## Multiplying and Dividing Rational Expressions



Start by navigating to the Online Lesson for instructions.

## **Objectives**

- Simplify the product or quotient of rational expressions.

## Why?

Working with rational expressions is a foundational step to developing skills and understanding rules for later lessons and levels. Rational expressions extend what you have learned about fractions to include fractions with variables in the numerator and denominator.

# ॐ Warm Up

Factor to solve.

1) 
$$2x^2 - 10x = 0$$

**2)** 
$$4x^2 + 12x + 9 = 0$$

Simplify.

3) 
$$\frac{5a^3b}{15ab^2}$$

**4)** 
$$\frac{4x^2}{9y^2} \cdot \frac{y}{24x}$$

# A Explore

## **(A)** Restrictions for Rational Expressions

Fill in the notes as you watch the video in the Online Lesson.

	A	is a ratio between two polynomial										
	expre	essions in the form $\frac{p}{q}$ where $p$ and $q$ are polynomials and										
	Notic	e that $\boldsymbol{q}$ cannot equal zero. A denominator of zero makes the expression undefined.										
	This i	s called a or excluded value.										
•	There	efore, any values that result in a rational expression with a denominator of zero are										
		or restricted from the expression.										
-	To fin	o find the restrictions for a rational expression:										
	1)	the numerator and denominator.										
	2)	Set each expression in the denominator equal to										
	3)	The resulting values are the excluded values, or restrictions.										
	Use t	he symbol to name restrictions on the denominator										

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### **Example 1**

(b) Complete the example as you watch the video in the Online Lesson.

Determine the restrictions for the denominator.

$$\frac{20x^2 + 13}{3x^2 - 12}$$

#### Plan

Factor the denominator

Set each expression in the denominator equal to zero

Solve for the restrictions of the denominator

**Implement** 

$$3x^2 - 12$$

$$3(x^2-4)$$

$$3(x-2)(x+2)$$

**Explain** 

- ▶ Factor
- Solve
- ► Write the restrictions (or excluded values) for *x*

### Example 2

© Complete the example as you watch the video in the Online Lesson.

State all numbers that make the given expression undefined.

$$\frac{11}{3x^3 - 19x^2 + 6x}$$

### Implement

#### Explain

- ▶ Factor the denominator
- ► Solve each expression in the denominator
- Write the values that make the expression undefined (in other words, the restrictions or excluded values)

## $\ oxed{ extstyle }$ Checkpoint: Restrictions for Rational Expressions

Determine the restrictions for the denominator.

$$\frac{5x - 8}{40x^3 + 11x^2 - 2x}$$



### h Simplifying Rational Expressions with Multiplication and Division

- (b) Fill in the notes as you watch the video in the Online Lesson.
- A \_\_\_\_\_\_ rational expression has \_\_\_\_\_ between the numerator and denominator other than 1 or -1.
- To simplify a rational expression:
  - 1) \_\_\_\_\_ the numerator and denominator completely.
  - 2) Determine if there are any \_\_\_\_\_ on the domain.
  - 3) common factors out of the expression.
- Rational expressions can have \_\_\_\_\_\_ that are monomials, binomials, trinomials, etc.
- To simplify monomials:
  - Use the rules you have already learned to simplify \_\_\_\_\_\_
  - $\frac{{}^{1}\cancel{8}x^{9}}{\cancel{24}x^{5}} = \frac{1x^{9-5}}{3} = \frac{x^{4}}{3}$ In this example, the \_\_\_\_\_\_ were simplified, and the \_\_\_\_\_

were subtracted.

- To simplify polynomials:
  - The numerator and denominator must have \_\_\_\_\_\_ factors in order to divide them out from the rational expression.
  - In this example, once factored, only the  $\frac{(x-3)(x+7)}{(2x-3)(x+7)} = \frac{x-3}{2x-3}$ can be simplified out of the expression.
  - It may be helpful to classify expressions by \_\_\_\_\_ and \_\_\_\_\_verbally to help determine if they are identical.

### Example 3

( ) Complete the example as you watch the video in the Online Lesson.

