Lesson 20

NAME:

Graphing Functions and Their Inverses

Start by navigating to the Online Lesson for instructions.

Objectives

- Graph a function and its inverse on the same coordinate plane and determine if they are functions.
- Use the horizontal line test to determine if the graph of an inverse is a function.

Why?

Functions are shown as equations and graphs. Representing a function and its inverse as a graph allows you to connect the domain and range and the coordinate plane.

Warm Up

1) Name the domain and range of R^{-1}.

$R = \{(-2, -6), (-1, -3), (0, 0), (1, 3), (2, 6)\}$

2) Use the relation to write a function rule $f(x)$, and its inverse $f^{-1}(x)$.

To continue, return to the Online Lesson.

Explore

Graphing Functions and Their Inverses

Fill in the notes as you watch the video in the Online Lesson.

- When the graph of a function is given, graph the inverse function by _____ _____ and plotting them on the coordinate plane.
- If the given function is defined as $f(a) = b$ and graphed as (a, b), then the inverse function can be defined as _____ and graphed as _____.

- When the inverse of a function is graphed it is _____ over the line $y = x$ because the inverse of $y = x$ is _____.
- The line of _____, $y = x$, is used to confirm visually that the function and its inverse are correct on the coordinate plane. It is optional, and so can be drawn as a _____.
- Recall that the _____ is a visual representation that determines if a graph on the coordinate plane is a function by running a vertical line across the graph.
- If the vertical line touches _____ at a time, the graph is not a function.
- The VLT can also be used to determine if the inverse of the given function is a _____.

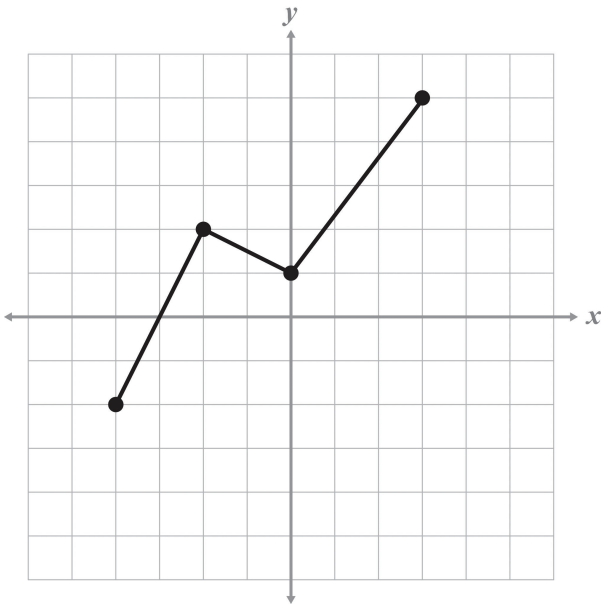
Example 1

Complete the example as you watch the video in the Online Lesson.

Graph the inverse on the coordinate plane.

Plan

Label the given points
Graph $y = x$ (line of reflection)
Graph (b, a)
Connect the points



Example 2

Complete the example as you watch the video in the Online Lesson.

Graph the inverse on the coordinate plane. Explain whether or not the graph and its inverse represent functions.

Plan

Label the given points
Graph $y = x$ (line of reflection)
Graph (b, a)
Connect the points
Use VLT

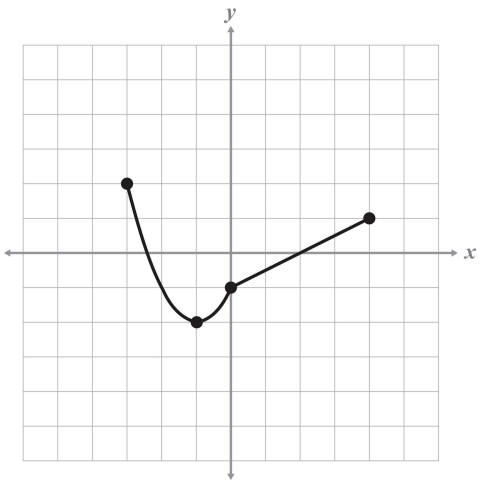
The given graph _____ a function

because it _____ the VLT.

