

# Lesson 31

## Operations with Functions

NAME:

 Start by following the instructions in the Online Lesson.

### Objectives

- ✓ Evaluate a function.
- ✓ Determine the sum, difference, product, and/or quotient of functions.
- ✓ Name the excluded domain values, if any, for combinations of functions.

### Why?

In the future, you may need to model a real-life scenario. Such modeling usually means you need to combine a number of functions while also excluding values that do not make the scenario true.

### Warm Up

Evaluate using  $f(x) = x^2 - 3x$  and  $g(x) = -2x^3 + 1$ .

1)  $f(5)$

2)  $g(-3)$

 To continue, return to the Online Lesson.

### Explore

#### Evaluating Functions

 Fill in the notes as you watch the video in the Online Lesson.

- Until now, you have used the substitution property to replace a \_\_\_\_\_ in an expression or equation with another \_\_\_\_\_ (a number, variable, or expression).
- Now, you can substitute an entire \_\_\_\_\_ into another function.

Substitution with Variables	Substitution with Functions	
$f(x) = 2x + 3$	$f(x) = 2x + 3$	$g(x) = x + 4$
$f(4) = 2(4) + 3$	$f(g(x)) = 2(x + 4) + 3$	
$f(4) = 11$	$f(g(x)) = 2x + 8 + 3$	
	$f(g(x)) = 2x + 11$	

**Example 1**

 Complete the example as you watch the video in the Online Lesson.

Given  $h(x) = -2x^2 + ax$  where  $a$  is a constant and  $h(-5) = -65$ , determine  $h(5)$ .

**Implement****Explain**

- ▶ Determine the value of  $a$
- ▶ Write the equation with the value of  $a$
- ▶ Evaluate for  $h(5)$

**Example 2**

 Complete the example as you watch the video in the Online Lesson.

Evaluate the functions  $f$  and  $g$  with the expression  $2n - 1$ .

$$f(x) = x^2 + 4 \quad g(x) = -x^2 + 6x$$

$$f(2n - 1) = (2n - 1)^2 + 4$$

 **Checkpoint: Evaluating Functions**

Evaluate  $j(x - 1)$  for  $j(x) = x^3 - 2x$ .



To continue, return to the Online Lesson.

## Combinations of Functions

 Fill in the notes as you watch the video in the Online Lesson.

- When functions are combined, it creates a \_\_\_\_\_.
- Functions can be combined through \_\_\_\_\_, \_\_\_\_\_, \_\_\_\_\_, and \_\_\_\_\_.

	$(f + g)(x)$	
	$(f - g)(x)$	
	$(fg)(x)$	
	$\left(\frac{f}{g}\right)(x)$	

- The \_\_\_\_\_ of the combination needs to include the restrictions of \_\_\_\_\_ functions being combined.
- Pay special attention to \_\_\_\_\_ and \_\_\_\_\_ functions because these have more restrictions on the domains.
- The commutative property ( $a + b = b + a$ ,  $ab = ba$ ) does not hold true for \_\_\_\_\_.
- Therefore, make sure to \_\_\_\_\_ the functions, particularly when determining the difference or quotient.

### Example 3

 Complete the example as you watch the video in the Online Lesson.

**Find  $(f + g)(x)$  and  $(f - g)(x)$  when  $f(x) = 3x^2 - x - 1$  and  $g(x) = x + 6$ . Determine the domain for each function.**

$$(f + g)(x) = f(x) + g(x)$$

$$= (3x^2 - x - 1) + (x + 6)$$

$$\text{Domain}_f \{x | x \in \mathbb{R}\}$$

$$\text{Domain}_g \{x | x \in \mathbb{R}\}$$

**Example 4**

 Complete the example as you watch the video in the Online Lesson.

Find  $(gh)(x)$  and  $\left(\frac{h}{g}\right)(x)$  when  $g(x) = 2x + 1$  and  $h(x) = \frac{1}{x - 2}$ . Determine the domain for each function.

