Lesson 4
Polynomial Identities

NAME:

Start by navigating to the Online Lesson for instructions.

Objectives

- Determine if a polynomial identity exists.

- Determine the value of an unknown to make a polynomial expression or equation true.

Why?

Being able to prove that both sides of an equation are equal strengthens mathematical understanding and use of the algebraic properties.

☼ Warm Up

1) Multiply.
$$(2x - 3)^2$$

2) Multiply.
$$(4x - 3)(x - 3)$$

3) Solve.
$$7x - \frac{1}{2}(2x - 6) = 6x + 3$$

To continue, return to the Online Lesson.

👓 Explore

👓 Polynomial Identities

▷ *Fill in the notes as you watch the video in the Online Lesson.*

- An _____ is an equation that will be true for any value of variable(s) in

  the equation.

- This means that the _____ will be true for _____ that

  replaces the variable.

- Recognizing a polynomial identity can help simplify _____ of

  an equation.

- This is because when the identity exists, the sides of the equation are

  _____ because they are equal.

- Some polynomial identities that you are already familiar with are:

  - Difference of two squares: _____

  - Perfect square trinomials:

    _____

    _____

  - Sum of cubes: _____

  - Difference of cubes: _____

Example 1

▷ *Complete the example as you watch the video in the Online Lesson.*

Determine if a polynomial identity exists. Explain.

$$\left(x^2+y^2\right)^2=\left(x^2-y^2\right)^2+\left(2xy\right)^2$$

Implement   **Explain**

Left side	Right side
$\left(x^2+y^2\right)^2$	$\left(x^2-y^2\right)^2+\left(2xy\right)^2$
$\left(x^2+y^2\right)\left(x^2+y^2\right)$	
$x^4+x^2y^2+x^2y^2+y^4$	
$x^4+2x^2y^2+y^4$	

▶ Write each side of the problem

▶ Expand expressions

▶ Distribute

▶ Combine like terms

When written in the same form, the left and right sides of the equation have identical terms, which

means _____.

Example 2

▷ *Complete the example as you watch the video in the Online Lesson.*

Determine if a polynomial identity exists. Explain.

$$(x-5)^2 = (x-4)(x+4) + (x-3)(x+3)$$

Left side	Right side
$(x-5)^2$	$(x-4)(x+4) + (x-3)(x+3)$

Example 3

▷ *Complete the example as you watch the video in the Online Lesson.*

Determine if a polynomial identity exists. Explain.

$$(a-b)^3 = a^3 - b^3 - 3ab(a-b)$$

Left side	Right side
$(a-b)^3$	$a^3 - b^3 - 3ab(a-b)$

☑ **Checkpoint: Polynomial Identities**

 Determine if a polynomial identity exists. Explain.

$(4x)^2 + 24x + 9 = (4x + 3)^2$

🖥 **To continue, return to the Online Lesson.**

🔭 Finding Unknowns in Polynomial Equations
   ▷ *Fill in the notes as you watch the video in the Online Lesson.*

- Finding an _____ coefficient or constant in a polynomial equation combines

  the operations with polynomials and solving equations.

- When comparing like terms in an equation, it is important to look at

  _____ of the equation.

- To check your answer, _____ the values back into the given equation.

  Then make sure both sides of the equation are _____.

Example 4

▷ *Complete the example as you watch the video in the Online Lesson.*

Find the missing values, M and N.

$$(2x + M)(Nx - 1) = 8x^2 + 10x - 3$$

Implement

$$2Nx^2 - 2x + MNx - M = 8x^2 + 10x - 3$$

Explain

▶ Distribute on the left side of the equation

▶ Solve for N using 2nd degree terms

▶ Solve for M using constant terms

▶ Substitute M and N into the given problem

▶ Check using mental math

Example 5

▷ *Complete the example as you watch the video in the Online Lesson.*

Find the missing value, P.

$$(x^2 - Px + 7) - (6x^2 - 5x + 3) = -5x^2 - 3x + 4$$

$$x^2 - Px + 7 - 6x^2 + 5x - 3 = -5x^2 - 3x + 4$$

☑ **Checkpoint: Finding Unknowns in Polynomial Equations**

Find the missing value when P is a whole number.

$$(Px - 2)(8x + P) = 24x^2 - 7x - 6$$

To continue, return to the Online Lesson.

