Lesson 26
Distance and Midpoint Formulas

NAME:

▣ Start by navigating to the Online Lesson for instructions.

Objectives

⊘ Determine the distance between two points on the coordinate plane.

⊘ Solve for midpoints and endpoints using the midpoint formula.

⊘ Apply distance and midpoint formulas to solve problems.

Why?

You may need to find the length of a line segment or the distance between points on a line segment that is neither vertical nor horizontal. Using distance and midpoint formulas, you can solve these problems.

☼ Warm Up

1) Determine the slope between the points $(14.2, -6.3)$ and $(-5.8, -7.9)$.

2) Determine the missing x-value given $(6.2, 1)$, $(x, 0.7)$, and $m = 0.3$.

▣ **To continue, return to the Online Lesson.**

ᛘ Explore

ᛘ The Distance Formula

▷ *Fill in the notes as you watch the video in the Online Lesson.*

■ The distance between any two points on a number line is calculated as $|a - b|$ or $|b - a|$ because distance _____ be a _____ value.

■ The _____ is used to determine the distance between

two _____ on the coordinate plane.

- The distance formula is: $d = \sqrt{\left(x_2 - x_1\right)^2 + \left(y_2 - y_1\right)^2}$

- It uses _____ and is a derivation of the Pythagorean Theorem.

- The distance formula allows you to solve *some* problems without having to _____ them on the coordinate plane.

- Distance, or length, is not typically written in radical form because that is not how they are measured. Instead, enter the entire formula into a _____.

- The result is likely a _____ because most numbers are not perfect squares.

Example 1

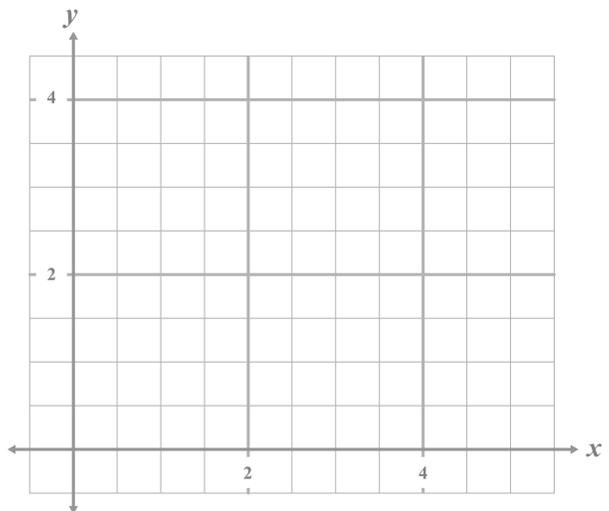▷ *Complete the example as you watch the video in the Online Lesson.*

Plot and connect the ordered pairs. Then find the perimeter of the figure. Round to the nearest hundredth.

$\{P\,(0, 0),\, Q\,(2.5, 4),\, R\,(4, 1.5)\}$

$d = \sqrt{\left(x_2 - x_1\right)^2 + \left(y_2 - y_1\right)^2}$

$PQ = \sqrt{(0 - 2.5)^2 + (0 - 4)^2} = 4.716$

$PQ = 4.72$ units

Example 2

▷ *Complete the example as you watch the video in the Online Lesson.*

Find the distance between the vertex and the *y*-intercept of the parabola. Round to the nearest hundredth.

$$y = (x + 5)^2 - 8$$

Implement

Explain

▶ Determine the vertex (h, k)

▶ Determine the *y*-intercept $(0, c)$

▶ Distance Formula

☑ **Checkpoint: The Distance Formula**

Determine the distance between the given points. Round to the nearest hundredth.

$$\{A\,(-7, -2.3),\ B\,(-3.4, 8)\}$$

🖱 **To continue, return to the Online Lesson.**

🔭 The Midpoint Formula

▷ *Fill in the notes as you watch the video in the Online Lesson.*

- The midpoint formula is used to find the point _____ from two endpoints on the coordinate plane.