 **Checkpoint: Combinations of Functions**

Find  $(hj)(x)$  and  $\left(\frac{j}{h}\right)(x)$  when  $h(x) = x^2 + 2x - 15$  and  $j(x) = x - 3$ . Determine the domain for each function.



To continue, return to the Online Lesson.

**Evaluating Combinations of Functions**

▶ Fill in the notes as you watch the video in the Online Lesson.

- The value of a combination of functions is determined when the \_\_\_\_\_ is given.
- \_\_\_\_\_ the  $x$ -value into all of the functions and \_\_\_\_\_.

**Example 5**

▶ Complete the example as you watch the video in the Online Lesson.

**Evaluate.**

$$f(x) = 3x^2 - x - 1 \quad g(x) = 2x + 1 \quad h(x) = \frac{1}{x - 2}$$

$$(fh)(0) \qquad \left(\frac{gh}{f}\right)(-1)$$

**Example 6**

▶ Complete the example as you watch the video in the Online Lesson.

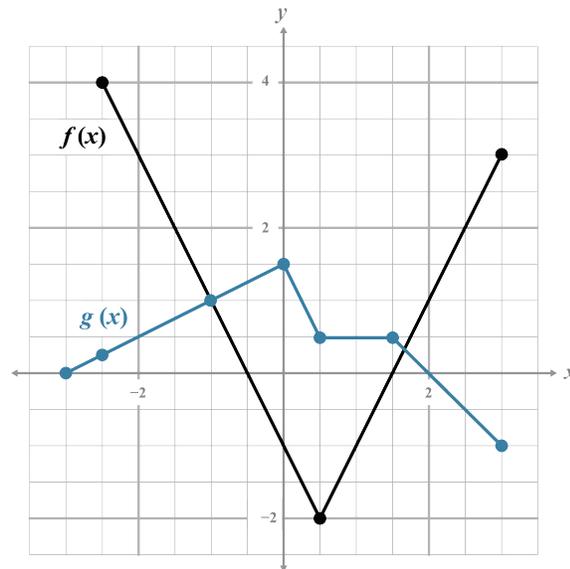
**Evaluate.**

$$(f + g)(-2)$$

$$(f + g)(-2) = f(-2) + g(-2)$$

$$(g - f)(0)$$

$$(fg)(1.5)$$

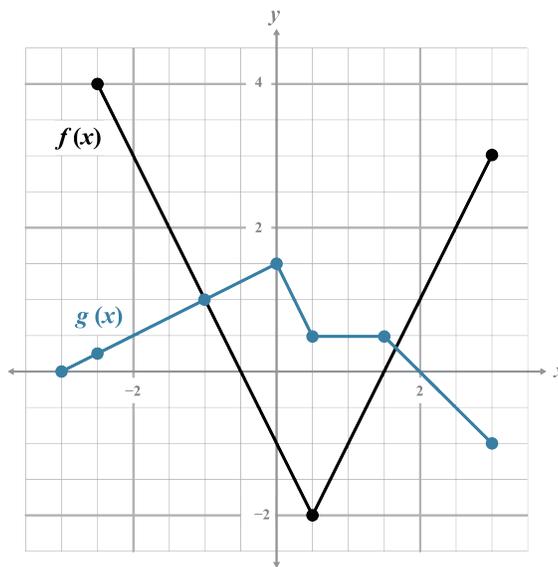


$$\left(\frac{g}{f}\right)(0.5)$$

**Checkpoint: Evaluating Combinations of Functions**

**Evaluate.**

$$(f - g)(3), \left(\frac{f}{g}\right)(0), (f + g)(-1)$$



To continue, return to the Online Lesson.

 **Practice 1**

Complete problems on a separate sheet of paper.

