Lesson 37
Exponential Functions

NAME:

Start by navigating to the Online Lesson for instructions.

Objectives

- Graph an exponential function.
- Describe transformations on exponential functions.
- Transform an exponential function.
- Write an exponential function given two points.

Why?

Exponential functions show a numerical base raised to a variable power. The values increase or decrease quickly (or exponentially) forming a curved graph. Exponential functions are the foundation of many applications at the end of this unit.

Warm Up

Simplify.

1) -3^4

2) $(-3)^4$

3) $\left(-\frac{2}{3}\right)^4$

4) $-\left(\frac{2}{3}\right)^{-4}$

Solve the system of equations.

5) $2x - 5y = 18$

$y = -0.5x$

To continue, return to the Online Lesson.

Explore

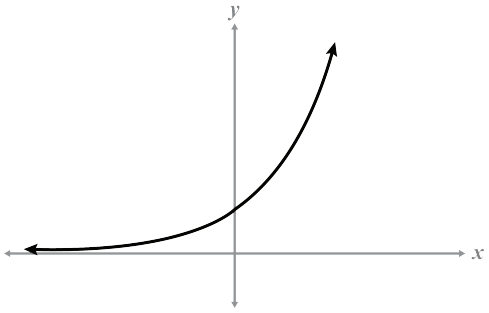
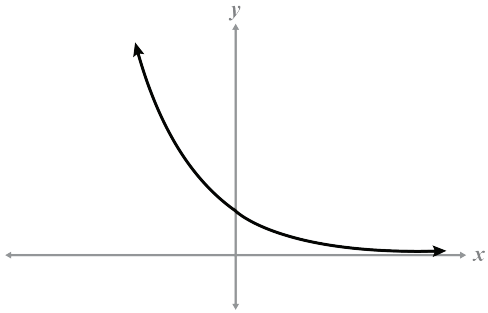
Graphing Exponential Functions

Fill in the notes as you watch the video in the Online Lesson.

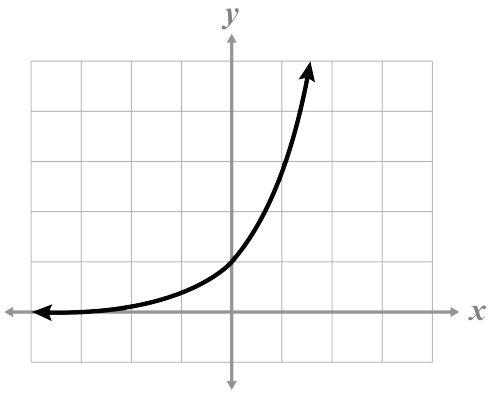
- The exponential _____ is: $y = b^x$, $b > 0$, $b \neq 1$.
  - $b > 0$: The base, b, cannot be _____.
  - $b \neq 1$: The base cannot equal one because one raised to any power is one, which results in _____.
- This equation has two forms:
  - _____
  - _____
- When the exponent, x, is _____, the _____ determines whether the function represents exponential growth or decay, without creating a graph.
- And so, the value of b is also called the _____.
- Because there are an infinite number of _____, there are infinite _____ _____.

Pay attention to $-b^x$ compared to $(-b)^x$. Without parentheses, the problem would read $-1 \cdot b^x$, which would mean the base is not negative.

Recall that when an exponent is negative, take the reciprocal of the base.

Growth Function	Decay Function
$b > 1$ $y = b^x, b > 0, b \neq 1$	$0 < b < 1$ $y = b^x, b > 0, b \neq 1$
	
Asymptote: End behavior: As $x \to +\infty$, _____, and as $x \to -\infty$, _____.	Asymptote: End behavior: As $x \to +\infty$, _____, and as $x \to -\infty$, _____.
Domain: Range:	

- The _____ exponential function, $f(x) = e^x$, is a special exponential function.
  - In the function, e equals $2.718281\ldots$, e is an _____ but well-rounded number.
  - Because e is an irrational number, the approximation _____ will be used (rather than $2.718281\ldots$).



It is recommended that you use a calculator when working with e.

Example 1

Complete the example as you watch the video in the Online Lesson.

Name the value of b and if it represents growth, decay, or neither.