- The midpoint formula is: $\left(\dfrac{x_1 + x_2}{2},\ \dfrac{y_1 + y_2}{2} \right)$

- ■ The midpoint formula calculates the _____ of the x-values and the _____ of the y-values written as an ordered pair.

- ■ If you have the midpoint and an endpoint, you can determine an _____ by working backward.

- ■ _____ all known values into the formula and solve for the unknown values.

Example 3

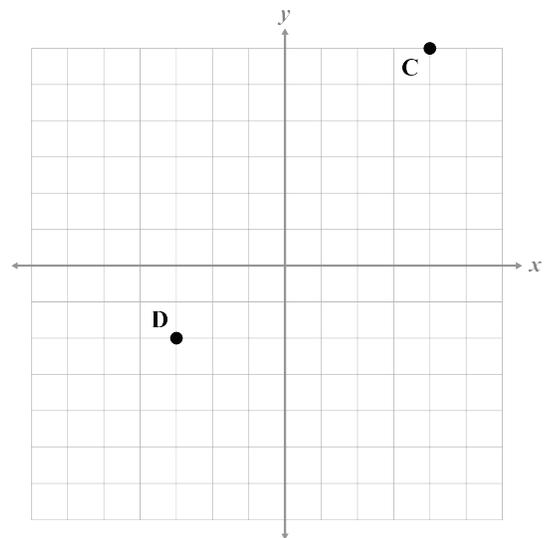▷ *Complete the example as you watch the video in the Online Lesson.*

A) **Calculate the midpoint between points CD. Mark the midpoint, M, on the graph.**

B) **Find the endpoint, E, with endpoint M and point D as the midpoint.**

A) $C\,(4, 6),\ D\,(-3, -2)$

$$\left(\frac{x_1 + x_2}{2},\ \frac{y_1 + y_2}{2}\right)$$

$$\left(\frac{4 + -3}{2},\ \frac{6 + -2}{2}\right)$$

B) $M\,(0.5, 2),\ D\,(-3, -2)$

$$\left(\frac{x_1 + x_2}{2},\ \frac{y_1 + y_2}{2}\right)$$

$$\left(\frac{x + 0.5}{2},\ \frac{y + 2}{2}\right) = (-3, -2)$$

Example 4

▷ *Complete the example as you watch the video in the Online Lesson.*

Mini's Mini Golf Course has a putting green for golfers to practice on before their games. Currently, there are three tees and two holes on the green. The plan is to add another hole *between* the existing holes.

**A)**   Find the midpoint between the existing holes.

B)   Determine the midpoint, horizontal and vertical, for the entire course from the middle tee (#2). Explain.

Horizontal:

The _____ is halfway between the left and

right side walls of the green. Because that is the

dividing line, _____ calculation is needed.

Vertical:

C)   Determine the distance in feet from the middle of the putting green to the new hole from part A. Explain.
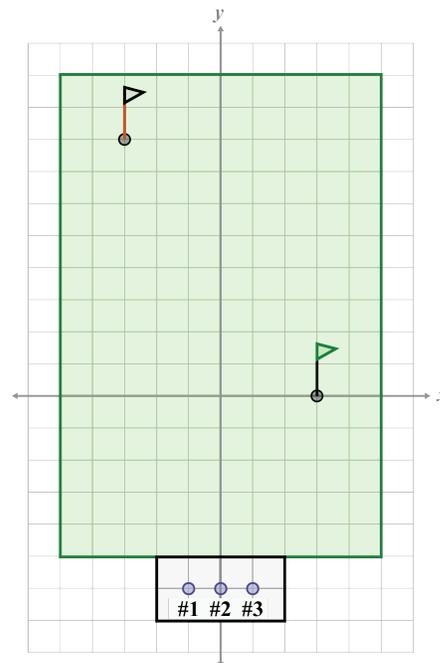
☑ Checkpoint: The Midpoint Formula

Find the endpoint, Q, given P (–7, –9.4) and midpoint M (–10, –3.55). Then show your answer is correct using the midpoint formula.

