Lesson 9
Solving Rational Equations

NAME:

Start by navigating to the Online Lesson for instructions.

Objectives

- Determine the excluded values for the rational equation.
- Solve rational equations that are proportions.
- Solve rational equations by finding the least common denominator (LCD).
- Apply rational equations to word problems.

Why?

How long will your group project take to complete? Knowing how to work with rational equations allows you to solve problems like this. Additionally, in the study of science, there are many formulas that require an understanding of rational equations to solve problems.

Warm Up

1) Isolate the variable k in the equation

$$PE = \frac{1}{2}kx^2$$

2) The ratio of string players to percussionists in the orchestra is 7 to 2. If there are 42 string players, how many percussionists are in the orchestra? Write and solve a proportion.

To continue, return to the Online Lesson.

Explore

Rational Equations: Proportions

Fill in the notes as you watch the video in the Online Lesson.

- To solve rational equations, determine the ______________________________.
- When the degree of the variable is __________, there will be ____________________ solution.

- When the degree of the variable is ______________________________, that degree is the ___________________ number of possible solutions.
- However, because there can be ______________________ on the denominator, it is possible to have fewer solutions than the degree.
- Therefore, it is important to check for restrictions on the denominator as well as ______________________ solutions.
- When one rational expression is set equal to another, this creates a _________________.
- Solve proportions by finding the ____________________________.

Example 1

Complete the example as you watch the video in the Online Lesson.

Solve.

Implement

$$\frac{h+2}{h}=\frac{2h+1}{h+2}$$

$h \neq -2, 0$

$(h+2)(h+2)=h(2h+1)$

Explain

- Restrictions
- Cross-product
- Distribute
- Move terms to one side
- Factor
- Solve for h

Because the solutions do not include values that would make the denominator zero, there are no extraneous solutions.

Example 2

Complete the example as you watch the video in the Online Lesson.

Solve.

Implement

$$\frac{3}{x-2}=\frac{5}{x+2}$$

Explain

- Restrictions
- Cross-product
- Distribute
- Isolate the variable

Checkpoint: Rational Equations: Proportions

Solve.

$$\frac{a-2}{a+1}=\frac{a-1}{a-2}$$

To continue, return to the Online Lesson.

Rational Equations: LCD

Fill in the notes as you watch the video in the Online Lesson.

- When rational equations are not proportions, determine the ______________________________ ____________________________. Then rewrite the problem using the LCD.
- Before solving for the variable, determine the restrictions on the ___________________________.

- If a restriction is also a solution, this value is ________________________ because the denominator would be undefined.
- After all parts of the problem are written with the LCD, use the ______________________ to solve the problem for the missing value(s) of the variable.

Example 3

Complete the example as you watch the video in the Online Lesson.

Solve. Check your solutions for extraneous values.

$$\frac{n}{n+1}+3=\frac{n^2-2}{n+1}$$

Plan

Determine the LCD
Simplify the numerator (distribute, combine like terms)
Solve for the unknown
Check

Implement

$$\frac{n}{n+1}+\frac{3(n+1)}{n+1}=\frac{n^2-2}{n+1}$$

$$n+3(n+1)=n^2-2$$

Explain

- Name the LCD and restrictions on the denominator
- All terms have LCD
- Simplify the numerator
- Factor
- Check for extraneous solutions

To continue, return to the Online Lesson.

Rational Equations: LCD (cont.)

Fill in the notes as you watch the video in the Online Lesson.

- Write ______________________________ solutions with the math shorthand, $\mathbb{R}$.
- Some rational equations may have _____________________ , which occurs when:
  - all ___________________ have simplified out of the equation, and
  - the remaining ___________________ are not equal. (i.e. $7 = 12$).

Example 4

Complete the example as you watch the video in the Online Lesson.

Solve. Check your solutions for extraneous values.

$$\frac{x}{x+6}+\frac{6}{x-6}=\frac{x^2+36}{x^2-36}$$

LCD: $(x-6)(x+6)$, $x \neq \pm 6$

Example 5

Complete the example as you watch the video in the Online Lesson.

Solve. Check your solutions for extraneous values.

$$\frac{6}{x+1}=\frac{4}{x-1}+\frac{2}{x-2}$$

Checkpoint: Rational Equations: LCD

Solve. Check your solutions for extraneous values.

$$n+\frac{n}{n+4}=\frac{6}{n+4}$$

To continue, return to the Online Lesson.