$f(x) = (0.2)^x$ $q(x) = 4^x$ $g(x) = (0.4)^{-x}$ $y = (-8)^x$ $h(x) = 10^{-x}$ $y = 1^x$ $k(x) = e^x$

Exponential Growth	Exponential Decay	Neither
These functions represent exponential growth because _____.	These functions represent exponential decay because _____.	These are not exponential functions because _____ _____.

Example 2

Complete the example as you watch the video in the Online Lesson.

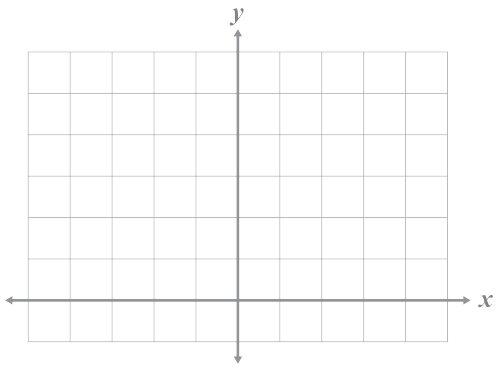
Graph both functions on the same coordinate plane. Name the end behavior and the domain and range.

$f(x) = 5^x$ $g(x) = \left(\frac{3}{2}\right)^{-x}$

x	$f(x)$	$g(x)$
-1		
0		
1		

Domain:

Range:



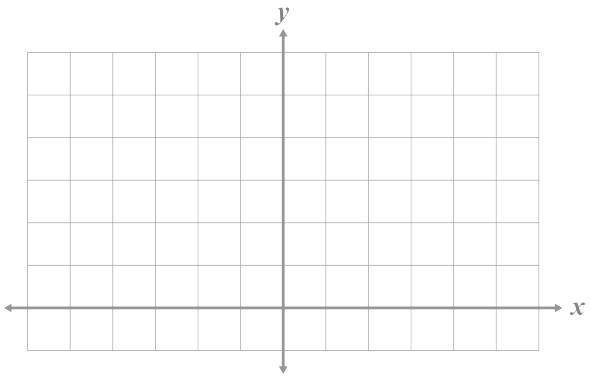
As $x \to +\infty$, _____, and as $x \to -\infty$, _____.

As $x \to +\infty$, _____, and as $x \to -\infty$, _____.

Checkpoint: Graphing Exponential Functions

Graph the exponential function. Name the value of b and explain if it represents growth or decay.

$y = 4^{-x}$



To continue, return to the Online Lesson.

Transforming Exponential Functions

Fill in the notes as you watch the video in the Online Lesson.

- Before transforming an exponential function, _____ to determine if the function represents growth or decay.
- As with all other transformations of functions, the _____ translate an exponential function in the same way for: $y = ab^{x-h} + k$
- _____: reflection, vertical stretch $|a| > 1$ or compression $0 < |a| < 1$
- _____: horizontal shift/translation
- _____: vertical shift/translation, the value of k is the horizontal asymptote
- The value of k will also help determine the _____ of the exponential function.

Example 3

Complete the example as you watch the video in the Online Lesson.

Describe the transformation from $f(x)$ to $g(x)$. Name the domain and range.

$f(x) = 2(8)^{x+4} + 6$ $g(x) = -2(8)^{x-3} + 1$

Plan

Determine if the function is growth or decay
Name a, h, k
Compare

Implement

$f(x)$ **$g(x)$**

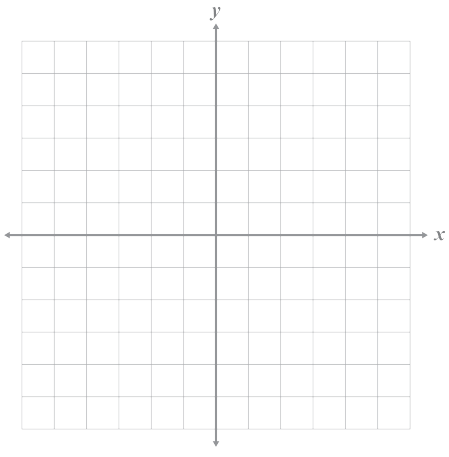
From $f(x)$, the function $g(x)$ _____, shifts _____, and _____ spaces.

Example 4

Complete the example as you watch the video in the Online Lesson.

Graph the exponential function. Name the domain and range.

$y = (0.5)^{x+2} - 3$



Domain:

Range:

To extend your learning, check out the More to Explore in the Online Lesson.