**To continue, return to the Online Lesson.**

✏ Practice 1

Complete problems on a separate sheet of paper.

Calculate the distance between the points. Round answers to the nearest hundredth.

1) $(-3, 5), (9, -1)$

2) $(4.25, 5.6), (1.7, -0.3)$

**3)** $(14, -10), (20, -5)$

4) $(6.75, 2), (0.75, -6)$

For problems 5–6, use the given graph (top right).

5) Calculate the midpoint between points AB. Mark the midpoint, M, on the graph.

6) Find the endpoint, C, with point M and point B as the midpoint.
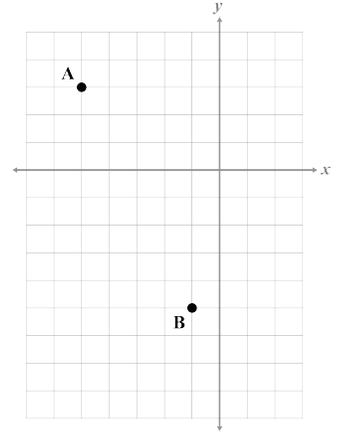
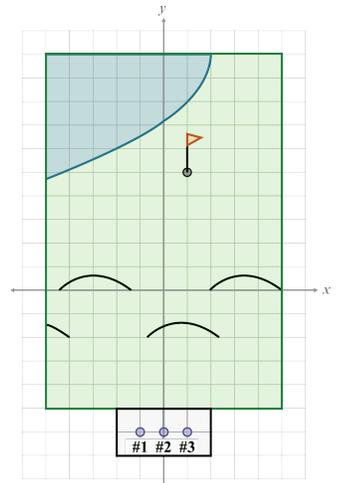**For problems 7–8, use the graph of triangle ABC.**

7) Determine the midpoint of each side of the triangle ABC.

8) Connect all of the midpoints to form a new triangle. Find the perimeter of the new triangle.
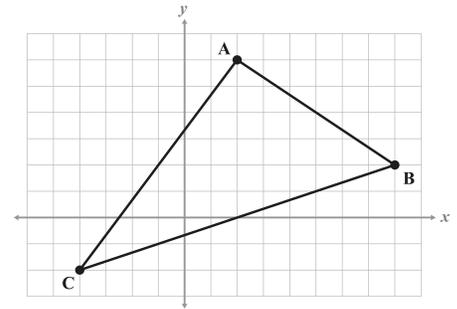
For problems 9–11, use the given graph of a miniature golf course hole.

9) On this mini golf course hole, Collin decided to take a straight shot from tee #3 to the hole. What is the distance to the hole?

10) Nelly also used tee #3 and decided to bank (or deflect) her shot from the left boundary wall at the midpoint. Find the midpoint of the left boundary wall.

11) Lauren took the same shot as Nelly and made a hole in one, what was the total distance for the shot?

For problems 12–14, use the equation $y = x^2 - 4x - 3$.

12) A line intersects the parabola $y = x^2 - 4x - 3$, at the points $(-1, 2)$ and $(5.5, 5.25)$. Determine the point, M, equidistant from the given points on the line.

13) Calculate the distance between the y-intercept and point M found in the previous problem.

14) Determine the value of the discriminant. Explain if it is possible to determine a real number distance between point M and either of the x-intercepts of the parabola.

Each square represents 1 square foot.

⟱ **To continue, return to the Online Lesson.**

☑ Mastery Check

✐ Show What You Know

Ayaka, Nancy, and Bryson, were playing a round of mini golf. Each player approached this hole a different way, but all shot a hole-in-one. They decided that the hole winner would be the player whose ball traveled the shortest distance. Determine the hole winner by calculating each player's shot distance.

