Lesson 35

NAME:

The FUNdamental Theorem of Algebra

Start by following the instructions in the Online Lesson.

Objectives

- Determine the number of roots for a given polynomial function using the Fundamental Theorem of Algebra.
- Locate and estimate the relative minimum and relative maximum of a function (including the use of technology).
- Sketch a graph given turning points and real zeros (distinct and multiple roots).
- Name the increasing and decreasing intervals across a function.

Why?

The Fundamental Theorem of Algebra tells you that every polynomial equation has at least one solution in the set of complex numbers. Understanding this theorem helps you develop problem-solving skills and think more abstractly about math.

Warm Up

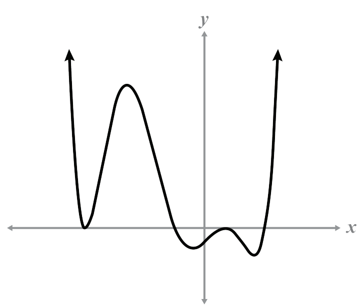
Name the complex expression to form a conjugate pair.

1) $7-4i$

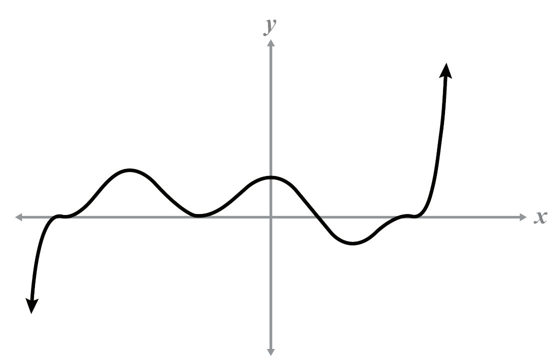
2) $1+i$

Determine the possible maximum degree from the graph.

3)



4)



This lesson requires technology to solve problems. The More to Explore for this lesson provides helpful tips and guidance in using technology for these types of problems. You may benefit from completing the More to Explore before continuing.

To continue, return to the Online Lesson.

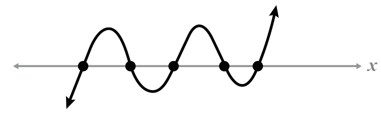
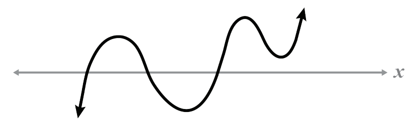
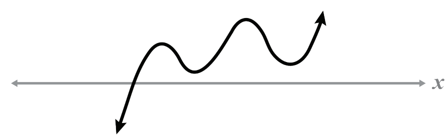
Explore

The FUNdamental Theorem of Algebra

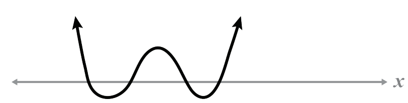
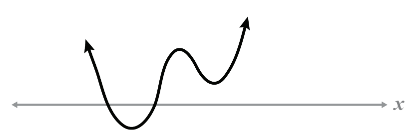
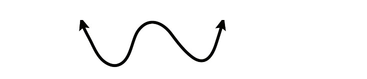
Fill in the notes as you watch the video in the Online Lesson.

- The Fundamental Theorem of Algebra states: Every polynomial equation with ________________ coefficients and ________________ degree n has _________________ complex roots.
- Which means:
    - _________, and there are _______________ complex roots (including multiplicities).
    - If _________ is a root, then _________ must also be a root, when a and b are real numbers and $b \uparrow 0$.
        - In other words, complex roots occur in _________________________.
- The FTA tells you the ____________________________________.
    - Polynomials can have real and non-real, complex roots, but the roots must have a __________________________________ the n^{th} degree of the polynomial.
    - You must continue to solve until you find _________________________, remembering to include multiplicities.
- Solving for roots with the FTA allows you to determine:
    - All ______________.
    - If a root is ______________ and will intersect the x-axis.
    - If a root is __________________________________, and will not intersect the x-axis.