Checkpoint: Transforming Exponential Functions

Describe the transformation from $f(x)$ to $g(x)$.

$f(x) = -3(0.67)^x + 1$ $g(x) = 3(0.67)^{x+1} - 4$

To continue, return to the Online Lesson.

Writing Exponential Equations

Fill in the notes as you watch the video in the Online Lesson.

- Applying what you learned about systems of equations, you can write the equation to an exponential function from _____.
- Use repeated _____ to determine the values of _____.

To write an exponential equation given two points:

1) Substitute _____ into $y = ab^x$.

2) _____ a.

3) Substitute _____ and _____ into $y = ab^x$.

4) Solve for the _____.

5) Solve for the _____.

6) Write the function by substituting _____ into $y = ab^x$.

Example 5

Complete the example as you watch the video in the Online Lesson.

Write an exponential equation that passes through the points $(-2, -8)$ and $\left(3, -\frac{1}{4}\right)$.

Checkpoint: Writing Exponential Equations

Write an exponential equation that passes through the points $(-1, 0.6)$ and $(2, 75)$.

To continue, return to the Online Lesson.

Practice 1

Complete problems on a separate sheet of paper.

Name the value of b and if this represents growth, decay or neither.

1) $f(x)=\left(\frac{3}{7}\right)^x$

2) $g(x)=9^x$

3) $f(x)=(3.2)^{-x}$

4) $v(x)=(0.3)^{-x}$

Describe the transformation from $f(x)$ to $g(x)$.

5) $f(x)=4^x,\ g(x)=-4^x$

6) $f(x)=2^{-x},\ g(x)=2^x$

7) $f(x)=5^x,\ g(x)=5^{x-1}+2$

8) $f(x)=8^{x+2},\ g(x)=2(8)^{x+5}$

Sketch the graph using technology. Name the end behavior.

9) $f(x)=2\left(\frac{1}{3}\right)^x$

10) $g(x)=e^x$

11) $h(x)=(1.5)^{-x}$

12) $f(x)=5^{x+2}-3$

Describe the transformation. Determine if the function represents growth or decay. Name the domain and range.

13) $f(x)=12(e)^{-x}+4$

14) $g(x)=-32(7)^{x+3}-2$

15) $h(x)=4(5)^x$

16) $r(x)=-10\left(\frac{2}{5}\right)^{-x}-7$

Write the equation in the form $y=ab^x$ using the given points.

17) When planted from seed, the sprouting rate of radish seedlings appears exponential. At the end of week zero, two radish seedlings are visible. After two weeks, eighteen seedlings are visible. Write the exponential equation that models the growth of the seedlings. (x: time, y: seedlings)

18) $(0, -5)$ and $\left(2, -\frac{5}{16}\right)$

19) $(1, 4)$ and $\left(-2, \frac{1}{16}\right)$

20) Two hours ago, twelve mold spores were counted under the microscope. One hour ago, six mold spores remained. Write an exponential equation to model the decline in the mold spores. (x: time, y: amount of mold spores)

To continue, return to the Online Lesson.

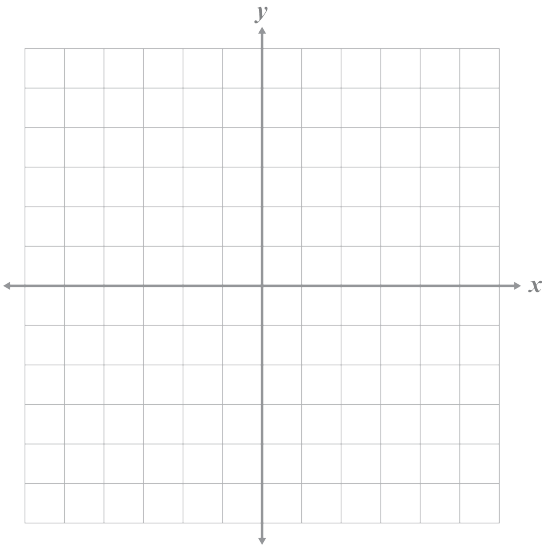
Mastery Check

Show What You Know

A) Write the equation of an exponential equation that passes through $\left(2, \frac{9}{4}\right)$ and $\left(-1, \frac{1}{12}\right)$.

