Lesson 40

NAME:

Logarithmic Properties (Properties of Logs)

Start by navigating to the Online Lesson for instructions.

Objectives

- ✓ Rewrite logarithmic expressions using the logarithmic properties.

- ✓ Solve logarithmic equations.

Why?

In order to apply logarithms to real-life scenarios, you first need to learn the properties of logs and be able to expand and contract logarithmic expressions and equations.

☼ Warm Up

Solve for the variable.

1) $4^8 = 4^3 \cdot 4^n$

2) $3^{12} = \dfrac{3^z}{3^7}$

3) $\sqrt[3]{25} = 5^{\frac{x}{3}}$

4) $2^{24} = \left(\dfrac{2^9}{2^h}\right)^3$

To continue, return to the Online Lesson.

🔭 Explore

🔭 Properties of Logs

▶ *Fill in the notes as you watch the video in the Online Lesson.*

- The logarithmic properties are used to _____ and _____ logarithmic expressions and equations.

- The _____, _____, and _____ rules will be used along with the foundational properties of logs.

- The goal for expanding a logarithmic expression is to allow for algebraic _____ and _____.

Properties of Logs

For all rules of logs, the variables a, b, and c are positive real numbers, n is a real number, and $a \neq 1$.

Properties of Logs	Logarithm Rule(s)	Related Exponent Rule
Foundational Properties	If $a^0 = 1$, then $\log_a 1 = 0$ If $a^n = a^n$, then $\log_a\left(a^n\right) = n$ If $a^1 = a$, then $\log_a\left(a\right) = 1$	
	$\log_a \dfrac{b}{c} = \log_a b - \log_a c$	
	$\log_a bc = \log_a b + \log_a c$	
	$\log_a b^n = n \cdot \log_a b$	

- Rewrite radical expressions as _____ before expanding with the properties of logs.

> For this lesson, assume all variables and variable expressions are positive values.

Example 1

▷ *Complete the example as you watch the video in the Online Lesson.*

Expand. Name the property of logs used.

A) $\log_2 5xyz$

B) $\log_x\left(4x^2 + 12x + 9\right)$

C) $\log_b \dfrac{3x - 7}{2x + 1}$

Example 2

▷ *Complete the example as you watch the video in the Online Lesson.*

Expand. Explain by listing the logarithmic property used for each step.

$$\log_c \frac{3x\sqrt{yz}}{a^3 b^2}$$

Implement

$$\log_c \frac{3x(yz)^{\frac{1}{2}}}{a^3 b^2}$$

Explain

▶ Radical to fractional exponent

▶ Quotient rule

▶ Product

▶ Power

▶ Product

☑ **Checkpoint: Properties of Logs**

Expand.

$$\log_a \frac{4x^2}{y}$$

🖥 **To continue, return to the Online Lesson.**

🔭 Contract Expressions with the Properties of Logs

▶ *Fill in the notes as you watch the video in the Online Lesson.*

- The goal for contracting a logarithmic expression is to _____ all logs of the

  _____ into a _____.

- Work _____ from multiple terms to one term using the properties of

  logarithms.

- It is recommended that you contract logs using the properties in this order:

  - _____ Rule

  - _____ Rule

  - _____ Rule

- Instructions for contracting expressions might say "_____" or "_____

  _____."

In this lesson, all logs have the same base.

Example 3

▷ *Complete the example as you watch the video in the Online Lesson.*

Rewrite as a single log.

A) $\dfrac{3}{4}\log_b x + \dfrac{1}{4}\log_b y$

Implement	Explain
	▶ Power Rule
	▶ Product Rule
	▶ Fractional exponent to a radical

B) $\log_b 5 + 2\log_b x - \log_b y - 4\log_b z$

Implement	Explain
	▶ Power Rule
	▶ Product Rule
	▶ Quotient Rule

Example 4

▷ *Complete the example as you watch the video in the Online Lesson.*

Contract.

$2\log_9(x+4) + 3\log_9(x-2) - 4\log_9 x$

☑ **Checkpoint: Contract Expressions with Properties of Logs**

Rewrite as a single log.

$3\log_2 x - 6\log_2 y - 4\log_2 z$

🖥 **To continue, return to the Online Lesson.**

🔍 Solve Logs with Properties

▶ *Fill in the notes as you watch the video in the Online Lesson.*

- Before solving logarithmic equations, it is important to remember the names of each part of

  the equation and their restrictions.

  - Restrictions on the base: _____

  - Restrictions on the argument: _____

$$\log_b y = x$$

- Check your work to make sure an _____ is not included as part

  of the solution because the argument must be _____.

To solve logs of the same base when the equation is equal to...

a single term or number:	one or more logs:
1) Contract and/or isolate the _____. 2) Write as an _____ equation. 3) Solve. 4) Check.	1) Contract logs _____, as needed. 2) Set _____ equal to one another. 3) Solve. 4) Check.

Example 5

▷ *Complete the example as you watch the video in the Online Lesson.*

Solve.