$f(x) = Ax^5 + Bx^4 + Cx^3 + Dx^2 + Ex + F$

Degree	Real	Non-Real, Complex	Sketch (with multiplicities of 1)
			
5			
			

$g(x) = Ax^4 + Bx^3 + Cx^2 + Dx + E$

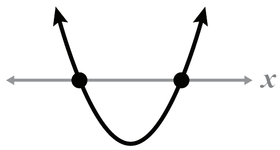
Degree	Real	Non-Real, Complex	Sketch (with multiplicities of 1)
			
4			
			

Example 1

Complete the example as you watch the video in the Online Lesson.

Given the sketch of a 2nd degree polynomial, determine if a is greater than or less than zero, and if the roots are real or non-real, complex.

A)



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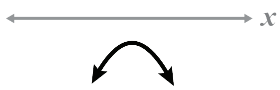

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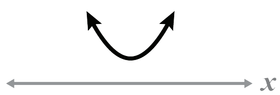

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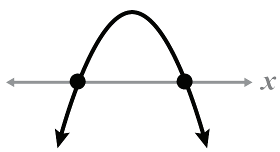
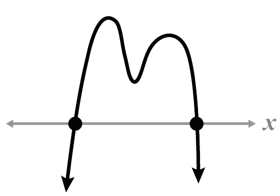


Example 2

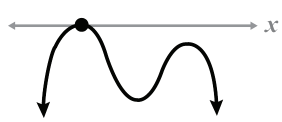
Complete the example as you watch the video in the Online Lesson.

Given a 4th degree polynomial $(n = 4)$ where $a < 0$, identify the sketch(es) with 2 real roots with multiplicities of 1. If a sketch does not meet these criteria, explain why.

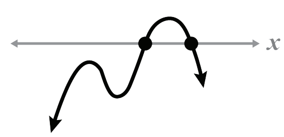
A)



B)

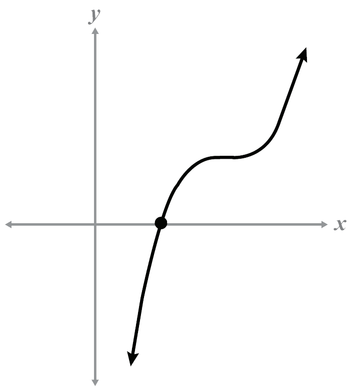


C)



Checkpoint: The FUNdamental Theorem of Algebra

Determine if a is greater than or less than zero, and the number of real and non-real, complex roots for the sketch of a 3rd degree polynomial.

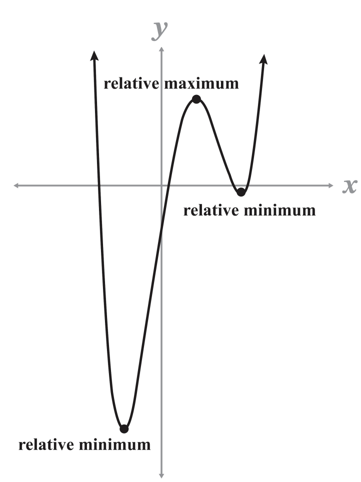


To continue, return to the Online Lesson.

Turning Points with Technology

Fill in the notes as you watch the video in the Online Lesson.

- A ______________________________ occurs when a graph changes directions (up to down or down to up).
- A graph's maximum number of turning points *must* be less than the degree of the polynomial, or __________________________.
- If a graph has fewer than $n - 1$ turning points, _________________________ may occur (but not always).
- Turning points are also called ____________________________________ or ____________________________________ points based on their positions on the graph.
- Graphs can have more than one turning point, and likewise _________________________ relative minimum and relative maximum.



- In this level, you will determine the largest or smallest turning point across an ______________ of a continuous function.
- A turning point may also be a ___________.

Steps for Identifying Turning Points

1) Use _____________________ to locate and estimate turning points on a graph.

2) Use the ____________________________________ features to accurately determine and name all turning points.

3) Sketch graphs that include ____________________________, ___________, and __________________________.

4) _____________ the numbers on your sketch to provide more information about the graph.

Remember, a sketch does not reflect the exact scale of a graph.
It gives a general idea of what is happening at key points on the graph.