The inverse of the graph _____ a

function because it _____ the VLT.

Student view:

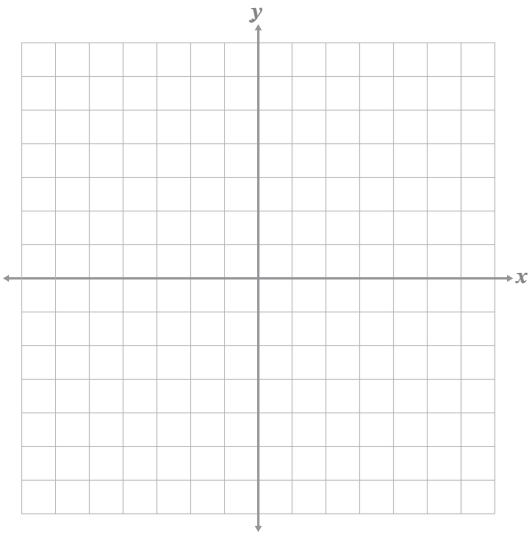


Example 3

Complete the example as you watch the video in the Online Lesson.

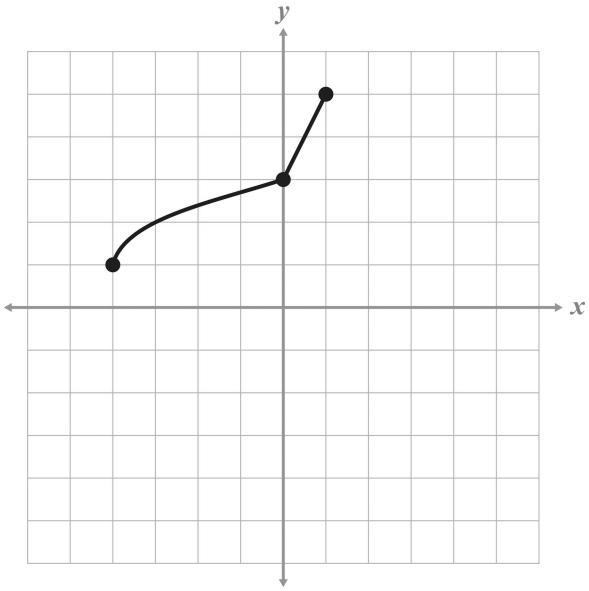
Find the inverse of the function algebraically. Then graph the function and its inverse on the coordinate plane. Explain whether or not the graph and its inverse represent functions.

$y = \frac{1}{3}x$



Checkpoint: Graphing Functions and Their Inverses

Graph the inverse on the coordinate plane.



To continue, return to the Online Lesson.

The Horizontal Line Test

Fill in the notes as you watch the video in the Online Lesson.

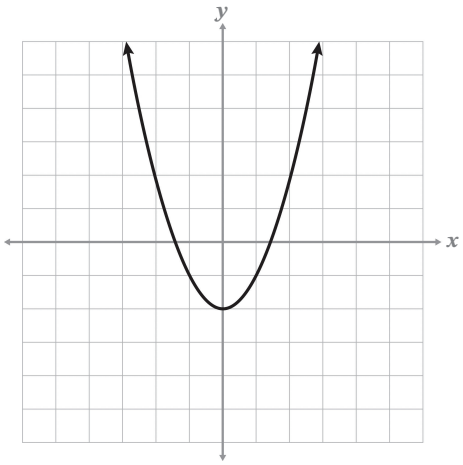
- The _____ is a visual representation that determines if the _____ of a graph on the coordinate plane is a function by running a horizontal line ↔ across the graph.
- If the horizontal line touches _____ at a time, the inverse of the graph is not a function.
- The HLT also determines if a function is _____.
- A one-to-one function has _____ for each input. (In other words, the domain AND range values are both unique for a one-to-one function.)

Example 4

Complete the example as you watch the video in the Online Lesson.

