

Lesson 6

Synthetic Division

NAME:



Start by navigating to the Online Lesson for instructions.

Objectives

- ✓ Use synthetic division with polynomial expressions.
- ✓ Evaluate polynomials using synthetic substitution.
- ✓ Use the Remainder Theorem to prove polynomial division is correct.

Why?

Using the most efficient method to solve problems is a foundation of mathematics. In that vein, synthetic division is a more efficient way to work with divisors that are linear binomials.



Warm Up

Write the expression in standard form, including all terms from degree n to degree 0, the constant. Name all of the coefficients and constants.

1) $7x^2 - x^3 + 5x^4 + 8$

2) $12 - 2x - 7x^3 + 10x$

Solve using mental math.

3) $3x - 1 = 0$

4) $5x + 3 = 0$



To continue, return to the Online Lesson.

🔍 Explore

🔍 Synthetic Division with Integers

🎧 Fill in the notes as you watch the video in the Online Lesson.

■ Synthetic division is a shorthand for dividing a _____ dividend by a _____ divisor.

■ Here are the steps for solving with synthetic division:

- 1) Set the linear divisor equal to _____ and solve.
- 2) Place the _____ of the linear divisor in the top left corner.
- 3) List the _____ and constant of the polynomial dividend (remembering to use 0 for any missing degree).

$(2x^3 - 5x^2 - x + 3) \div (x + 3)$
 $x + 3 = 0$
 $x = -3$

-3	2	-5	-1	3
	2	-11	32	-93

↓ ↗ ↗ ↗
multiply +(-6) +(33) +(-96)

Solution: $2x^2 - 11x + 32 + \frac{-93}{x+3}$

- 4) Bring down the _____ coefficient under the line.
- 5) Multiply the _____ by the first coefficient and place it under the second coefficient.
- 6) _____ the column vertically.
- 7) Repeat until _____ values are left.
- 8) The value in the bottom right corner represents the _____. If the remainder is zero, then the linear divisor is a factor of the polynomial.
- 9) Write the solution as:

Example 1

▶ Complete the example as you watch the video in the Online Lesson.

Simplify using synthetic division.

$$(5x^4 + 12x^3 - 6x^2 - 14x - 8) \div (x + 2)$$

Implement

$$x + 2 = 0$$

$$x = -2$$

-2	5	12	-6	-14	-8

$$5x^3 + 2x^2 - 10x + 6 - \frac{20}{x + 2}$$

Explain

- 1) Set the linear divisor equal to zero and solve
- 2) Place the zero of the linear divisor in the top left corner
- 3) List the coefficients and constant of the polynomial dividend
- 4) Write the first coefficient under the line
- 5) Multiply the constant r by the first coefficient and place it under the second coefficient
- 6) Add the column vertically
- 7) Repeat until no values are left
- 8) Determine if there is a remainder or if the divisor is a factor
- 9) Write the quotient with the remainder

Example 2

▶ Complete the example as you watch the video in the Online Lesson.

Simplify using synthetic division.

$$\frac{9x^3 - 36x^2 - 4x + 16}{x - 4} \quad x - 4 = 0$$

$$x = 4$$

4	9	-36	-4	16

$$9x^2 - 4 \text{ or } (3x - 2)(3x + 2)$$

Remember to write the final expression in standard form, factoring completely when possible.

Example 3

▶ Complete the example as you watch the video in the Online Lesson.

Find the quotient using synthetic division.

$$(3p^3 - 8p + 14) \div (p - 6)$$

Checkpoint: Synthetic Division with Integers

Find the quotient using synthetic division.

$$(x^4 - 6x^3 + 21x + 31)(x - 4)^{-1}$$



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📺 Synthetic Division with Non-Integers

🎥 Fill in the notes as you watch the video in the Online Lesson.

- When the divisor has a coefficient other than _____, you need to complete an additional step to use synthetic division.
- First, divide **every** term of the dividend and the divisor by the _____ of the divisor.
- Then follow the steps of _____ division.

Example 4

▶ Complete the example as you watch the video in the Online Lesson.

Simplify using synthetic division.

$$\frac{18x^4 - 15x^3 + 30x^2 - 21x + 1}{3x - 1}$$

Implement

$$\frac{(18x^4 - 15x^3 + 30x^2 - 21x + 1) \div 3}{(3x - 1) \div 3}$$

$$\frac{6x^4 - 5x^3 + 10x^2 - 7x + \frac{1}{3}}{x - \frac{1}{3}}$$

$$x - \frac{1}{3} = 0$$

$$x = \frac{1}{3}$$

Explain

- ▶ Divide every term by the coefficient of the linear divisor (In this example, the coefficient of $3x$ is 3.)
- ▶ Simplify each term
- ▶ Find the zero of the divisor
- ▶ Write the remainder using the given divisor (Multiply all terms of the remainder by the number you divided out.)
- ▶ Write the quotient with the remainder

Example 5

 Complete the example as you watch the video in the Online Lesson.