✎ Practice 1

Complete problems on a separate sheet of paper.

Determine if the given expressions will form an identity.

1) $(a+b)^3$ and $a(a^2+ab)+2ab(a+b)+b^2(a+b)$

Left side: $(a+b)^3$ Right side: $a(a^2+ab)+2ab(a+b)+b^2(a+b)$

2) $-6x^2-2(25x+28)$ and $3x(-2x-14)-8(x+7)$

3) $\left(\dfrac{1}{2}n-7\right)(n+8)$ and $\dfrac{1}{2}n(n+26)-56(n+1)$

4) $(3g)^2+10(3g+2)+1$ and $(3g)^2+10(3g+3)-9$

5) $x(5x+8)(5x-8)$ and $x(5x)^2-8^2$

6) $(2ab)^2+4(5ab)+(5ab)^2$ and $4ab(ab+4)+ab(25ab+4)$

Find the missing value(s) in the given equation.

7) $(Rx+4)(2x-R)=6x^2-x-12$

8) $(Bx^2-Px+8)+(4x^2+1x-15)=10x^2-14x-7$

9) $3x(Wx+Q)+8(Wx+Q)=12x^2+35x+8$

10) $(Ax+3)^2=25x^2+30x+9$

11) $(5x^2-Gx+1)-(9x^2-Mx+2)=-Gx^2+3x-1$

12) $(Rx+K)(Rx-K)=36x^2-81$; Assume R and K are whole numbers.

⌨ **To continue, return to the Online Lesson.**

☑ Mastery Check

✑ Show What You Know

A)   A student was asked to show their work to prove the equation represented a polynomial identity. Find their error and correct it to prove the identity exists. Indicate the line(s) in which the error occurs.

$$x^4 - y^4 = (x^2 + y^2)(x + y)(x - y)$$

Simplifying the right side

$$(x^2 + y^2)(x + y)(x - y)$$

$$x^3 + x^2y + x^3 - x^2y$$

$$\underline{\hspace{3cm}} + xy^2 + y^3 + xy^2 - y^3$$

$$2x^3 + 2xy^2$$

$$x^4 - y^4 \neq 2x^3 + 2xy^2$$

Student Response

I distributed the 1st binomial across the 2nd and 3rd binomial.

This is NOT an identity because the left and right sides of the equation are NOT equal.

B) Determine the non-zero value of Q that will form a polynomial identity. Then rewrite the polynomial using the value you found for Q.

$$(Qx + 1)^2 = Qx^2 + (Qx + 2)(5x + 1) - 4x - 1$$

·ıl|ıl· Say What You Know

In your own words, talk about what you have learned using the objectives for this part of the lesson and your work on this page.

To continue, return to the Online Lesson.

✎ Practice 2

Complete problems on a separate sheet of paper.

Determine if the given expressions will form an identity.

1) $(2z + 1)^2 - 2(z - 1)$ and $2z(2z + 1) - 1$

2) $(x + 2)(x + 5) - (x - 7)(x - 3)$ and $(x - 1)((x + 7) - (x - 10)) + 6$

3) $2x^2y^2 - 4x^2 - 9y^2$ and $x^2(y + 2)(y - 2) + y^2(x + 3)(x - 3)$

4) $a(b + 3) - b(b + 3)$ and $b(a - b) + 3(a - b)$

5) $(a + 2)^3$ and $(a + 2)(a^2 - 2a + 4) + 6a(a + 2)$

6) $(x^2 - 4)(x^2 - 9) = (x + 2)(x - 3)(x^2 - x - 6)$

Find the missing value(s) of the given equation.

7) $(Bx - 5)(2x - C) = 6x^2 - 11x - 35$

8) $((Ax)^3 + B^3) = (5x + 4)(25x^2 - 20x + 16)$

9) $(Qx^2 + 3x + Y) - (4x^2 - 5x - 1) = 9x^2 + 8x + 7$

10) $(Gx - 4)(x + 5) - B = 3x^2 + 11x + 8$

11) $(Wx + 1)(2x - 5) + (3x - R)(x + 1) = 7x^2 - 7x - 7$

12) $5x^2 + Hx - 2 - (Vx^2 - Hx - 3) = 14x + 1$

🖥 **To continue, return to the Online Lesson.**