For the given graph:

- **Name the domain and range for the graph and its inverse.**
- **Explain whether or not the graph represents a function.**
- **If the graph is a function, determine if it is one-to-one.**
- **If the graph is a function, determine if the inverse is also a function.**



The parabola is _____ because it passes the VLT. However, it is

_____ because it _____. This also means that the

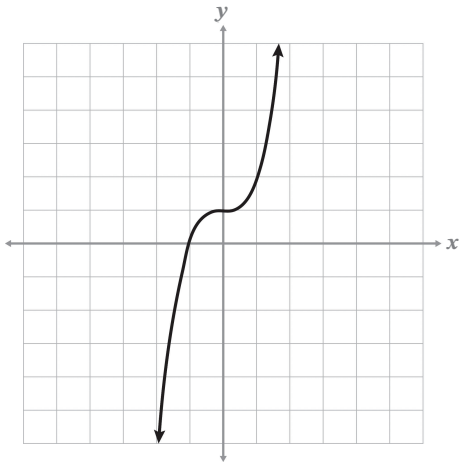
inverse is _____.

Example 5

Complete the example as you watch the video in the Online Lesson.

For the given graph:

- **Name the domain and range for the graph and its inverse.**
- **Explain whether or not the graph represents a function.**
- **If the graph is a function, determine if it is one-to-one.**
- **If the graph is a function, determine if the inverse is also a function.**

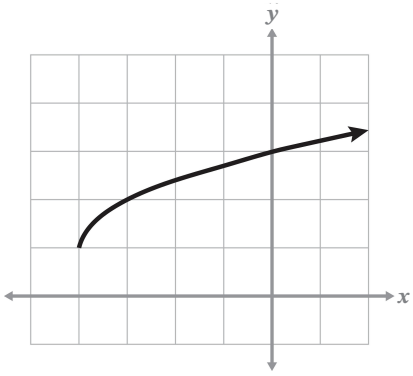


Example 6

Complete the example as you watch the video in the Online Lesson.

For the given graph:

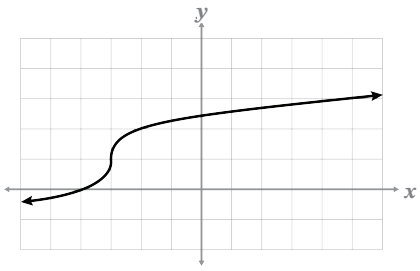
- **Name the domain and range for the graph and its inverse.**
- **Explain whether or not the graph represents a function.**
- **If the graph is a function, determine if it is one-to-one.**
- **If the graph is a function, determine if the inverse is also a function.**



Checkpoint: The Horizontal Line Test

For the given graph:

- **Name the domain and range for the graph and its inverse.**
- **Explain whether or not the graph represents a function.**
- **If the graph is a function, determine if it is one-to-one.**
- **If the graph is a function, determine if the inverse is also a function.**



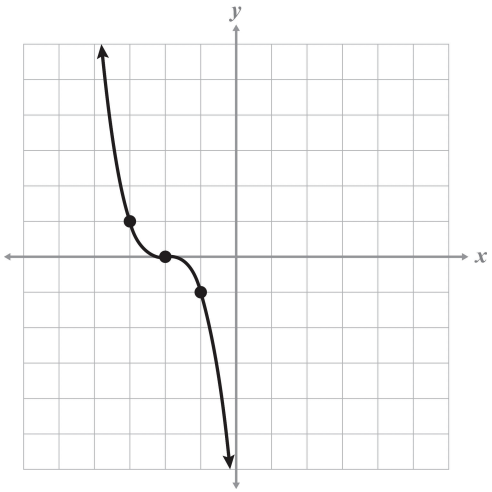
To continue, return to the Online Lesson.

Practice 1

Complete problems on a separate sheet of paper, except where coordinate planes are provided.

Graph the inverse on the provided coordinate plane.