Applications of Rational Equations

Fill in the notes as you watch the video in the Online Lesson.

- The rate for a work problem is thought of as:
- A rate of work problem is a transformation of the equation ______________.
  - d is ____________________ and is the job being completed
  - r is ________________
  - t is ________________
- When the rate of work for one person is ____________________ with the rate of work for another person, the task that needs to be completed can be finished ________________ because those rates of work are combined using $d = rt$.
- Using a table to set up problems can help determine the information you have and the information you need to solve for:

	Person 1	Person 2	Working Together
time:			
rate of work:	+	=	

Example 6

Complete the example as you watch the video in the Online Lesson.

Solve.

Natalee and Grant are shoveling snow from a very long driveway after a snow storm. It takes Natalee 6 hours to completely shovel the driveway alone. Grant is able to shovel the driveway alone in 10 hours. How long would it take if Natalee and Grant work together?

Plan

Identify key information in the problem
Organize time and rate in a chart
Write and solve an equation

Implement

	Natalee		Grant		Together
time:					
rate:		+		=	

$$\frac{1}{6}+\frac{1}{10}=\frac{1}{t} \qquad \text{LCD}(6,\,10,\,t)=30t$$

$$30t\left(\frac{1}{6}+\frac{1}{10}=\frac{1}{t}\right) \quad \text{or} \quad \frac{1}{6}\left(\frac{5t}{5t}\right)+\frac{1}{10}\left(\frac{3t}{3t}\right)=\frac{1}{t}\left(\frac{30}{30}\right)$$

Explain

Checkpoint: Applications of Rational Equations

Solve.

Lindsey and Haven are painting a fence in their backyard. Alone, Lindsey can paint the fence in 9 hours. Together they are able to paint the fence in 5 hours. How long would it take Haven to paint the fence alone?

	Lindsey	Haven	Together
time:			
rate:	+	=	

To continue, return to the Online Lesson.

Physics Applications of Rational Equations

Fill in the notes as you watch the video in the Online Lesson.

- In physics, you often begin with the definition of a term. Then you apply solving techniques to create a new equation by ________________________ a particular variable.

- The average velocity, v, can be defined as the change in displacement, Δx, divided by time, t:
  - $v =$ average _______________
  - $\Delta x =$ change in ________________________________
  - $t =$ ____________
- Assuming constant acceleration, the average velocity can also be defined as the sum of the final and initial velocities divided by two:
  - $v_f =$ ________________ velocity
  - $v_i =$ _________________ velocity
  - $v = \dfrac{v_f + v_i}{2}$

$v =$ average velocity

$v_f =$ final velocity

$v_i =$ initial velocity

Since both definitions are the average velocity, we can set these two rational equations equal to each other.

Velocity equation in terms of time, t.

- _________________________ can be defined as the change in velocity over time:
- The change in velocity is defined as:
  - $a = \dfrac{v_f - v_i}{t}$

$v_f =$ ___________ velocity

$v_i =$ ____________ velocity

$t =$ __________

  - $t = \dfrac{v_f - v_i}{a}$

Acceleration equation in terms of time, t.

- Since both equations are equal to _____ they can be set _____________ to each other to obtain a new formula:

Example 7

Complete the example as you watch the video in the Online Lesson.

Mark is driving at 20 meters per second ($20\ \mathrm{m/s}$) and sees an obstruction in the road. He then slams on the brakes and decelerates at 6 meters per second squared ($-6\ \mathrm{m/s^2}$) until the car is stopped.

Calculate the distance, Δx, traveled until the car stops using the formula: $\dfrac{2\Delta x}{v_f + v_i} = \dfrac{v_f - v_i}{a}$

Plan

Identify the given values
Solve for Δx

Implement

$v_i = 20\ \mathrm{m/s}$

$v_f = 0\ \mathrm{m/s}$ (Mark comes to a full stop)

$a = -6\ \mathrm{m/s^2}$

$$\frac{2\Delta x}{0+20} = \frac{0-20}{-6}$$

Example 8

Complete the example as you watch the video in the Online Lesson.

A car traveling at 15 meters per second ($15\ \mathrm{m/s}$) accelerates at 1 meter per second squared ($1\ \mathrm{m/s^2}$) for a distance of 32 meters.

Find the final velocity using the formula: $\dfrac{2\Delta x}{v_f + v_i} = \dfrac{v_f - v_i}{a}$

Plan

Identify the given values
Solve for v_f.

