

Lesson 19

NAME:

Finding Inverses Algebraically



Start by navigating to the Online Lesson for instructions.

Objectives

- ✓ Determine if functions are inverses of one another algebraically.
- ✓ Find the inverse of a simple function algebraically.

Why?

Inverses allow relationships to be built working forward and backward. Checking that your work makes sense forward and backward is a model many businesses use to complete projects.



Warm Up

Use the given function to complete problems 1 and 2.

$$f(x) = \frac{2}{5}x - \frac{1}{5}$$

1) Evaluate the function at $f(5)$.

2) Evaluate the function at $f(3a + 1)$.



To continue, return to the Online Lesson.



Explore



Inverse Relations and Functions

▶ Fill in the notes as you watch the video in the Online Lesson.

- Recall that a _____ is a set of ordered pairs.
- A function is a relation in which there is _____ for every input.

- _____ can be applied to relations and functions.
- When the inverse of a relation (or function) R^{-1} is found, the relationship between the domain and range is _____.
- This means that the coordinates are _____, or reflected over the line $y = x$ when graphed.
- Two relations R and Q are inverses of one another, if and only if every ordered pair in R is (a, b) and every ordered pair in Q is (b, a) where _____.
 - The _____ of R is the _____ of Q .
 - The _____ of R is the _____ of Q .

Example 1

🎥 Complete the example as you watch the video in the Online Lesson.

Represent relation R as a table. Represent the inverse of the relation R^{-1} as a mapping.

$R = \{(-2, -3), (1, -1), (0, 2), (3, 5)\}$

R

x Domain	y Range
-2	-3
0	2
1	-1
3	5

R^{-1}

Domain

Range

Compare the domain and range of R and R^{-1} . Explain if either R or R^{-1} are functions.

The domain of R is the range of R^{-1} and the range of R is _____.

_____ because each domain is unique.

Example 2

▶ Complete the example as you watch the video in the Online Lesson.

A small town surveyed its community to determine the type of transportation townspeople use in a typical week. The survey collected the type of transportation people used and the number of wheels associated with the transportation. A relation was written to compile the data.

Find the inverse of the relation. Explain how you know if the relation and its inverse are functions or not.

$$W = \{(\text{bicycle}, 2), (\text{car}, 4), (\text{truck}, 4), (\text{semi-truck cab}, 10), (\text{motorcycle}, 2), (\text{feet}, 0), (\text{SUV}, 4)\}$$

 W

Domain

Range

The relation W is a function because each type of transportation, or the domain, _____.

 W^{-1}

Domain

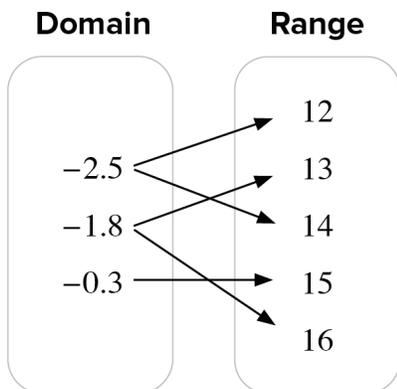
Range

W^{-1} is _____

because multiple types of transportation have the same number of wheels, which means the domain has repeated values.

Checkpoint: Inverse Relations and Functions

Write the inverse of relation P as a table. Explain if the inverse represents a function or not.

 P 

To continue, return to the Online Lesson.

📺 Verifying Inverses

▶ Fill in the notes as you watch the video in the Online Lesson.

- Two functions, f and g , are _____ of one another when $f(a) = b$ and $g(b) = a$.
- Any ordered pair (a, b) in the given function is _____.

Example 3

▶ Complete the example as you watch the video in the Online Lesson.

Verify that the functions $f(x)$ and $g(x)$ are inverses of one another using $f(-12)$.

$$f(x) = 8(x + 14)^3 - 18$$

$$g(x) = \frac{\sqrt[3]{x + 18}}{2} - 14$$

Plan

Find $f(a) = b$

Find $g(b)$

Implement

$$x = -12$$

$$f(-12) = 8((-12) + 14)^3 - 18$$

$$f(-12) = 8(2)^3 - 18$$

$$f(-12) = 8(8) - 18$$

$$f(-12) = 64 - 18$$

$$f(-12) = 46$$

Explain

▶ Substitute

▶ Evaluate

The functions are inverses of one another by definition since $f(-12) = 46$ and $g(46) = -12$.