1)



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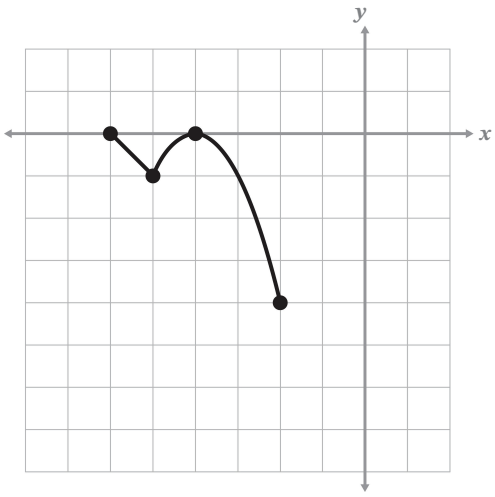

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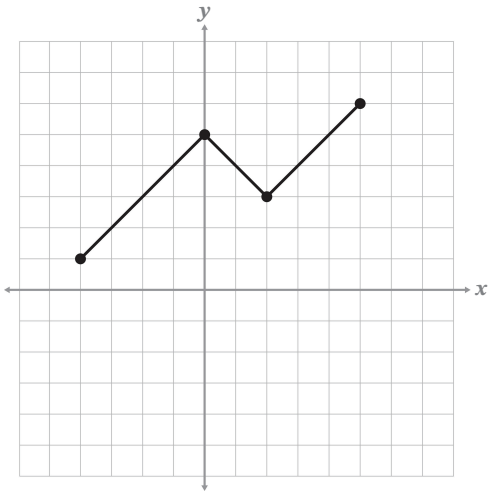

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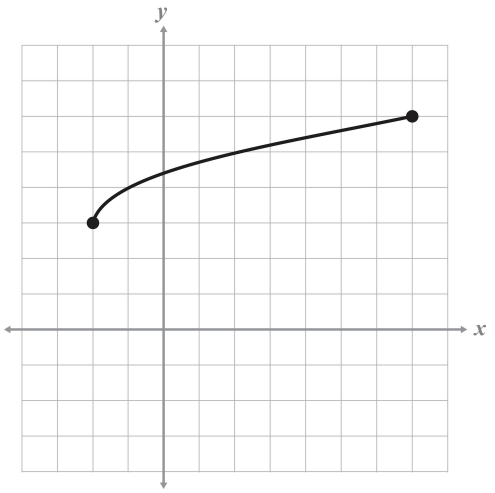
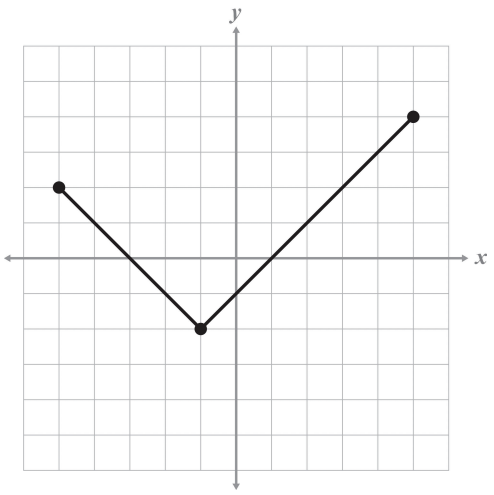


Graph the inverse on the coordinate plane. Explain whether or not the graph and its inverse represent functions.

5)



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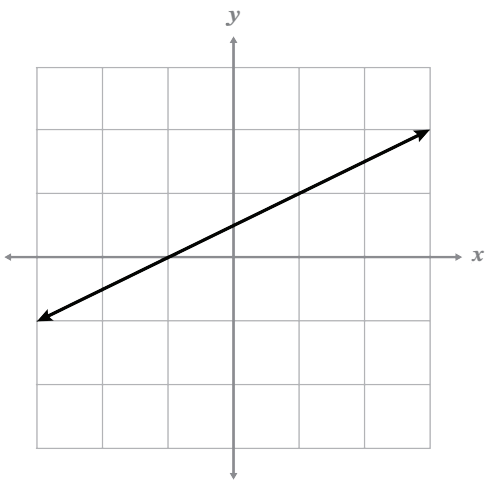

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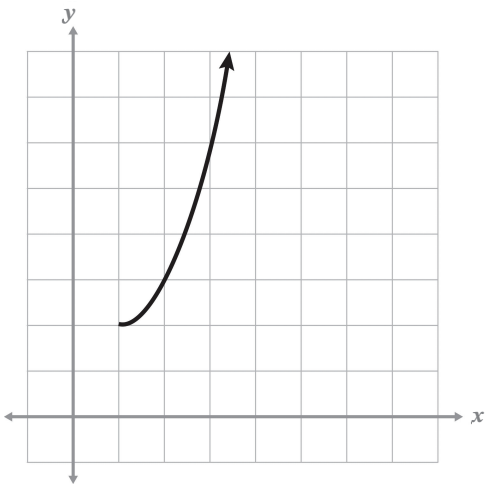


Find the inverse of the function algebraically. Then graph the function and its inverse on the coordinate plane. Explain whether or not the graph and its inverse represent functions.