Example 3

Complete the example as you watch the video in the Online Lesson.

A) **Sketch the equation. Include roots and turning points.**

B) **What is the maximum number of turning points that can occur in the graph? Explain why the graph does or does not have the maximum number of turning points.**

C) **Name the relative maximum point on the interval $[-2, 2]$.**

$p(x) = (x - 0.5)^3(x + 2)(x + 1)$

Example 4

Complete the example as you watch the video in the Online Lesson.

A) Sketch the equation. Include roots and turning points.

B) Name all real and non-real, complex roots.

C) Name the relative maximum and minimum over the interval [–4, 1.5].

$g(x) = -(x^4 - 1)(x+3)^2$

Checkpoint: Turning Points

A) Sketch the equation. Include roots and turning points.

B) Name the relative minimum and maximum over the interval [–1, 3].

$f(x) = (4x+1)(x^2 - 2x + 1)$

To continue, return to the Online Lesson.

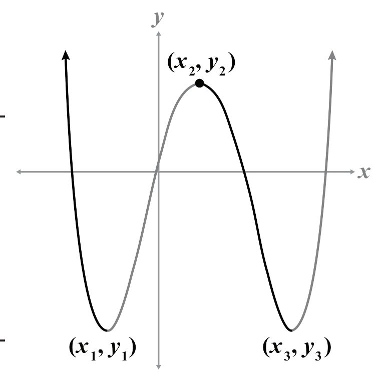
Increasing and Decreasing Intervals with Technology

Fill in the notes as you watch the video in the Online Lesson.

- Because turning points are neither increasing nor decreasing, they can be used to ____________________________ across the x-axis.

How to Describe Intervals Using Turning Points (and End Behaviors)

- Label __________________________ on the graph.
- Moving left to right, define intervals (in interval notation) using turning points and end behaviors, such as:
  - From an end behavior to a turning point ________________
  - From a turning point to a turning point ________________ ________________
  - From a turning point to an end behavior ________________

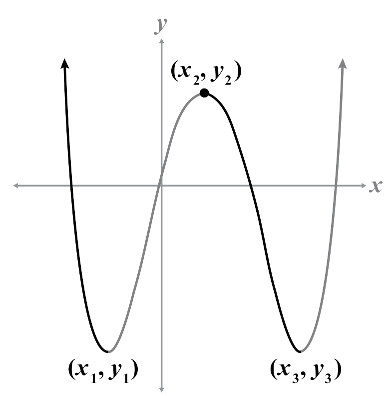


- Then add a description of the x-values for __________________________________ as increasing, decreasing, or constant.

Interval Description	x	y	Interval Notation
x- and y-values increase across the interval			
x-values increase and y-values decrease across the interval			
x-values increase and y-values remain the same			

How to Express Interval Descriptions

- The graph of the polynomial function ____________________ for x-values across the intervals $(-\infty, x_1)$ and (x_2, x_3).
- The graph of the polynomial function ____________________ for x-values across the intervals (x_1, x_2) and $(x_3, +\infty)$.

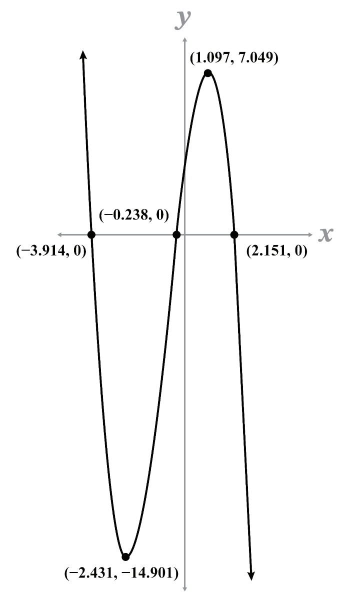


Example 5

Complete the example as you watch the video in the Online Lesson.

Name the interval(s) in which the function increases and decreases.

- The graph of the polynomial function increases across the interval ____________________.
- The graph of the polynomial function decreases across the intervals ____________________ and ____________________.