Implement

$\Delta x = 32$ meters

$v_i = 15\ \mathrm{m/s}$

$a = 1\ \mathrm{m/s^2}$

$$\frac{2(32)}{v_f + 15} = \frac{v_f - 15}{1}$$

$$\left(v_f + 15\right)\left(v_f - 15\right) = 64$$

$$v_f^2 - 225 = 64$$

Checkpoint: Physics Applications of Rational Equations

Brian and Steven were sledding down a hill. Brian gave Steven's sled a push such that his initial velocity was $3\ \mathrm{m/s}$. He accelerated down the hill at $4\ \mathrm{m/s^2}$. Steven traveled 40 meters down the hill.

Substitute the known values into the equation: $\dfrac{2\Delta x}{v_f + v_i} = \dfrac{v_f - v_i}{a}$

And set it equal to zero. Do not solve.

To continue, return to the Online Lesson.

Practice 1

Complete problems on a separate sheet of paper.

Solve. Check your solutions for extraneous values.

1) $\dfrac{x+4}{-x-2}=\dfrac{3x}{3x-10}$

2) $\dfrac{5x-9}{2x+1}=\dfrac{3x-7}{x+2}$

3) $\dfrac{4}{a^2-2a-8}=\dfrac{12}{a^2-a-6}$

4) $\dfrac{7}{4x}-\dfrac{1}{3}=\dfrac{2}{9x}$

5) $\dfrac{5}{g+6}+\dfrac{4}{g-3}=\dfrac{12g}{g^2+3g-18}$

6) $\dfrac{5}{x}-\dfrac{2}{x+1}=\dfrac{3}{x+6}$

7) $\dfrac{x}{x+4}-\dfrac{4}{4-x}=\dfrac{x^2+16}{x^2-16}$

8) $\dfrac{3h}{h+6}-\dfrac{h}{h+5}=-\dfrac{4}{h^2+11h+30}$

9) The community garden can be harvested alone in 10 hours by Farmer Frank. If Garden Gail also helps, it takes only 6 hours to harvest. How long would it take Garden Gail to harvest alone?

10) The sum of the reciprocals of two numbers is equal to $\frac{2}{3}$. The difference between the two numbers is 4. Find the numbers.

11) Bryson owns a lawn mowing business. If Bryson works alone, he can get all of the lawns mowed for a given day in 8 hours. If he hires his brother, Aaron, he can mow all the lawns in 10 hours. How long would it take to mow all the lawns if they work together? Round to the nearest half hour.

12) A car, traveling at an initial velocity of 26 $\mathrm{m/s}$, brakes and slows down (decelerates) at a rate of $-6\ \mathrm{m/s^2}$.

Find the final velocity when the change in distance is 40 meters, using the formula:

$$\frac{2\Delta x}{v_f+v_i}=\frac{v_f-v_i}{a}$$

Δx = change in distance, v_f = final velocity, v_i = initial velocity, a = acceleration

To continue, return to the Online Lesson.

Mastery Check

Show What You Know

Two tanker trucks are needed to fill an Olympic-sized swimming pool at Sandy's Swim Club. The first truck can fill the pool in x hours. The second truck takes twice as long to fill the pool alone. When the trucks fill the pool together it takes $3\frac{1}{3}$ hours.

A) Make a chart to show time and rate of work for Truck A, Truck B, and Together.

B) Use part A to write and solve an equation to find the time it takes each truck to fill the pool alone.

The average person at Sandy's Swim Club can swim the length of the pool, 50 meters, in 70 seconds. The club's Olympic swimmer can swim the length of the pool in $2p - 3$ seconds. It takes three times longer for the average swimmer to swim the length of the pool.

C) Determine how long it takes an Olympic swimmer to swim 50 meters. Show your work.

Say What You Know

In your own words, talk about what you have learned using the objectives for this part of the lesson and your work on this page.

To continue, return to the Online Lesson.

Practice 2

Complete problems on a separate sheet of paper.

Solve. Check your solutions for extraneous values.