Find the quotient using synthetic division.

$$(32x^3 + 8x^2 - 4x + 5) \div (4x + 3)$$

Checkpoint: Synthetic Division with Non-Integers

Simplify using synthetic division.

$$\left(2x^3 - 11x^2 + 13x - \frac{3}{2}\right)(2x - 3)^{-1}$$



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Remainder Theorem for Polynomials

 Fill in the notes as you watch the video in the Online Lesson.

- The Remainder Theorem: If the polynomial $P(x)$ is divided by $(x - k)$, then the _____ is equal to the value, $P(k)$.
- Rather than using _____ for all values of x , use synthetic division to find the value of the remainder.
- Synthetic division is more _____ than substitution because you do not need to raise any term to a power.

Example 6

 Complete the example as you watch the video in the Online Lesson.

Determine if $P(5)$ and $P(-5)$ are factors of $x^3 + x^2 - 17x + 15$.

$P(5)$

$P(-5)$

$$P(5) = 80$$

OR

$$P(5) = (5)^3 + (5)^2 - 17(5) + 15 = 80$$

_____ because the remainder is not zero.

The value -5 is a root of the polynomial because _____; therefore,

$(x + 5)$ is _____.

Example 7

 Complete the example as you watch the video in the Online Lesson.

Find the missing value when $P(n) = -5$ for $P(x) = x^2 - 6x + 3$.

Plan

Substitute the values into the synthetic division frame

Complete synthetic division

Solve for n

 Checkpoint: Remainder Theorem for Polynomials

Find the value of n for $P(6) = 5$ for $P(x) = 3x^3 - 19x^2 + 10x + n$.

6	3	-19	10	n
				5



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 **Practice 1**

Complete problems on a separate sheet of paper.

Find the quotient using synthetic division.

1) $(5a^3 + 14a^2 - 7a + 9) \div (a + 4)$

2) $\frac{x^2 - 8x + 12}{x - 3}$

3) $2x - 1 \overline{)4x^3 - 6x^2 + 10x + 2}$

4) $(x^3 - 3x^2 + 5x - 6) \div (x - 2)$

5) $\frac{9x^4 + 6x^3 - 12x^2 - 8x + 4}{3x + 2}$

6) $(x^4 - 13x^2 + 36) \div (x + 3)$

7) $\frac{b^3 - 8}{b - 2}$

8) $x - 6 \overline{)x^4 - 12x^2 - 8x - 76}$

Use the Remainder Theorem to determine $P(k)$.

9) $P(x) = 2x^3 + x^2 - 4x + 3; P(-1)$

10) $P(x) = x^5 - 4x^3 + x^2 - 5; P(5)$

11) $P(x) = 9x^3 + 13x^2 - 6x + 8; P(-2)$

12) $P(x) = 3x^2 - 2x - 1; P\left(-\frac{1}{3}\right)$

Find the missing value.

13) $P(3) = 2; P(x) = 2x^3 - 3x^2 - 5x + n$

14) $P(-1) = 5; P(x) = x^3 + nx - 8$



To continue, return to the Online Lesson.

 **Mastery Check** **Show What You Know**

Use synthetic division to find the missing value.

A) Find the value of n so that the remainder is 8 when $(x^3 + 5x^2 + 12x + n) \div (x + 4)$

B) Find the value of n so that the remainder is 3 when $(4x^2 + nx - 7)(x - 1)^{-1}$

C) Find the value of n so that the remainder is -17 when $(x^2 - 5x - 11) \div (x - n)$

 **Say What You Know**

In your own words, talk about what you have learned using the objectives for this part of the lesson and your work on this page.



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 Practice 2

Complete problems on a separate sheet of paper.

Find the quotient using synthetic division.

1) $(3y^3 + 17y^2 + 22y + 8) \div (y + 4)$

2) $(9x^2 - 7x - 40) \div (x + 5)$

3) $(x^4 - 3x^2 + x - 5)(x + 1)^{-1}$

4) $2y - 1 \overline{)4y^2 - 8y + 3}$

5)
$$\frac{2y^3 - 5y - 2}{y + 1}$$

6) $(2x^3 + 13x^2 - x - 110)\left(x - \frac{5}{2}\right)^{-1}$

7)
$$\frac{3x^4 - 2x^2 + 5x - 2}{x - 3}$$

8) $(3x^2 + 4x - 9) \div (3x + 1)$

Use the Remainder Theorem to determine $P(k)$.

9) $P(x) = 4x^3 + 5x^2 - 3; P(-3)$

10) $P(x) = x^5 + 5x^4 - 10x^3 + 10x^2 - 5x - 1; P(1)$

11) $P(x) = -2x^2 + 5x + 3; P\left(-\frac{1}{2}\right)$

12) Find the remainder using $(x + 7)$ for $P(x) = 9x^4 + 62x^3 - x^2 + 22x - 100$

Find the missing value.

13) $P(2) = -1; P(x) = x^4 - 3x^2 + 4x + n$

14) $P(-4) = 6; P(x) = 5x^3 + 21x^2 - nx + 7$



To continue, return to the Online Lesson.