8) $k(x) = 3(x + 1) - 4$

9) $h(x) = x^2$

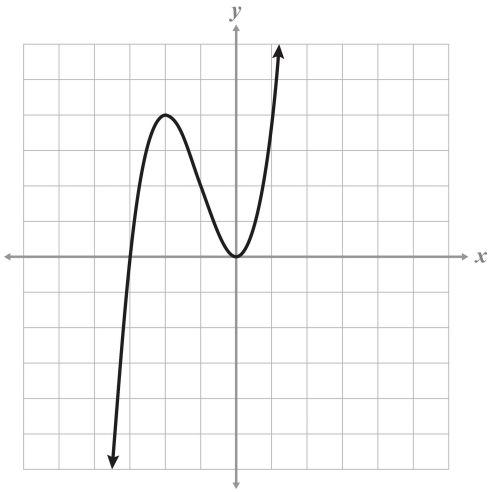
10) $f(x) = 2$

11) $g(x) = -\frac{4}{3}x - 4$

For the given graph:

- **Name the domain and range for the graph and its inverse.**
- **Explain whether or not the graph represents a function.**
- **If the graph is a function, determine if it is one-to-one.**
- **If the graph is a function, determine if the inverse is also a function.**

12)



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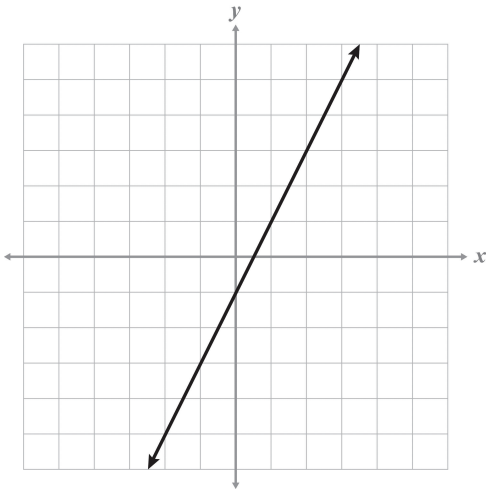

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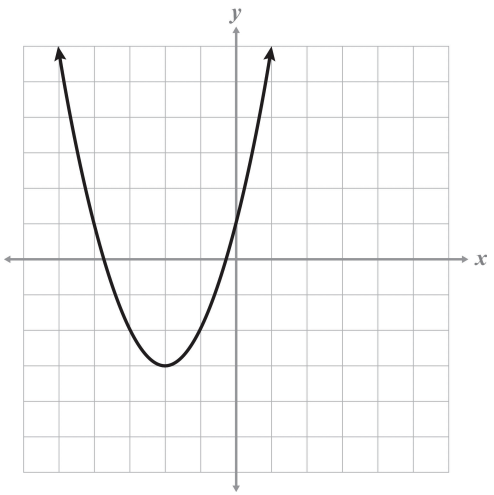

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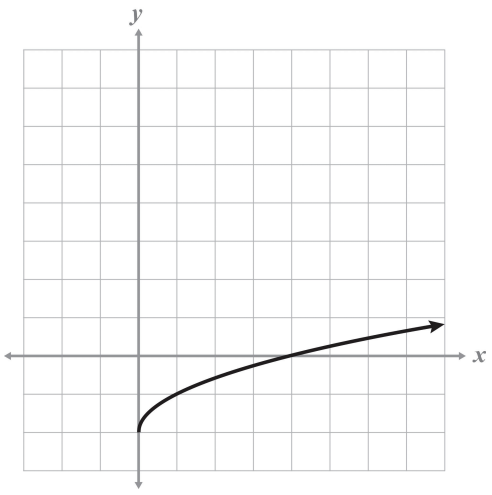