Simplify. State the restrictions on the denominator.

$$\frac{2x(x+3)(3x+11)}{8x^2(2x-5)(x+3)}$$

#### Plan

Solve for the restrictions on the denominator

Simplify the rational expression

#### Implement

 $\frac{2x(x+3)(3x+11)}{2x(4x)(2x-5)(x+3)}$ 

$$\frac{(x+3)(3x+11)}{4x(2x-5)(x+3)}$$

$$\frac{3x+11}{4x(2x-5)},$$

#### **Explain**

- ▶ Solve for the *x*-values in the denominator to name the restrictions
- Simplify the monomial  $\frac{2x}{8x^2}$
- ▶ Simplify *identical* binomials
- ▶ Write the answer with the restrictions



To continue, return to the Online Lesson.

### h Simplifying Rational Expressions with Multiplication and Division (cont.)

- (b) Fill in the notes as you watch the video in the Online Lesson.
- When a division symbol is present in a rational expression:
  - Find the \_\_\_\_\_ of the fraction after the symbol (the divisor).
  - And change the operation to \_\_\_\_\_\_\_

■ Therefore, when rational expressions are \_\_\_\_\_\_\_, state the restrictions for the \_\_\_\_\_\_.

Which means, you must determine all the values that are excluded from the numerator and the denominator of the \_\_\_\_\_\_.

### Example 4

(b) Complete the example as you watch the video in the Online Lesson.

Simplify the expression. State the restrictions on the denominator.

$$\frac{2x+10}{12x-28} \div \frac{x^2-25}{6x-14}$$

**Implement** 

$$\frac{2(x+5)}{4(3x-7)} \div \frac{(x-5)(x+5)}{2(3x-7)}$$

divisor (numerator and denominator)

$$3x-7=0$$
  $x-5=0$   $x+5=0$   $3x-7=0$ 

#### **Explain**

- ▶ Factor all parts of the expression
- ► State the restrictions for first denominator and the *entire* divisor
- Write the fraction after the division symbol as its reciprocal.
- ► Write as one large expression Simplify like terms and expressions
- ▶ Write answer with the restrictions

#### **Example 5**

(b) Complete the example as you watch the video in the Online Lesson.

#### Simplify. State when the given expression is undefined.

$$\frac{x^2 + 6x - 16}{2x^2 + 5x - 3} \div \frac{3x^2 - 5x - 2}{4x^2 - 36} \cdot \frac{x^3 + 3x^2 - 10x}{6x^2 + 12x - 90}$$

#### **Implement**

#### **Explain**

- ► Factor all parts of the expression
- ► Factor completely
- State the restrictions First and last denominator, middle expression: numerator and denominator
- Write the fraction after the division symbol as its reciprocal
- Write as one large expression
  Simplify like terms
  and expressions

oxedge Checkpoint: Simplifying Rational Expressions with Multiplication and Division Simplify. State when the given expression is undefined.

$$\frac{x^2 - 1}{7x^2 + 5x - 2} \cdot \frac{21x^2 - 6x}{10x^2 - 7x - 3}$$





## Practice 1

Complete problems on a separate sheet of paper.

State the restrictions on the denominator.

$$1) \quad \frac{8x^2}{x^3 + 2x^2 + x}$$

$$2) \quad \frac{r+1}{2r^3 + 3r^2 - 2r - 3}$$

$$3) \quad \frac{4p+5}{4p^2-16}$$

**4)** 
$$\frac{x+3}{x^4-81}$$

Simplify. State the restrictions on the denominator.

**5)** 
$$\frac{6x^2 + x - 1}{2x^2 - 13x + 6} \cdot \frac{16x^2 + 40x + 25}{12x^2 + 11x - 5}$$

**6)** 
$$\frac{x^2 + 7x + 12}{x^2 - x - 12} \div \frac{x^2 + 3x + 2}{x^2 - 9}$$

Simplify.

