Lesson 24
Completing the Square

NAME:

📄 Start by navigating to the Online Lesson for instructions.

Objectives

✓ Complete the square when $a = 1$.

✓ Complete the square when $a \neq 0, 1$.

Why?

Solving by completing the square is used to rewrite conic equations in vertex form. This allows you to visualize transformations of conics later in this unit and for the rest of upper-level mathematics. Whether or not a quadratic equation can be factored, solving quadratics by completing the square always works.

🌤 Warm Up

Factor.

1) $x^2 - 2xn + n^2$

2) $\dfrac{1}{4}x^2 + 5x + 25$

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🔭 Explore

🔭 Writing Perfect Square Trinomials

▷ *Fill in the notes as you watch the video in the Online Lesson.*

- When quadratic trinomials are _____ trinomials, they can be written

  as: $a^2 + 2ab + b^2 =$

- With a perfect square trinomial in _____, $\left(ax^2 + bx + c\right)$, the terms a

  and c are perfect squares and:

■ You can prove this formula is true by using a perfect square trinomial when the value of c is known. For example: $x^2 + 8x + 16 = (x + 4)^2$

Plan

Determine a, b, and c using _____ form

Find the value of _____ using the formula:

$b = 2\sqrt{ac}$

Implement

$a = 1,\ b = 8,\ c = 16$

$b = 2\sqrt{(1)(16)} = 2\sqrt{16} = 2 \cdot 4 = 8$

$c = \left(\dfrac{b}{2}\right)^2 = \left(\dfrac{8}{2}\right)^2 = (4)^2 = 16$

■ When you know the values of a and b, you can use this formula to find the value of c:

Example 1

ⓟ *Complete the example as you watch the video in the Online Lesson.*

Solve for the value that will make the expression a perfect square trinomial.

$x^2 - 11x + c$

Example 2

ⓟ *Complete the example as you watch the video in the Online Lesson.*

Solve for the value that will make the expression a perfect square trinomial.

$x^2 + bx + \dfrac{16}{49}$

☑ **Checkpoint: Writing Perfect Square Trinomials**

Solve for the value that will make the expression a perfect square trinomial.

A) $x^2 + bx + 64$

B) $x^2 + \dfrac{3}{2}x + c$

To continue, return to the Online Lesson.

Solving to Complete the Square when $a = 1$

▷ *Fill in the notes as you watch the video in the Online Lesson.*

- _____ is used to solve quadratic equations, particularly

  those that cannot be factored.

- To solve using completing the square, follow these steps:

  - Calculate the term that will make the expression _____ a perfect

    square trinomial.

  - Take the square root of _____ of the equation.

- Following these steps exactly shows that you can solve for the *x*-intercepts of quadratic

  equations _____.

- Here are some important tips to remember when solving by completing a square:

  - When finding a square root, write the _____ before the

    square root symbol because you are solving.

- Perform the _____ operation on _____ sides of the equation so that equality is maintained. (You can see this especially in steps 2, 3, 4, and 7 in Example 3.)

  - Answers are the _____ and are also called solutions or roots.

  - Answers can be _____.

  - The steps are the same for _____ completing the square problem but the expressions will be _____ depending on the values of a, b, and c.

Example 3

▷ *Complete the example as you watch the video in the Online Lesson.*

Solve by completing the square.

$x^2 - 31 = 8x$

Implement

$a = 1$, continue to step 3

$x^2 - 8x + \boxed{} = 31 + \boxed{}$

$(x - 4)^2 =$

Explain (in Steps)

1) Write equation in standard form

2) Divide all terms by leading coefficient, a (If $a = 1$, this step can be skipped)

3) Add $-\dfrac{c}{a}$ to both sides of equation

4) Simplify and write $+\square$ after terms on both sides Calculate value that makes LEFT side of equation a perfect square trinomial

5) Write left side of the equation as product of a binomial squared: $\left(x - \dfrac{b}{2}\right)^2$ Simplify right side of equation

6) Solve for x by taking square root of entire equation

7) Isolate the variable

Remember $\left(\dfrac{b}{2}\right)^2 = \left(\dfrac{1}{2}b\right)^2$ because dividing by 2 is the same as multiplying by $\dfrac{1}{2}$.