B) Write a new equation for the exponential function in part A, translated right two spaces and down three spaces.

C) Graph part A and part B on the same coordinate plane.



Say What You Know

In your own words, talk about what you have learned using the objectives for this lesson and your work on this page.

To continue, return to the Online Lesson.

Practice 2

Complete problems on a separate sheet of paper.

Name the value of b and if this represents growth, decay or neither.

1) $p(x) = -2\left(\frac{5}{6}\right)^{-x}$

2) $h(x) = \left(-\frac{3}{4}\right)^{x}$

3) $b(x) = (1)^{8x}$

4) $f(x) = 4(2.75)^{-x}$

Describe the transformation from $f(x)$ to $g(x)$.

5) $f(x) = (3)^{x}, g(x) = 2(3)^{x-4} + 6$

6) $f(x) = \left(\frac{4}{5}\right)^{-x}, g(x) = \left(\frac{5}{4}\right)^{x}$

7) $f(x) = (7)^{x} - 3, g(x) = -4(7)^{-x} + 1$

8) $f(x) = (6)^{x-5}, g(x) = (6)^{x+5}$

Sketch the graph using technology. Name the end behavior.

9) $f(x) = -(2)^{x} + 4$

10) $g(x) = e^{(x+1)} - 3$

11) $h(x) = \left(\frac{1}{4}\right)^{x}$

12) $p(x) = \left(\frac{5}{4}\right)^{-x}$

Describe the transformation. Determine if the function represents growth or decay. Name the domain and range.

13) $f(x) = -13\left(\frac{1}{4}\right)^{-x} - 6$

14) $g(x) = 15(e)^{x}$

15) $h(x) = 9\left(\frac{1}{8}\right)^{x-1} + 2$

16) $q(x) = (1)^{x}$

Write the equation in the form $y = ab^{x}$ using the given points.

17) $(-2, 1)$ and $(1, 8)$

18) $(1, -3)$ and $(-2, -81)$

19) $\left(2, \frac{2}{3}\right)$ and $(-1, 144)$

20) One year ago, Ringo purchased a $\$0.20$ comic book at a yard sale. When he showed the comic to a collector, he was told to hold onto it because it would increase in value. In two more years, the comic is worth $\$25$. Write an exponential equation to model the value of the comic book.

To continue, return to the Online Lesson.

Targeted Review

Complete items on a separate sheet of paper.

Find the inverse of the function.

1) $f(x) = \dfrac{3}{x-2}$

2) $g(x) = \sqrt{x+3}$

Evaluate when $x = -5$.

3) 2^{x+9}

4) x^{8+x}

Evaluate when $x = -3$.

5) $5^{x}\left(5^{3-x}\right)$

6) $\left(\dfrac{2}{3}\right)^{x} x^{x}$

Write the equation, given the verbal description.

7) A square root function translated 2 units right and one unit up from the parent function

8) A quadratic function reflected across the x-axis, translated up 5 units and left 8 units

Multiple Choice

_____ **9)** Simplify $\sqrt[3]{5x^7y^6}$ using rational exponents.

A) $x^2y^2\sqrt[3]{5x}$

B) $5^{\frac{1}{3}}x^{2\frac{1}{3}}y^2$

C) $5^{\frac{1}{3}}x^{\frac{7}{3}}y^{\frac{6}{3}}$

D) cannot be simplified

Multiple Choice

_____ **10)** Simplify: i^{67}

A) i **B)** 1

C) $-i$ **D)** -1

_____ **11)** Simplify: $\dfrac{a^5 b^{-4}}{b c^{-2}}(a\,c)^{-2}$

A) $\dfrac{a^3}{b^4}$ **B)** $\dfrac{a^3}{b^5}$

C) $\dfrac{a^3 c^3}{b^5}$ **D)** $\dfrac{1}{a^{10} b^5 c^4}$

_____ **12)** Simplify: $\left(2x^3 y\right)^4$

A) $2x^{12}y^4$ **B)** $8x^{12}y^4$

C) $16x^7y^5$ **D)** $16x^{12}y^4$

Problem	1	2	3	4	5	6	7	8	9	10	11	12
Origin	L19	L19	—	—	—	—	L18	L18	L11	L15	A1	A1

L = Lesson in this level, A1 = Algebra 1: Principles of Secondary Mathematics

To continue, return to the Online Lesson.