A) Ayaka: From tee #1 banks (deflects) off the right boundary wall at $(5, 0)$

B) Nancy: From tee #2 banks off the midpoint of the right boundary wall

C) Bryson: From tee #3 banks off the right boundary wall at $(5, 7)$
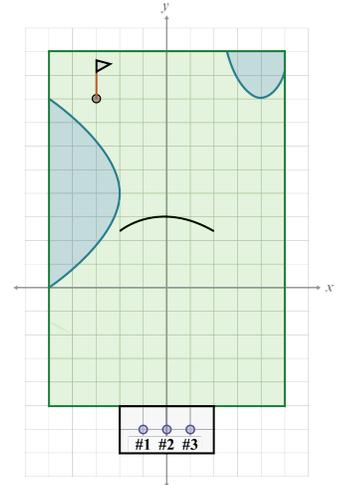
� Say What You Know

In your own words, talk about what you have learned using the objectives for this lesson and your work on this page.

To continue, return to the Online Lesson.

✎ Practice 2

Complete problems on a separate sheet of paper.

1) Find the distance between (3.6, –9.1), (8.2, –7.4). Round to the nearest hundredth.

2) Find the midpoint between (3.6, –9.1), (8.2, –7.4).

3) Find the midpoint between (7, 14) and (19, 3).

For problems 4–6, use the given graph of a miniature golf course hole.
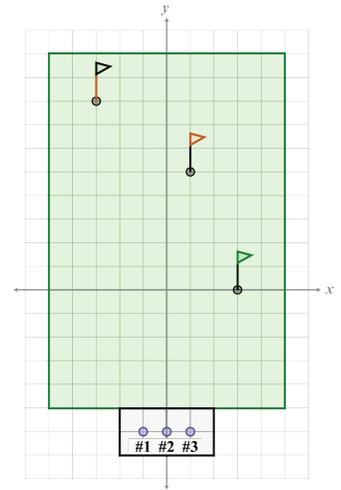
4) What is the distance between the hole in quadrant 2 and the hole on the axis?

5) What is the midpoint between the holes in quadrants one and two?

6) How much further would a player using tee #3 have to putt to the hole in quadrant 2 than a player using tee #1?

7) The definition of a rhombus is a parallelogram with four equal sides. Susan drew a figure with the points A (–3, –2), B (0, 2), C (3, –2), D (0, –5) and determined it was a rhombus. Show and explain your work to prove or disprove this claim.

8) Marie drew the figure and created a quadrilateral with the points A (–3, –2), B (0, 2), C (3, –2), D (0, –6) then found the midpoints of diagonals $\overline{AC}$ and $\overline{BD}$. Describe the relationship between the midpoints of $\overline{AC}$ and $\overline{BD}$.

For problems 9–11, use the table.

Diameter of a Circle	Radius of a Circle
• Has two endpoints on the circle • Goes through the center of the circle	• Is the distance from the center to an endpoint on the circle

9) The endpoints of the diameter of a circle are at (–4, –5) and (–7, 1). Find the center.

10) Find the length of the radius. Round to the nearest hundredth.

11) Find the area of the circle using 3.14 for π. Round your answer to the nearest hundredth.

For problems 12–13, use the given graph.

12) Find the area of the rectangle $WXYZ$. Round the final solution to a whole number.

13) Find the perimeter of the rectangle $WXYZ$. Round to the nearest hundredth.
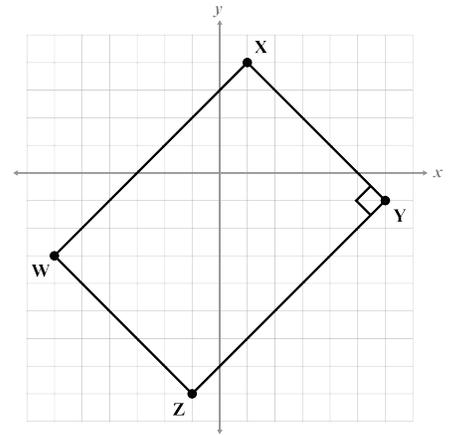
14) Given the equation $f(x) = (x + 4)^2 - 3$, determine the distance between the vertex and the y-intercept. Round to the nearest hundredth.

15) Given the equation $g(x) = (x - 2)^2 - 9$, find the distance between the vertex and one of the roots. Round to the nearest hundredth.

To continue, return to the Online Lesson.