7) 
$$\frac{2z^2 + 13z + 15}{z^2 + 4z + 4} \cdot \frac{2z^2 + z - 6}{z^2 - 25}$$

8) 
$$\frac{b^2-9b+20}{4b^2+6b} \div \frac{2b^2-32}{4b^2+12b+9} \cdot \frac{8b+8}{b^2-3b-4}$$

9) 
$$\frac{g^4 - 13g^2 + 36}{g^2 - 10g + 25} \div \frac{g^2 - 5g + 6}{g^2 - g - 30} \cdot \frac{g^2 - 25}{g^2 + 7g + 10}$$
 10)  $\frac{7x}{x^2 - 4x - 21} \div \frac{x^3 - 49x}{x^2 + 9x + 18}$ 

**10)** 
$$\frac{7x}{x^2 - 4x - 21} \div \frac{x^3 - 49x}{x^2 + 9x + 18}$$

11) 
$$\frac{14c^4 - c^3 - 3c^2}{9c^3 - 25c} \cdot \frac{9c^2 - 9c - 10}{49c^2 + 42c + 9}$$

**12)** 
$$\frac{4y^2 + 4y - 35}{3y^2 - 20y - 7} \cdot \frac{y^2 - 11y + 28}{6y^2 + 19y - 7} \div \frac{y^2 - 16}{9y^2 - 1}$$

Find the missing side.

- 13) The area of a triangle is  $\frac{2x+1}{5x+2}$  m<sup>2</sup> and the area of a rectangle is  $\frac{4x^2+2x}{5}$  m<sup>2</sup>. Find the probability of the triangle.  $P(\text{triangle}) = \frac{\text{area of triangle}}{\text{area of rectangle}}$  What is the probability of the triangle if x = 1?
- Find the height of a trapezoid when the sum of the bases is  $\frac{4x+5}{x^2+9x-10}$  inches and the area is  $\frac{12x^2+19x+5}{x^2-100}$  inches<sup>2</sup>. Then explain if x=1 is a possible solution.
- 15) All numbers that make the denominator of a rational expression equal to \_\_\_\_\_ must be excluded as possible values.
- **16)** A rational expression can also be described as the \_\_\_\_\_\_ of two polynomials, since the fraction bar represents a division problem.



## 

### **Show What You Know**

The surface area and volume of a cylinder are used in the efficiency ratio  $\frac{SA}{V}$ .

$$SA = 2\pi rh + 2\pi r^2 \quad V = \pi r^2 h$$

A) Simplify the efficiency ratio for a cylinder.

**B)** The restriction for the radius r is  $x \neq \frac{1}{2}$ . The restriction for the height h of the cylinder is x = 3. Write a binomial expression for the radius and height of the cylinder.

C) Use parts A and B to write a simplified expression for the efficiency ratio  $\frac{SA}{V}$ .

### ு∥⊩Say What You Know

In your own words, talk about what you have learned using the objectives for this part of the lesson and your work on this page.

## Practice 2

Complete problems on a separate sheet of paper.

Simplify. State when the given expression is undefined.

1) 
$$\frac{3x}{x^2 + 7x + 10} \cdot \frac{x+2}{x^2 + 5x}$$

2) 
$$\frac{b^2 - 9b + 14}{3b + 18} \div \frac{3b^2 - 15b - 72}{6b + 36} \cdot \frac{b^2 + 6b + 9}{b^2 - b - 2}$$

3) 
$$\frac{2n-3}{n^3+4n^2+n+4} \div \frac{4n^2-9}{4n^2+22n+24}$$

**4)** 
$$\frac{k^2 - j^2}{k - j} \div \frac{k^4 - j^4}{k^3 - j^3}$$

Simplify.

$$5) \frac{\frac{x^2 + 4x - 5}{x^2 - 3x - 18}}{\frac{x^2 + 6x + 5}{x^2 - 4x - 12}}$$

**6)** 
$$\frac{3x^2 + x - 14}{2x^2 + 13x + 20} \cdot \frac{3x^2 - 48}{10x^2 - 24x + 8} \div \frac{18x^2 - 98}{10x^2 + 21x - 10}$$