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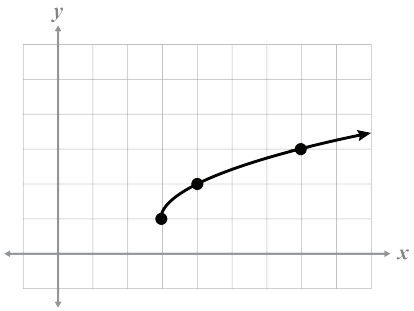
Mastery Check

Show What You Know

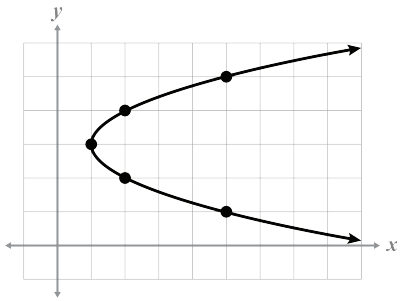
Two students were given the equation $y = x^2$ and instructed to translate it so that $h = 1$, and $k = 3$. Then they needed to graph the inverse of the translated function. Both students have incorrect answers. Explain their errors and show the correct answer.

Student A



A) Student A error analysis:

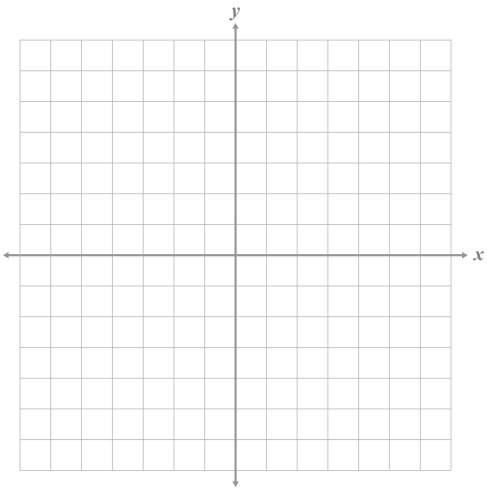
Student B



B) Student B error analysis:

C) Equation for the translated function

D) Graph of the inverse



Say What You Know

In your own words, talk about what you have learned using the objectives for this part of the lesson and your work on this page.

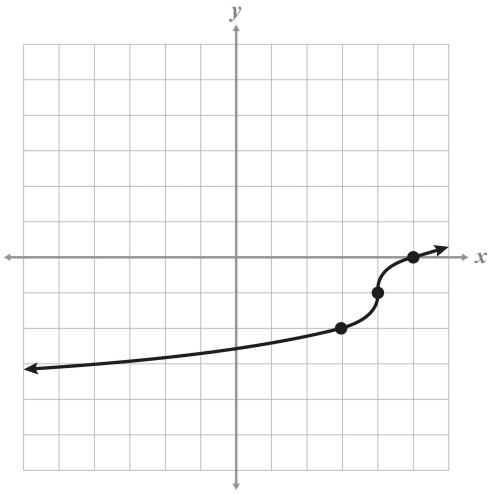
To continue, return to the Online Lesson.

Practice 2

Complete problems on a separate sheet of paper, except where coordinate planes are provided.

Graph the inverse on the coordinate plane.

1)



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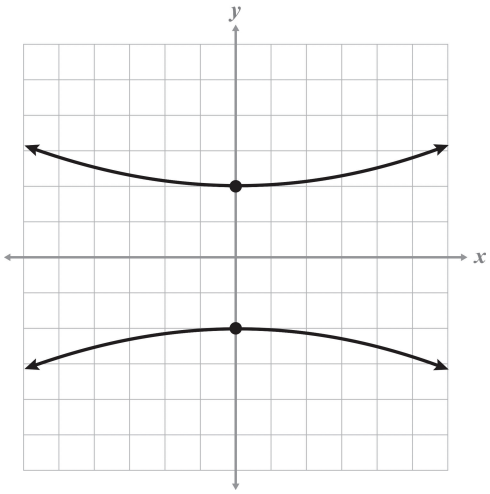