Example 6

Complete the example as you watch the video in the Online Lesson.

$h(x) = x(x+1)(x-3)\left(x-\sqrt{5}\right)\left(x+\sqrt{5}\right)$

A) Name the interval(s) in which the function increases and decreases.

B) Name the relative minimum and maximum *over the interval* $[-3, 1]$.

x	y

A) Increases across the intervals:

____

Decreases across the intervals:

____

B) Relative minimum: ________________________

C) Relative maximum: ________________________

Checkpoint: Increasing and Decreasing Intervals

Name the interval(s) in which the function increases and decreases.

$g(x) = x^2(2x - 1)(x + 1.75)$

To continue, return to the Online Lesson.

Practice 1

Complete problems on a separate sheet of paper.

1) Explain the number of complex roots that can occur for a ninth degree polynomial.

2) Explain if it is possible for a tenth degree polynomial to have three non-real, complex roots.

Sketch a graph that matches the given information.

3) A fifth degree polynomial function with a positive leading coefficient, with a single and a double real root

4) A sixth degree polynomial function where $a < 0$, and no real roots

5) A fourth degree polynomial function where $a > 0$, and one real root

6) A third degree polynomial function with a positive leading coefficient, with three real roots

7) What is the maximum number of turning points that can occur in a polynomial graph?

8) Explain why a polynomial graph will not always have the maximum number of turning points.

For problems 9–12:

A) Sketch the equation. Include roots and turning points.
B) Name the interval(s) in which the function is increasing and decreasing.
C) Name the relative minimum and maximum across the given interval.

9) $g(x) = (x-1)^3(x+1)^2(2x+1)$ across $[-1, 2]$

10) $p(x) = x^3 - 19x + 30$ across $[-10, 0]$

11) $f(x) = -x^4 + x^3 + 12x^2$ across $[-3, 4]$

12) $y = 6x^3 + x^2 - 19x + 6$ across $[-2, 2]$

For problems 13–14:

A) Sketch a graph that matches the equation.
B) Name all real and non-real, complex roots.

13) $k(x) = (x+2)(2x^2 + 2x + 1)$

14) $f(x) = (x^2 - 1)(x^2 - 9)$

To continue, return to the Online Lesson.

Mastery Check

Show What You Know

Use the equation $k(x) = x^5 - 6x^4 + 10x^3 - 6x^2 + 9x$ to answer each part.

A) Explain how many roots and what type can occur for $k(x)$.

B) Explain how many turning points can occur for $k(x)$. Why would a graph have fewer turning points?

C) Sketch the graph of $k(x)$. Label all turning points and roots.

D) Determine all roots for $k(x)$. Describe the multiplicities as real or non-real, complex roots.

Say What You Know

In your own words, talk about what you have learned using the objectives for this lesson and your work on this page.

To continue, return to the Online Lesson.

Practice 2

Complete problems on a separate sheet of paper.

1) Explain if it is possible for any polynomial to have an odd number of non-real, complex roots.

2) Describe what is happening to the x- and y-values when the function decreases across an interval.

3) Explain why it is important to identify the turning points on the graph of a polynomial function.

4) Describe the Fundamental Theorem of Algebra.

Sketch a graph that matches the given information.

5) A seventh degree polynomial function where $a < 0$, with five real roots

6) A fourth degree polynomial function where $a > 0$ and a real root with a multiplicity of two

7) A third degree polynomial with a positive leading coefficient, and a non-real, complex conjugate pair

8) A sixth degree polynomial with a negative leading coefficient, and four real roots

For problems 9–10:

A) Sketch a graph that matches the equation.
B) Name all real and non-real, complex roots.

9) $h(x) = -\left(x^2+4\right)\left(x^2-x-2\right)$

10) $m(x) = \left(3x^3-x^2\right)\left(x^2+6\right)$

For problems 11–14:

A) Sketch the equation. Include roots and turning points.
B) Name the interval(s) in which the function is increasing and decreasing.
C) Name the relative minimum and maximum across the given interval.

