Lesson 41
Common Logs

NAME:

Start by navigating to the Online Lesson for instructions.

Objectives

- Approximate the value of logarithm expressions.
- Solve logarithmic equations with common logs.

Why?

Not all logarithmic expressions and equations are written with the same base. And until now, you were not able to solve them. Using the Change of Base Rule, you can solve logarithmic problems with more than one base.

Warm Up

Rewrite the expression with fractional exponents and prime factors. Do not simplify.

1) $\sqrt[5]{6x^4y^8}$

2) $\sqrt[3]{27x^4y^2}$

Rewrite the equation in logarithmic form.

3) $81^{\frac{3}{4}} = 27$

4) $5^x = 21$

To continue, return to the Online Lesson.

Explore

Common Logs

Fill in the notes as you watch the video in the Online Lesson.

- The common logarithm has a ____________________. Recall that when the base is not written for a logarithm, it is understood to be 10.
- It can be written as $\log_{10} x$ or, most commonly, _______________.
- You can ___________________ the approximate value of a log without technology when you ask yourself:
  - "Ten to what power is ________________ to the given argument?"

    or
  - "________________ which two whole numbers is the argument?"

Example 1

Complete the example as you watch the video in the Online Lesson.

The approximate value of $\log 72$ is between which two integers? To which integer is the value closest? Explain. Do not use technology.

Plan

Name the powers of 10 the log is between
Decide if the log is closer to the lower or higher power of 10
Explain

Implement

Explain

- The approximate value of $\log 72$ is between _______________.
- It is closer to _____________________________ ______________________________________.

Example 2

Complete the example as you watch the video in the Online Lesson.

The approximate value of $\log \frac{1}{450}$ is between which two integers? Explain.

For $\log \frac{1}{450}$, x is between ____________ because $\frac{1}{450}$ is between $\frac{1}{100}$ and $\frac{1}{1000}$.

Checkpoint: Common Logs

The approximate value of $\log 1331$ is between which two integers? To which integer is the value closest? Explain.

To continue, return to the Online Lesson.

Approximate Logarithms with Technology

Fill in the notes as you watch the video in the Online Lesson.

- Use technology to ______________________ the value of logs.
- If needed, use ________________________________ to expand log expressions in order to approximate their value.
- To avoid errors, only round as the ____________________ step.

Example 3

Complete the example as you watch the video in the Online Lesson.

Write $\log 24$ in terms of X and Y when $\log 2 = X$ and $\log 3 = Y$.

Plan

Express the argument as its prime factorization
Expand with the rules for logs
Substitute in the given values

Example 4

Complete the example as you watch the video in the Online Lesson.

A) Write $\log 36$ in terms of X and Y when $\log 2 = X$ and $\log 3 = Y$.

B) Calculate the approximate value of $\log 36$ when $\log 2 \approx 0.301$ and $\log 3 \approx 0.4771$.

Plan

Express the argument as its prime factorization
Expand with the rules for logs
Substitute in the given values
Check

Checkpoint: Approximate Logarithms with Technology

A) **Write $\log 63$ in terms of P and Q when $\log 3 = P$ and $\log 7 = Q$.**

B) **Calculate the approximate value of $\log 63$ when $\log 3 \approx 0.4771$ and $\log 7 \approx 0.8451$.**

To continue, return to the Online Lesson.

Solve with Common Logs

Fill in the notes as you watch the video in the Online Lesson.

The Equality Rule

When a is a positive number not equal to one, $\log_a x = \log_a y$ if and only if $x = y$.

- The Equality Rule for logarithms allows you to rewrite ____________________ equations in terms of ____________________.
- Because of the phrase ________________________ the rule is true forward and backward.
- This rule determines the ______________ of a logarithm when x is ____________________ to a rational number, $\mathbb{Q}$.
- You can write the ____________ answer in terms of logarithms and, if necessary, _______________________ the value with technology.

Example 5

Ⓟ *Complete the example as you watch the video in the Online Lesson.*

Solve. Write the answer as a logarithm and as a number to four decimal places.

$5^x = 12$

Plan

Equality rule for logs
Isolate x
Approximate with technology

$5^x = 12$

$\log 5^x = \log 12$

Example 6

Ⓟ *Complete the example as you watch the video in the Online Lesson.*

Solve. Write the answer with common logarithms.

$3^{\frac{x}{5}} = 0.4$

Explain

- Equality rule
- Power rule
- Quotient rule
- Solve

Example 7

Complete the example as you watch the video in the Online Lesson.

Solve. Write the answer as a logarithm and as a number to four decimal places.

$3^{x-2} = 88$

Checkpoint: Solve with Common Logs

Solve. Write the answer as a logarithm and as a number to four decimal places.

$300^{4x} = 10$

To continue, return to the Online Lesson.

Change of Base Rule

Fill in the notes as you watch the video in the Online Lesson.

Change of Base Rule

The variables a, b, and X are positive real numbers, and $a \neq 1$, $b \neq 1$.

Log of any base: $\log_b X = \dfrac{\log_a X}{\log_a b}$

Common log: $\log_b X = \dfrac{\log X}{\log b}$

- The Change of Base Rule is used to ______________________________, so they have the same base.
- Use the rule when you:
  - have an expression or equation with _________________ bases.
  - need a _________________________ to approximate logs with technology.

Example 8

Complete the example as you watch the video in the Online Lesson.

Write the expression with common logs using the Change of Base Rule.