$\log_2 x + \log_2(x - 7) = 3$

Plan

Contract logs
Write as an exponential equation
Solve
Check

Implement

Example 6

▷ *Complete the example as you watch the video in the Online Lesson.*

Solve.

$$\log_3(x+6) + \log_3(x+4) = \log_3 3$$

Plan

Contract logs
Set arguments equal to one another
Solve
Check

Implement

	Option 1	**Option 2**	**Option 3**

Example 7

▷ *Complete the example as you watch the video in the Online Lesson.*

Solve.

$$\log_2(x-3) + \log_2 x = \log_2(x+2) + 2$$

$$\log_2(x-3) + \log_2 x - \log_2(x+2) = 2$$

Example 8

▷ *Complete the example as you watch the video in the Online Lesson.*

Solve.

$$-4\log(2x+1)+3=-5$$

☑ **Checkpoint: Solve Logs with Properties**

Solve.

$$\log_4(x+2)-\log_4(x-1)=1$$

⌨ **To continue, return to the Online Lesson.**

✎ Practice 1

Complete problems on a separate sheet of paper.

Expand.

1) $\log_5 7x^2$

2) $\log \dfrac{4x}{(2x+1)(x-2)}$

3) $\log_x \dfrac{x^3 y^5}{ab^2}$

4) $\log_v \dfrac{\sqrt[4]{x^3 y}}{z^5}$

Contract. Rewrite as a single log.

5) $3\log_x(y+2) - \log_x(y-1)$

6) $2\log_7 x + 4\log_7(x-1) - 3\log_7(x+1)$

7) $\dfrac{1}{4}\log x + \dfrac{3}{4}\log y - 2\log z$

8) $\dfrac{2}{3}\log_2(x+1) - 3\log_2(2x+1)$

Solve.

9) $\log_3(x-4) + 5 = 7$

10) $\log_2(x-6) + \log_2(x-4) - \log_2 x = \log_2 4$

11) $\log_2 3 + \log_2 x = 4$

12) $\log_2(x+1) + \log_2(x-1) = 3$

13) $\log(x+4) = \log x + \log 4$

14) $\log(x+2) - \log(4x+3) = \log\left(\dfrac{1}{x}\right)$

15) $3\log_2(4x-1) = 15$

16) $\log_6(5x+1) = \log_6(2x+3) + \log_6 2$

17) $\log_2(x+2) - \log_2(x-5) = 3$

18) $\log_6(x+5) + \log_6 x = 2$

19) $\log_5(x-2) - \log_5(x+3) = \log_5(x+1) - \log_5(x-7)$

20) $\log_7 x + \log_7 4 = \log_7 2$

🖥 **To continue, return to the Online Lesson.**

🗒 Mastery Check

✍ Show What You Know

A) Name the rule or property used to complete each step.

Explain

Given: $\log_b \dfrac{\sqrt[3]{(x-y)}}{3x}$

Step 1: $\log_b \dfrac{(x-y)^{\frac{1}{3}}}{3x}$

Step 2: $\log_b(x-y)^{\frac{1}{3}} - \left(\log_b 3x\right)$

Step 3: $\dfrac{1}{3}\log_b(x-y) - \left(\log_b 3x\right)$

Step 4: $\dfrac{1}{3}\log_b(x-y) - \left(\log_b 3 + \log_b x\right)$

Step 5: $\dfrac{1}{3}\log_b(x-y) - \log_b 3 - \log_b x$

B) Write the expression as a single logarithm.

$$\frac{1}{2}\log_a 5 + \frac{1}{2}\log_a x - \frac{1}{2}\log_a 7$$

C) Solve: $2\log_3 x - \log_3 4 = \log_3 25$

〰️ **Say What You Know**

In your own words, talk about what you have learned using the objectives for this lesson and your work on this page.

🖥 **To continue, return to the Online Lesson.**

✎ Practice 2
Complete problems on a separate sheet of paper.

Expand.

1) $\log_4 \dfrac{\sqrt[3]{b}}{4}$

2) $\log_3 \dfrac{10x^3\sqrt{y}}{z^5}$

3) $\log \dfrac{x^2 + 2x + 1}{4x^2 - 9}$

4) $\log_7 \dfrac{1}{5x}$

Contract. Rewrite as a single log.

5) $\dfrac{2}{3}\log(x - 4) - \log(x + 1)$

6) $5\log_8 x + 3\log_8 y - 2\log_8 z - 6\log_8 a$

7) $\dfrac{2}{3}\log_2 x + \dfrac{1}{3}\log_2 y + \dfrac{2}{3}\log_2 z$

8) $3\log(x - 5) + 2\log(x + 2) - 4\log 2x$

Solve.

9) $\log_3(4x + 5) - \log_3 x = 2$

10) $\log_4 x + \log_4(2x - 3) = \log_4 5$

11) $\log_9(3x - 7) = \log_9 x + \log_9 2$

12) $\log_{36}(4x - 3) = \dfrac{1}{2}$

13) $\log_2(x - 3) + \log_2(x - 5) = \log_2 3$

14) $\log_5(x - 1) + \log_5(x + 3) - 2\log_5 x = 0$

15) $\log_4(3x + 7) = 3$

16) $\log_5(x + 4) - \log_5(x - 2) = 2$

17) $\log(x - 5) - \log(x + 4) = \log(x - 1) - \log(x + 2)$

18) $2\log_4(x - 2) = \log_4 x + \log_4(x - 3)$

19) $\dfrac{1}{2}\log_2(x + 3) = 2$

20) $-5\log_3(2x + 5) = -15$

⌨ **To continue, return to the Online Lesson.**