1) $\dfrac{4x}{x-2}=\dfrac{3x}{x+3}$

2) $\dfrac{2h-7}{h^2-h-2}+\dfrac{3h}{h+1}=4$

3) $\dfrac{2}{b+2}-\dfrac{4}{b}=\dfrac{1}{b^2+2b}$

4) $\dfrac{5}{x-3}-\dfrac{2}{4-x}=\dfrac{7x}{x^2-7x+12}$

5) $\dfrac{100}{x^2-25}-\dfrac{2x}{x+5}=\dfrac{-x}{x-5}$

6) $\dfrac{5}{y+5}+1=\dfrac{7y+2}{y^2+y-20}$

7) $\dfrac{4}{x}+\dfrac{10}{x+5}=\dfrac{9}{2x-5}$

8) $\dfrac{3x}{x+4}-\dfrac{1}{x-4}=\dfrac{3x^2-13x-4}{x^2-16}$

9) Jacob and Elena need to paint their living room. Alone, Jacob can paint the room in 4 hours while Elena can paint the room by herself in 3 hours. Determine how long it will take if Jacob and Elena paint the room together. Round to the nearest quarter hour.

10) A bricklayer constructs a retaining wall in 12 hours when working alone. When the bricklayer and an apprentice work together, the retaining wall is constructed in 8 hours. How long would it have taken for the apprentice to do the job alone?

11) A bicyclist has an initial velocity of $4\ \mathrm{m/s}$ before they begin to travel downhill. If the hill is 32 meters long and they accelerate at $2\ \mathrm{m/s^2}$, how fast will the bicyclist be traveling when they reach the bottom of the hill? Use the formula $\dfrac{2\Delta x}{v_f+v_i}=\dfrac{v_f-v_i}{a}$, where Δx = change in distance, v_f = final velocity, v_i = initial velocity, a = acceleration.

12) The difference between the reciprocals of two numbers is $\dfrac{1}{10}$. The second number is 5 more than the first. Determine the value(s) of each number.

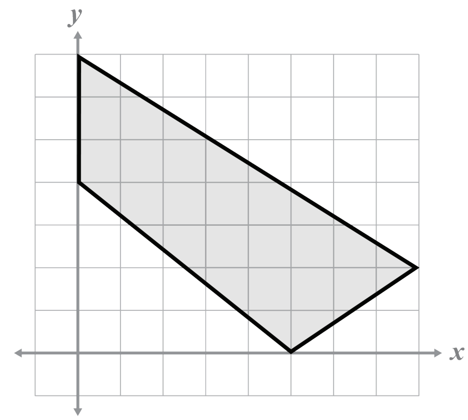
To continue, return to the Online Lesson.

Targeted Review

Complete items on a separate sheet of paper.

1) Name the vertices formed from the system of linear inequalities in the given graph.

2) Using the objective function $f(x, y) = 2x + y$, determine the minimum and maximum using the given graph.



Write the polynomial identity using the variables x and y.

3) Difference of two squares

4) Difference of cubes

5) Divide $15x^5 + 9x^3 + 2x^2 - 6$ by $6x$

6) Determine the value that would be used for synthetic substitution or synthetic division from the given binomials.

A) $a - 5$

B) $3b + 1$

C) $3 - c$

Simplify. Name any restrictions on the denominator.

7) $\dfrac{15x^4 - 15y^4}{5x^2 - 5y^2}$

8) $\dfrac{8x^3 + 12x}{12x^3 + 34x^2 + 24x}$

Multiple Choice

9) Select all that apply.

When a region is *unbounded* for linear programming this means:

- ☐ the region will not be completely enclosed.
- ☐ the region does not exist.
- ☐ the region will continue infinitely in at least one direction.
- ☐ nothing is known about the problem.

_____ **10)** Determine the restrictions on the denominator for a triangle with an area of:

$$\frac{25-4x^2}{3x^2-13x-10}$$

A) $-5, \frac{2}{3}$

B) $-2, \frac{5}{3}$

C) $-\frac{2}{3}, 5$

D) $\pm\frac{5}{2}$

_____ **11)** Factor completely: $6x^3-6xy^2+9x^2-9y^2$

A) $(6x+9)(x^2-y^2)$

B) $3(2x+3)(x+y)(x-y)$

C) $3(2x+3)(x+y)^2$

D) cannot be factored

_____ **12)** Use synthetic substitution to find $f(-3)$ for $f(x)=x^6+2x^5-x^4+x^3-10x^2+2$

A) -13

B) $1{,}073$

C) 47

D) -141

Problem	1	2	3	4	5	6	7	8	9	10	11	12
Origin	L1	L1	L3	L3	L5	L6	L7	L7	L1	L7	L3	L6

L = Lesson in this level, A1 = Algebra 1: Principles of Secondary Mathematics, FD = Foundational Knowledge

To continue, return to the Online Lesson.