For problems 1–10, use the functions:  $f(x) = x^2 - 9$      $g(x) = x + 3$      $h(x) = \frac{1}{2x - 6}$

Find the sum, difference, product, or quotient of the named functions. Determine the domain for each function.

- 1)  $\left(\frac{f}{g}\right)(x)$                       2)  $(fh)(x)$   
 3)  $(g + h)(x)$                       4)  $(f - g)(x)$

Evaluate.

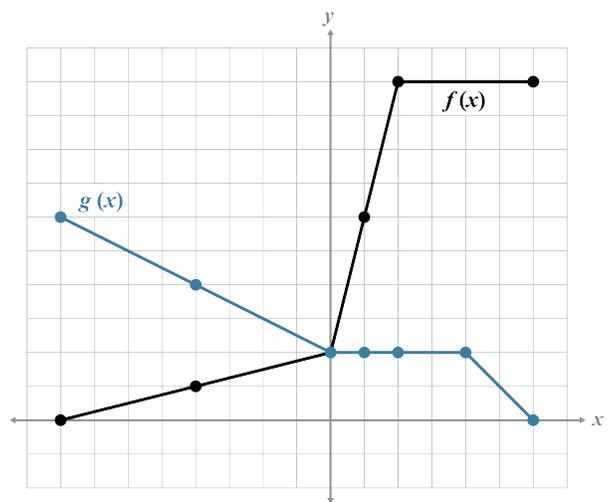
- 5)  $(g - h)(2)$                       6)  $f(2x - 1)$   
 7)  $\left(\frac{fh}{g}\right)(0)$                       8)  $g\left(\frac{1}{4x}\right)$   
 9)  $h(x^2 + 1)$                       10)  $(g - f)(-1)$

For problems 11–14, determine the value of  $a$  using the functions:  $f(x) = ax^2 + 4x + 5$      $g(x) = \frac{1}{ax - 1}$

- 11)  $f(-2) = 25$                       12)  $g\left(\frac{2}{3}\right) = 1$   
 13)  $f(1) = -1$                       14)  $g(-3) = 3$

For problems 15–20, evaluate using the graph.

- 15)  $\left(\frac{f}{g}\right)(-4)$   
 16)  $(f - g)(5)$   
 17)  $(fg)(0)$   
 18)  $\left(\frac{g}{f}\right)(-8)$   
 19)  $(g + f)(-8)$   
 20)  $(gf)(6)$



To continue, return to the Online Lesson.

 **Mastery Check** **Show What You Know**

**A)** Determine  $f\left(\frac{1}{2}\right)$  if  $f(-4) = 40$  and  $f(x) = ax^2 - 5x - 12$ .

**B)** Fill in  $f(x)$  based on your work in part A. Then use the functions  $f(x)$ ,  $g(x)$ , and  $h(x)$  to complete parts C, D, and E.

$$f(x) = \underline{\hspace{10em}} \quad g(x) = \frac{4}{2x+3} \quad h(x) = 3x - 12$$

**C)** Find the sum of two functions that result in a quadratic trinomial. Show your work.

**D)** Find the product of two functions that result in a linear binomial. Show your work.

**E)** Evaluate the new functions in parts C and D when  $x = 10$ .

 **Say What You Know**

In your own words, talk about what you have learned using the objectives for this lesson and your work on this page.



**To continue, return to the Online Lesson.**

 **Practice 2**

Complete problems on a separate sheet of paper.

For problems 1–10, use the functions:  $f(x) = \frac{1}{x-5}$      $g(x) = x^3 - 10x^2 + 25x$      $h(x) = x^2 - 4x - 5$

Find the sum, difference, product, or quotient of the named functions. Determine the domain for each function.

1)  $(fg)(x)$

2)  $\left(\frac{h}{g}\right)(x)$

3)  $(h-g)(x)$

4) Find  $f(x^2-4)$

Evaluate.