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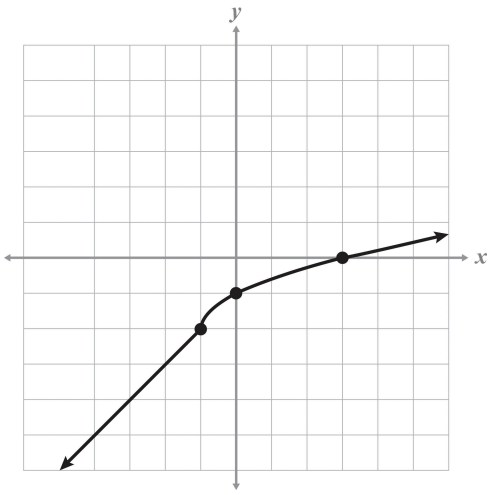

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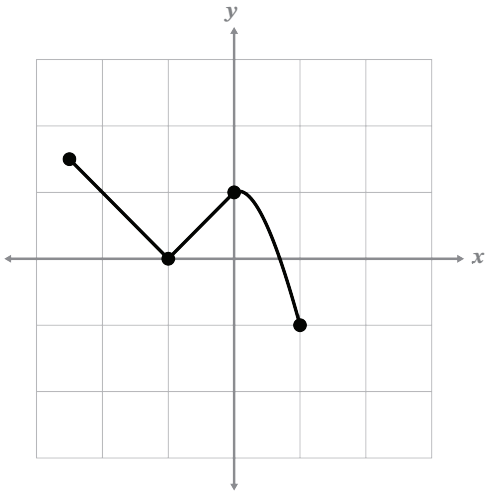
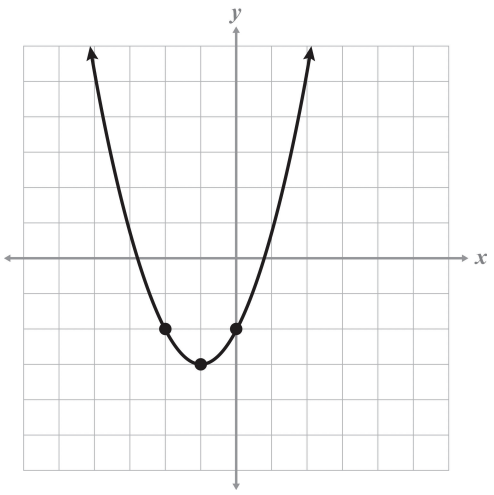


Graph the inverse on the coordinate plane. Explain whether or not the graph and its inverse represent functions.

5)



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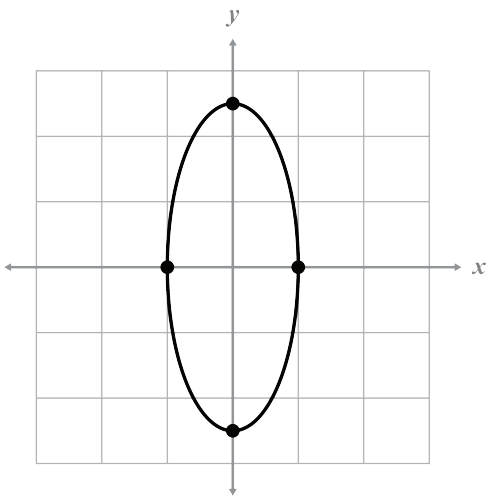

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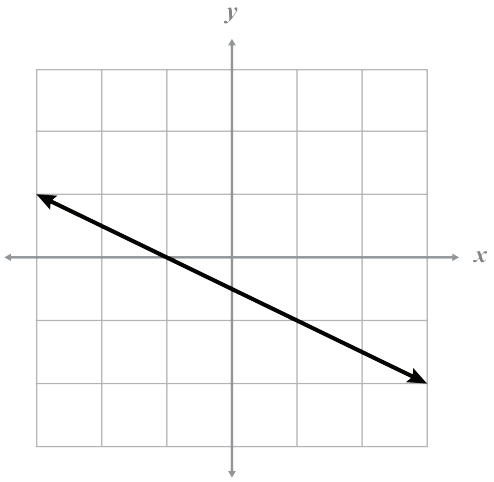

Find the inverse of the function algebraically. Then graph the function and its inverse on the coordinate plane. Explain whether or not the graph and its inverse represent functions.