Example 4

▷ *Complete the example as you watch the video in the Online Lesson.*

Solve by completing the square.

$x^2 + 3x + 2 = -2x$

$a = 1$, continue to step 3

☑ **Checkpoint: Solving to Complete the Square when $a = 1$**

Using Example 4, explain each step out loud to your instructor, describing what happened as you solved the problem.

🖥 **To continue, return to the Online Lesson.**

👀 Solving to Complete the Square when $a \neq 0, 1$

▷ *Fill in the notes as you watch the video in the Online Lesson.*

- When solving problems by completing the square, the result may include _____

  _____.

  - When complex solutions occur, remember to first _____ of any

    radial, and check that the denominator is _____.

  - Complex solutions can occur when _____, but is more common when

    _____.

Example 5

▷ *Complete the example as you watch the video in the Online Lesson.*

Solve by completing the square.

$3x^2 + 4x + 2 = 0$

Implement

Explain (in Steps)

1) Write equation in standard form

2) Divide all terms by leading coefficient, a
(If $a = 1$, this step can be skipped)

3) Add $-\dfrac{c}{a}$ to both sides of equation

4) Simplify and write $^+\square$ after terms on both sides
Calculate value that makes LEFT side of
equation a perfect square trinomial

5) Write left side of equation as product of a
binomial squared

Simplify right side of equation

6) Solve for x by taking the square root of entire
equation

7) Isolate the variable

Example 6

▷ *Complete the example as you watch the video in the Online Lesson.*

Solve by completing the square.

$3x^2 + 1 = 6x$

☑ **Checkpoint: Solving to Complete the Square when $a \neq 0,\ 1$**

Solve by completing the square.

$2x^2 - 20x + 74 = 0$

To continue, return to the Online Lesson.

✎ Practice 1

Complete problems on a separate sheet of paper.

Determine the value that will make the expression a perfect square trinomial. Show your work.

1) $x^2 + 16x + c$

2) $x^2 + 7x + c$

3) $x^2 + bx + 81$

4) $x^2 + bx + \dfrac{100}{81}$

5) $x^2 + \dfrac{5}{2}x + c$

Solve the quadratic equations by completing the square.

6) $x^2 - 6x + 8 = 0$

7) $x^2 - 2x + 17 = 0$

8) $x^2 = 7x + 4$

9) $2x^2 + 8x + 3 = 0$

10) $4x^2 + 5x + 4 = 0$

To continue, return to the Online Lesson.

☑ Mastery Check

✎ Show What You Know

Follow each direction below using the equation $ax^2 + bx + c = 0$ to show the work.

Direction	Show Work Using $ax^2 + bx + c = 0$
1) Subtract c from both sides of the equation.	
2) Divide all terms by the coefficient a.	
3) Add b to the second power divided by 4 times a to the second power to both sides.	
4) Write the left side of the equation as a binomial squared.	
5) Write the right side of the equation as one fraction with a common denominator, $4a^2$.	
6) Take the square root of both sides of the equation.	
7) Isolate the variable x.	
8) Write the right-side of the equation as one fraction.	

ᯤ Say What You Know

In your own words, talk about what you have learned using the objectives for this lesson and your work on this page.

🖥 **To continue, return to the Online Lesson.**

✎ Practice 2

Complete problems on a separate sheet of paper.

Determine the value that will make the expression a perfect square trinomial. Show your work.

1) $x^2 + bx + 49$

2) $x^2 + bx + \dfrac{4}{9}$

3) $x^2 - 24x + c$

4) $x^2 + 6x + c$

5) $x^2 + bx + 225$

Solve the quadratic equations by completing the square.

6) $2x^2 - 3x - 2 = 0$

7) $x^2 = 10x - 14$

8) $3x^2 + 9x + 5 = 0$

9) $x^2 + 20x + 94 = 0$

10) $x^2 - 2x = -38$

To continue, return to the Online Lesson