used to estimate the value of a logarithm.

As with evaluating logarithms by hand, when no base is given, the base, $b$, is $10$, which is the default base for calculators.

## Example 5

*Complete the example as you watch the video in the Online Lesson.*

**Evaluate with a calculator to the ten thousandths (four decimal places).**

**A)** $\log 8$

**B)** $\log 225$

## Checkpoint: Evaluating Logarithmic Expressions

**Evaluate.**

$\log_8 \frac{1}{2}$

The More to Explore for this lesson provides helpful tips and guidance in using technology for these types of problems.

You may benefit from completing the More to Explore before continuing.

**To continue, return to the Online Lesson.**

# Practice 1

*Complete problems on a separate sheet of paper.*

**Convert each equation to the equivalent logarithmic form.**

**1)** $27^{\frac{2}{3}} = 9$

**2)** $49^{-\frac{1}{2}} = \frac{1}{7}$

**3)** $15^2 = 225$

**4)** $9^0 = 1$

**Convert each equation to the equivalent exponential form.**

**5)** $\log_{16} 2 = \frac{1}{4}$

**6)** $\log_3 3 = 1$

**7)** $\log_5 \sqrt[3]{5} = \frac{1}{3}$

**8)** $\log_4 64 = 3$

**Evaluate.**

**9)** $\log_4 8$

**10)** $\log_{27} 81$

**11)** $\log_5 625$

**12)** $\log_{\frac{1}{5}} 25$

**13)** $\log_{49} \frac{1}{343}$

**14)** $\log_4 \frac{1}{32}$

**15)** $\log_9 27$

**16)** $\log_2 64$

**Evaluate with a calculator to the ten thousandths (four decimal places).**

**17)** $\log 100$

**18)** $\log 295$

**19)** $\log 16$

**20)** $\log 84$

**To continue, return to the Online Lesson.**

# Mastery Check

## Show What You Know

**A)** Evaluate the logarithmic expressions.

$\log_{125} 5$ $\log_{125} \frac{1}{5}$ $\log_{5} 125$ $\log_{5} \frac{1}{125}$

**B)** Draw a line matching the exponential expression to its correct logarithmic expression.

| | |
|---|---|
| $5^3$ | $\log_{125} 5$ |
| $5^{-3}$ | $\log_{125} \frac{1}{5}$ |
| $125^{-\frac{1}{3}}$ | $\log_{5} 125$ |
| $125^{\frac{1}{3}}$ | $\log_{5} \frac{1}{125}$ |

## Say What You Know

In your own words, talk about what you have learned using the objectives for this part of the lesson and your work on this page.

**To continue, return to the Online Lesson.**

# Practice 2

*Complete problems on a separate sheet of paper.*

**Convert the equation to its inverse (exponential to logarithmic or logarithmic to exponential).**

**1)** $5^{-2} = \frac{1}{25}$

**2)** $64^{\frac{2}{3}} = 16$

**3)** $49^{-\frac{3}{2}} = \frac{1}{343}$

**4)** $6^{-2} = \frac{1}{36}$

**5)** $\log_{23} 1 = 0$

**6)** $\log_{17} \sqrt{17} = \frac{1}{2}$

**7)** $\log_{196} 14 = \frac{1}{2}$

**8)** $\log_{20} \frac{1}{400} = -2$

**Evaluate.**

**9)** $\log_{11} \sqrt[4]{11}$

**10)** $\log_{6} 216$

**11)** $\log_{225} 15$

**12)** $\log_{12} \frac{1}{144}$

**13)** $\log_{25} 125$

**14)** $\log_{32} 64$

**15)** $\log_{81} 243$

**16)** $\log_{0.25} 4$

**Evaluate with a calculator to the ten thousandths (four decimal places).**

**17)** $\log 7$

**18)** $\log 1$

**19)** $\log (0.001)$

**20)** $\log 1225$

**To continue, return to the Online Lesson.**

# Targeted Review

*Complete items on a separate sheet of paper.*

**Simplify.**

**1)** $\sqrt{-12}+\sqrt{-75}$

**2)** $\sqrt{40}-\dfrac{2}{5\sqrt{6}}$

**Rewrite the expression so all terms are in the numerator. Assume all bases are positive.**

**3)** $\dfrac{x^5y^4z^{-9}}{x^{-6}y^{13}z^{-2}}$

**4)** $\left(\dfrac{3x^3y}{x^2y^5}\right)^{-2}$

**5)** Describe the transformation from $f(x)$ to $g(x)$.
$f(x)=(3)^x$, $g(x)=-5(3)^{x+8}-2$

**6)** Sketch the graph using technology: $g(x)=\dfrac{1}{2}(4)^{x+1}-3$

**Solve.**

**7)** $36^{\left(\frac{1}{2}x\right)}=216^{(2x+5)}$

**8)** $\left(\dfrac{1}{4}\right)^x<\left(\dfrac{1}{8}\right)^{x-2}$

**Multiple Choice**

**____ 9)** Name the value of $b$, and if it represents growth or decay for the exponential equation: $y=6(3)^{-x}$

**A)** 6, growth

**B)** 3, growth

**C)** $\dfrac{1}{3}$, decay

**D)** $\dfrac{1}{6}$, decay

**Multiple Choice**

_____ **10)** Find $[g \circ f](x)$ when $f(x) = 3x^2 - 2$ and $g(x) = -x + 6$.

**A)** $-3x^2 + 8$

**B)** $3x^2 - x + 4$

**C)** $3x^2 - 36x + 106$

**D)** $-3x^2 + 4$

_____ **11)** Name the domain and range for the inverse of the function: $g(x) = \frac{3}{x} + 2$

**A)** domain: $\{x \mid x \in \mathbb{R}\}$
range: $\{y \mid y \in \mathbb{R}\}$

**B)** domain: $\{x \mid x \in \mathbb{R}, x \neq 0\}$
range: $\{y \mid y \in \mathbb{R}, y \neq 2\}$

**C)** domain: $\{x \mid x \in \mathbb{R}, x \neq 2\}$
range: $\{y \mid y \in \mathbb{R}, y \neq 0\}$

**D)** domain: $\{x \mid x \in \mathbb{R}, x \neq 2\}$
range: $\{y \mid y \in \mathbb{R}\}$

_____ **12)** Write an exponential equation that passes through the points $(3, 128)$ and $\left(-1, \frac{1}{32}\right)$.

**A)** $y = 2^x$

**B)** $y = \frac{1}{2^{11}}(64)^x$

**C)** $y = 8\left(\frac{1}{4}\right)^x$

**D)** $y = \frac{1}{4}(8)^x$

| **Problem** | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **Origin** | 11 | 12 | – | – | 37 | 37 | 38 | 38 | 37 | 32 | 19 | 38 |

*L = Lesson in this level, A1 = Algebra 1: Principles of Secondary Mathematics, FD = Foundational Knowledge*

**To continue, return to the Online Lesson.**