7) 
$$\frac{8h^3 - 1}{6h^2 + 6h} \cdot \frac{6h^4 + 12h^3 + 6h^2}{4h^2 - 1}$$

8) 
$$\frac{r^2+3r-10}{r^2+6r+9} \div \frac{r^2+r-20}{20r^2-11r-3} \div \frac{5r^2-9r-2}{r^2-r-12}$$

9) 
$$\frac{4w^2}{w^2 + 8w + 16} \div \frac{16}{5w^2 - 80}$$

- 11) Find the length of a rectangle when the width is  $\frac{x^2 + 2x + 1}{x^2 + 8x + 15}$  meters and the area is  $\frac{x^2 - 1}{x^2 + 10x + 25}$  meters<sup>2</sup>. Explain whether x = 1 is a possible solution.
- 12) Find the area of a triangle with base  $\frac{2x^2+5x+3}{x-3}$  feet and the height of  $\frac{x^2-x-6}{2}$  feet. Then determine if a length of x = 3 is possible. Explain your reasoning.
- **13)** When rational expressions are divided, the \_\_\_\_\_\_ of the term \_\_\_\_\_ the division symbol must be written.
- 14) When a rational expression is \_\_\_\_\_\_, the denominator has a value of zero.

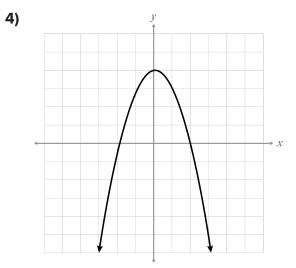
## © Targeted Review

Complete items on a separate sheet of paper.

1) Solve: 
$$\frac{4x+1}{6} = \frac{3x-4}{8}$$

2) Solve: 
$$\frac{3}{4} \left( 3x + \frac{1}{2} \right) = -1$$

Name the domain and range.



**5)** Solve: 
$$\frac{x-8}{5} = \frac{2x+5}{10}$$

**6)** Factor: 
$$x^3 + 64$$

Divide.

7) 
$$(3x^3 + x^2 + 2x + 10)(x^2 - 2x + 1)^{-1}$$

**8)** 
$$(4x^3 - 14x^2 - 2) \div (2x - 1)$$

#### **Multiple Choice**

\_\_\_\_\_ **9)** Describe the transformation of the parabola  $y = (x-2)^2 + 5$  as compared to the parent function.

- A) The parabola shifts 2 spaces right and 5 spaces up.
- **B)** The parabola shifts 2 spaces left and 5 spaces up.
- C) The parabola shifts 2 spaces right and 5 spaces down.
- **D)** The parabola shifts 2 spaces left and 5 spaces down.

\_\_\_\_\_ **10)** Solve:  $\frac{2}{3}n - 5 = \frac{5}{4}$ 

**A)** 
$$\frac{8}{75}$$

**B)** 
$$\frac{75}{8}$$

**c)** 
$$-\frac{45}{8}$$

**D)** 
$$\frac{55}{8}$$

#### Use the graph to answer problems 11–12.

**11)** Select all vertices of the system:

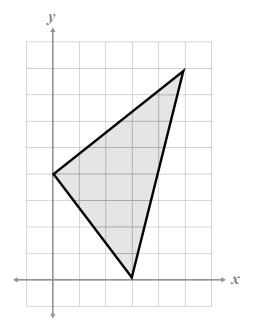


$$y \ge 4x - 12$$

$$y \ge -\frac{4}{5}x + 4$$

- $\square (4,0)$
- $\square$  (3,0)

- $\square$  (8, 5)



- **12)** Using the graph from problem 11, select the minimum and maximum values using the objective function f(x,y) = x 2y.
  - □ 4
  - □ 3
  - □ -8
  - □ -11
  - □ -2

Problem	1	2	3	4	5	6	7	8	9	10	11	12
Origin	A1	A1	A1	A1	A1	L03	L05	L06	A1	A1	L01	L01

L = Lesson in this level, A1 = Algebra 1: Principles of Secondary Mathematics, FD = Foundational Knowledge