A) $\log_m (8)^3$

Explain
- Change of Base rule
- Power rule

B) $\log_6 2x + \log_5 3$

Explain
- Change of Base rule
- Product rule

Checkpoint: Change of Base Rule

Write with common logs using the Change of Base Rule.

$\log_v xy$

To continue, return to the Online Lesson.

Practice 1

Complete problems on a separate sheet of paper.

The approximate value of the log is between which two integers?

1) $\log 154$
2) $\log 4$
3) $\log 6438$
4) $\log \frac{1}{25}$

For problems 5–8:

A) **Write the log in terms of X and Y when $\log 2 = X$ and $\log 3 = Y$.**
B) **Calculate the approximate value of the log when $\log 2 \approx 0.301$ and $\log 3 \approx 0.4771$.**

5) $\log 108$
6) $\log 96$
7) $\log 162$
8) $\log 144$

Solve. Write the answer as a logarithm and as a number to four decimal places.

9) $6^x = 30$
10) $81^{x-3} = 15$
11) $25^{3x+1} = 520$
12) $7^{2x} = 490$
13) $2^x = 56$
14) $32^{x+1} = 12$

Write with common logs using the Change of Base Rule.

15) $\log_5 9^4$
16) $\log_3 11x$
17) $\log_2 6 + \log_3 4$

Solve. Write as a common log.

18) $7^{\frac{x+2}{3}} = 84$
19) $2^{4-x} = 13$
20) $15^{\frac{x}{7}} = 25$

To continue, return to the Online Lesson.

Mastery Check

Show What You Know

A) If $\log_a b = y$, prove $\log_a b = \dfrac{\log b}{\log a}$.

B) Explain the steps you used in part A. Write them beside each step.

C) Calculate the value of $\dfrac{\log b}{\log a}$ when $a = 5$ and $b = 19$.

Say What You Know

In your own words, talk about what you have learned using the objectives for this lesson and your work on this page.

To continue, return to the Online Lesson.

Practice 2

Complete problems on a separate sheet of paper.

The approximate value of the log is between which two integers?

1) $\log 890$

2) $\log \frac{7}{500}$

3) $\log 0.2$

4) $\log 83$

For problems 5–8:

A) **Write the log in terms of X and Y when $\log 2 = X$ and $\log 5 = Y$.**

B) **Calculate the approximate value of the log when $\log 2 \approx 0.301$ and $\log 5 \approx 0.6990$.**

5) $\log 20$

6) $\log 200$

7) $\log 250$

8) $\log \frac{8}{25}$

Solve. Write the answer as a logarithm and as a number to four decimal places.

9) $17^{x+1} = 34$

10) $2^x = 99$

11) $6^{\frac{x}{4}+2} = 51$

12) $54^{2x} = 100$

13) $3^{1-x} = 10$

14) $7^{5x+4} = 56$

Write with common logs using the Change of Base Rule.

15) $\log_m 8x$

16) $\log_{25} 50$

17) $\log_b r$

Solve. Write as a common log.

18) $13^{\frac{x}{2}} = 7$

19) $8^{\frac{2x}{3}} = 100$

20) $9^x = 29$

To continue, return to the Online Lesson.

Targeted Review

Complete items on a separate sheet of paper.

1) Name the value of b for $h(x) = 12(0.45)^{-x}$ and state if this represents growth, decay, or neither.

2) Evaluate: $\log_{49}\frac{1}{343}$

3) Use the properties of logs to correct the right side of the equation.
$\log_b(2x)^3 = 3\log_b 2 + \log_b x$

4) Sketch the graph $y = \left(\frac{3}{4}\right)^{-x}$ using technology.

5) Graph the solution to $125^{2x-12} > 25^x$ on a number line.

6) Write the expression $\log_a 4 + 2\log_a x - \log_a y$ as a single logarithm.

Solve.

7) $4^{(2x-1)} = 32^x$

8) $\log_3(x-4) = 2$

Multiple Choice

_____ **9)** Rewrite the logarithmic equation $\log_{36}\frac{1}{216} = -\frac{3}{2}$ as an exponential equation.

A) $36^{-\frac{3}{2}} = \frac{1}{216}$

B) $36^{\frac{1}{216}} = -\frac{3}{2}$

C) $\frac{1}{216}^{-\frac{2}{3}} = 36$

D) $216^{\frac{2}{3}} = 36$

_____ **10)** Describe the transformation of the exponential function from $f(x)$ to $g(x)$ when $f(x) = (3)^x$ and $g(x) = (3)^{x-6} + 4$

A) $g(x)$ is translated right 4 units and up 6 units from $f(x)$

B) $g(x)$ is translated right 6 units and up 4 units from $f(x)$

C) $g(x)$ is translated left 4 units and up 6 units from $f(x)$

D) $g(x)$ is translated left 6 units and up 4 units from $f(x)$

Multiple Choice

_____ **11)** Solve: $\log_2(x+2)+\log_2(x+3)=2$

A) $1, 2$

B) -0.5

C) $-2, -1$

D) -1.5

_____ **12)** In the equation $\frac{3^{5x}}{3^7}=3^3$, which expression represents the left side of the equation written with all terms in the numerator?

A) 2

B) $3^{\frac{5}{7}x}$

C) 3^{5x+7}

D) 3^{5x-7}

Problem	1	2	3	4	5	6	7	8	9	10	11	12
Origin	L37	L39	L40	L37	L38	L40	L38	L40	L39	L37	L40	—

L = Lesson in this level, A1 = Algebra 1: Principles of Secondary Mathematics

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