8) $h(x) = \sqrt[3]{x-2} + 1$

9) $g(x) = 2\sqrt{x-2} + 1 \text{ for } \{x \mid x \geq 2\}$

10) $f(x) = -x^2 - 1$

11) $x = -2$

For the given graph:

- **Name the domain and range for the graph and its inverse.**
- **Explain whether or not the graph represents a function.**
- **If the graph is a function, determine if it is one-to-one.**
- **If the graph is a function, determine if the inverse is also a function.**

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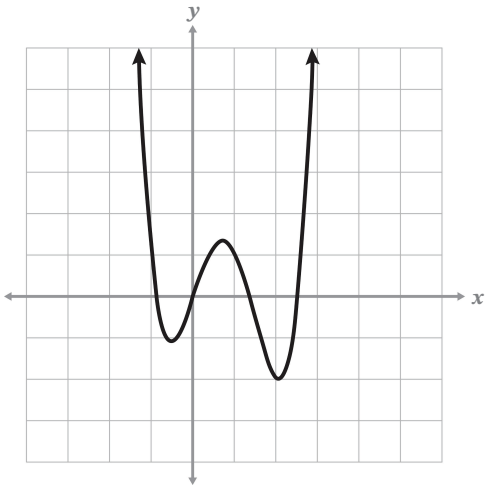

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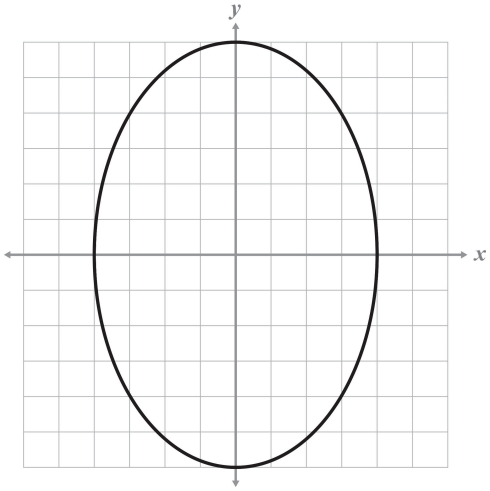

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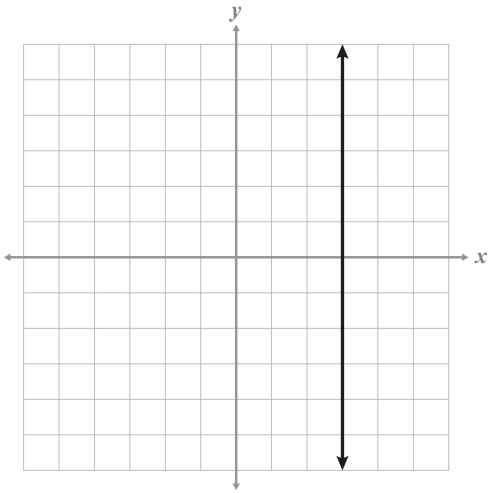

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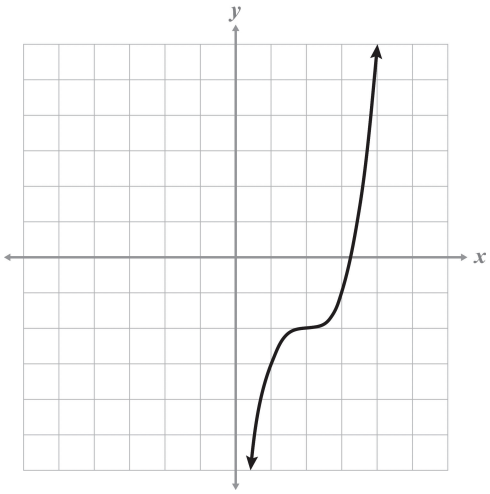

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