11) $f(x) = 3x^4 - 10x^3 - 24x^2 - 6x + 5$ across $[-2,\ 0]$

12) $g(x) = (x+2)(x+3)(x-1)$ across $[-0.5,\ 2]$

13) $h(x) = x^4 + 2x^3 - 8x^2$ across $[-6,\ 1]$

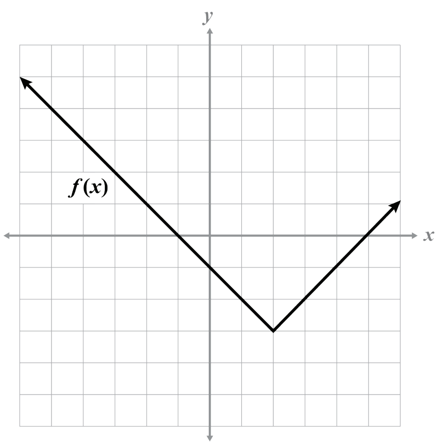
14) $q(x) = x^3 + 4x^2 - 5$ across $[-4,\ 3]$

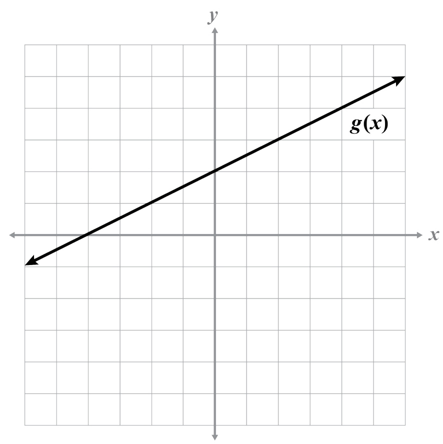
To continue, return to the Online Lesson.

Targeted Review

Complete items on a separate sheet of paper.

For problems 1–4, use the graphs of $f(x)$ and $g(x)$.





1) $(f+g)(-4)$

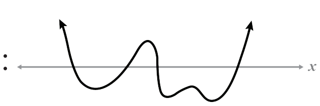
2) $(f \circ g)(2)$

3) $(g \circ g)(6)$

4) $\left(\dfrac{f}{g}\right)(-1)$

5) Determine all possible rational roots for $f(x)=3x^3+2x^2-19x+6$.

6) Write the polynomial from problem 5 as a product of its rational factors.

7) Identify the end behavior from the sketch of the function: 

8) State the roots and the multiplicities for $b(x)=x^2(x+7)^3(x-6)$.

Multiple Choice

_____ **9)** Determine the end behavior for $h(x) = x(x-3)^2(x+1)(x-5)$.

A) $x \to \pm\infty, f(x) \to -\infty$

B) $x \to \pm\infty, f(x) \to +\infty$

C) $x \to -\infty, f(x) \to -\infty$, and $x \to +\infty, f(x) \to +\infty$

D) $x \to -\infty, f(x) \to +\infty$, and $x \to +\infty, f(x) \to -\infty$

_____ **10)** Determine $g \circ f$ when $f(x) = 3x+1$, and $g(x) = \frac{-x+1}{2}$.

A) $-\frac{3}{2}x + \frac{5}{2}$

B) $-\frac{3}{2}x$

C) $\frac{3}{2}x + \frac{5}{2}$

D) $\frac{3}{2}x + 1$

_____ **11)** Determine $q(3)$ for the function $q(x)$, when $q(-2) = -\frac{1}{3}$.
$q(x) = ax^2 - 3a$

A) -2

B) $-\frac{1}{3}$

C) 2

D) 3

_____ **12)** Select the function that contains a double root at $f(5) = 0$.

A) $f(x) = (x-2)^5(x+5)^2$

B) $f(x) = (x-5)^2(x+2)$

C) $f(x) = (x+5)^2(x+2)$

D) $f(x) = x^2(x-2)^5$

Problem	1	2	3	4	5	6	7	8	9	10	11	12
Origin	L31	L32	L32	L31	L34	L34	L33	L34	L33	L32	L31	L34

L = Lesson in this level, A1 = Algebra 1: Principles of Secondary Mathematics

To continue, return to the Online Lesson.