5)  $h(x-2)$

6)  $(f+g)(-1)$

7)  $\left(\frac{fg}{h}\right)(x)$

8)  $(g-h)(-2)$

9)  $(f+h)(0)$

10)  $h(x^2)$

For problems 11–14, determine the value of  $a$  using the functions:  $f(x) = ax^2 - 10$      $g(x) = \frac{3}{ax+2}$

11)  $f(\sqrt{5}) = 0$

12)  $g(-2) = -\frac{3}{4}$

13)  $f(-1) = -13$

14)  $g\left(\frac{1}{2}\right) = \frac{3}{4}$

For problems 15–20, evaluate using the graph.

15)  $\left(\frac{f}{g}\right)(4)$

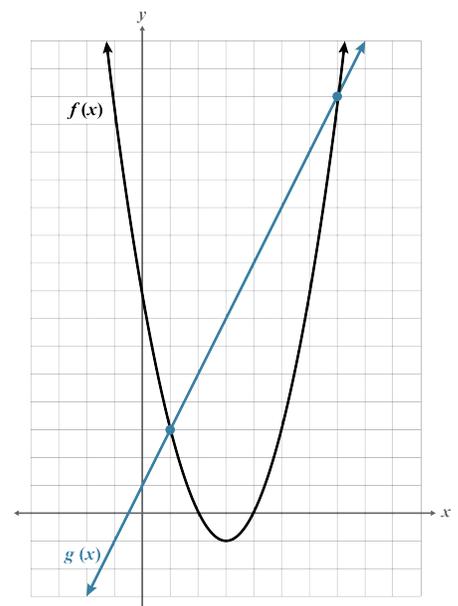
16)  $(g-f)(0)$

17)  $(fg)(-1)$

18)  $(f+g)(3)$

19)  $(f-g)(7)$

20)  $\left(\frac{g}{f}\right)(2)$



To continue, return to the Online Lesson.

# Targeted Review

Complete items on a separate sheet of paper.

- 1) Determine if the given functions are inverses.

$$f(x) = \sqrt[3]{4x+2} \quad g(x) = \frac{x^3-2}{4}$$

- 2) Solve by completing the square.

$$5x^2 - 10x + 15 = 0$$

- 3) Write the equation when the solution is  $\pm 6i$ .

- 4) Evaluate.

$$i =$$

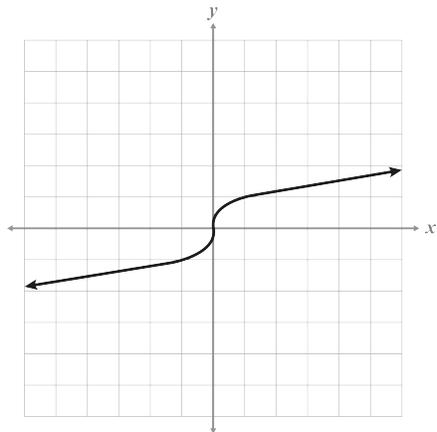
$$i^2 =$$

$$i^3 =$$

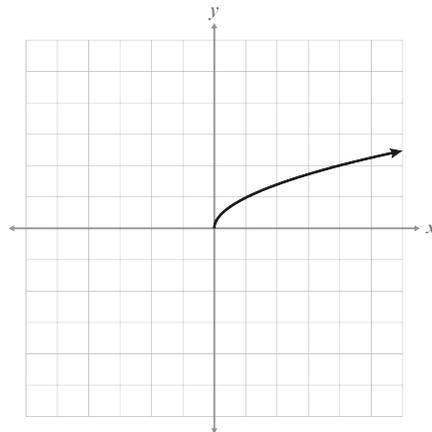
$$i^4 =$$

Name the parent graph and its end behavior.

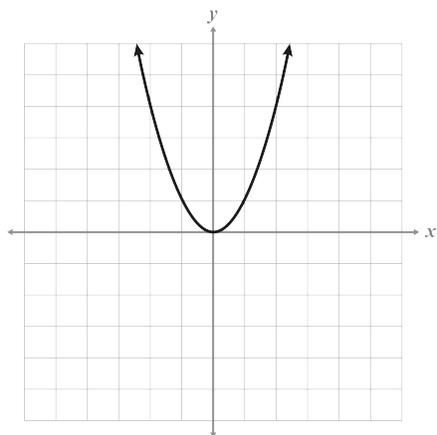
5)



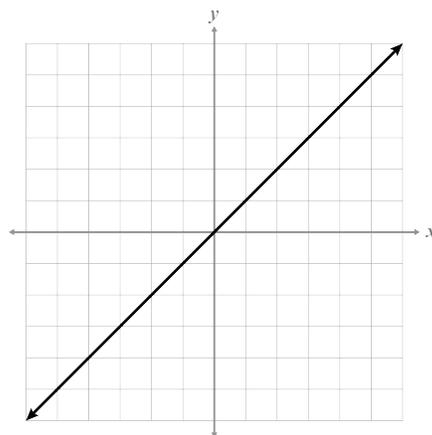
6)



7)



8)



**Multiple Choice**

\_\_\_\_\_ 9) Solve.

$$\left(x - \frac{2}{5}\right)^2 = \frac{16}{25}$$

A)  $\frac{6}{5}$

B)  $\frac{4}{5}$

C)  $-\frac{2}{5}, \frac{6}{5}$

D)  $-\frac{6}{5}, \frac{2}{5}$

\_\_\_\_\_ 10) Name the vertex of the parabola.

$$f(x) = x^2 - 6x + 13$$

A) (0, 13)

B) (-3, 4)

C) (3, 4)

D) (3, 22)

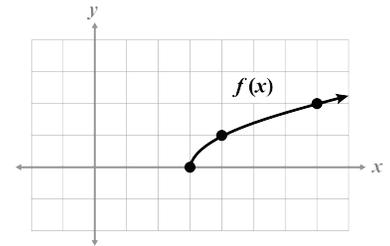
\_\_\_\_\_ 11) Name the domain and range of the *inverse* to the function.

A) Domain:  $\{x|x \in \mathbb{R}\}$ , Range:  $\{y|y \in \mathbb{R}\}$

B) Domain:  $\{x|x \in \mathbb{R}, x \geq 0\}$ , Range:  $\{y|y \in \mathbb{R}, y \geq 3\}$

C) Domain:  $\{x|x \in \mathbb{R}, x \geq 3\}$ , Range:  $\{y|y \in \mathbb{R}, y \geq 0\}$

D) Domain:  $\{x|x \in \mathbb{R}, x \leq 0\}$ , Range:  $\{y|y \in \mathbb{R}, y \leq 3\}$



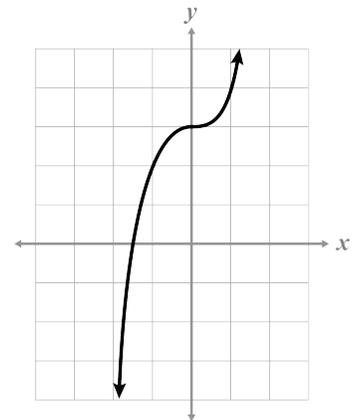
12) A student graphed a function through (0, 3) and knows that as  $x \rightarrow +\infty, y \rightarrow +\infty$ . Select all true statements about the function.

The graph represents a cube root function.

The graph represents a cubic function.

As  $x \rightarrow -\infty, y \rightarrow -\infty$

As  $x \rightarrow -\infty, y \rightarrow +\infty$



Problem	1	2	3	4	5	6	7	8	9	10	11	12
Origin	L19	L24	L23	L15	L17	L17	L17	L17	L13	L27	L21	L18

L = Lesson in this level, A1 = Algebra 1: Principles of Secondary Mathematics

 To continue, return to the Online Lesson.