Example 4

▶ Complete the example as you watch the video in the Online Lesson.

Verify that the given functions are inverses of one another for $h(1)$. Explain your reasoning.

$$h(x) = 4x - 8$$

$$j(x) = \frac{1}{4}x + 8$$

Example 5

 Complete the example as you watch the video in the Online Lesson.

Verify which function, $j(x)$ or $k(x)$, is the inverse of $g(x)$.

$$g(x) = \sqrt{x-3} - 1$$

$$j(x) = (x+3)^2 + 1, \{x|x \geq -3\}$$

$$k(x) = (x+1)^2 + 3, \{x|x \geq -1\}$$

$$g(3) = -1$$

Checkpoint: Verifying Inverses

Verify that the given functions are inverses of one another using $f(0)$. Explain.

$$f(x) = \frac{x-4}{x+2}$$

$$g(x) = \frac{2x+4}{1-x}$$



To continue, return to the Online Lesson.

Inverses of Functions Algebraically

 Fill in the notes as you watch the video in the Online Lesson.

- The inverse of a function can be found _____ by interchanging, or switching, the domain and range values.
- When given a function, find its inverse by _____ the variable _____ with the variable _____.
- Then solve for the inverse by _____ y in the new equation, which gives you the inverse of the original function.
- To check that you have found the inverse, determine if _____.
- Function notation $f(x)$ is used to represent _____ (when the inverse is also a function).
- The notation for the inverse of a function is _____, and is read “the inverse of the function” or “ f inverse.”
- The “-1” in the $f^{-1}(x)$ notation is _____; **not an exponent**. It cannot be written as a fraction. $f^{-1}(x) \neq \frac{1}{f(x)}$
- It is important to note that, as with relations, not every inverse of a function will also be _____.

Example 6

▶ Complete the example as you watch the video in the Online Lesson.

Find the inverse of the function. Then check your work using $f(6)$.

$$f(x) = \frac{3}{x-4}$$

Plan

Write as a “y =” equation

Switch x and y

Solve for y

Check

Implement

$$y = \frac{3}{x-4}$$

$$x = \frac{3}{y-4}$$

Explain

- ▶ Write as $y =$
- ▶ Switch x and y
- ▶ Solve for y

Check

- ▶ Find $f(6)$
- ▶ Find $f^{-1}\left(\frac{3}{2}\right)$

Example 7

▶ Complete the example as you watch the video in the Online Lesson.

Find the inverse of the function. Check your work.

$$h(x) = \frac{5}{8}x - 7$$

Example 8

▶ Complete the example as you watch the video in the Online Lesson.

Find the inverse of the function. Check your work.

$$g(x) = \frac{1}{4}x^2 + 5, \{x|x \geq 0\}$$

Checkpoint: Inverses of Functions Algebraically

Find the inverse of the function. Check your work.

$$q(x) = \frac{9}{2x - 1}$$



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 **Practice 1**

Complete problems on a separate sheet of paper.

- 1) Explain how the inverse of a function is found algebraically.

Given relation R , create a table of R and a mapping of R^{-1} . Explain whether the relation and its inverse are functions.

2) $R = \{(5, 2), (6, -7), (3, 1), (8, -10)\}$

3) $R = \{(-3, 4), (9, -2), (6, 1), (-4, 3), (2, 1)\}$

- 4) A class was asked to list their favorite summer activity. The results were: $R = \{(Maddie, hiking), (Hope, relaxing), (Stetson, swimming), (Austin, hiking), (Natalee, relaxing)\}$.

Verify that the given functions are inverses of one another using $f(4)$.

5) $f(x) = 9(x-2)^2 - 1$ $g(x) = \frac{1}{3}\sqrt{x+1} + 2$

6) $f(x) = \frac{5}{4}x - 2$ $g(x) = \frac{4}{5}x + 2$

7) $f(x) = 2\sqrt[3]{x-4} + 5$ $g(x) = \frac{1}{2}(x+5)^3 + 4$

Find the inverse of $f(x)$ algebraically.

8) $f(x) = 7x + 9$, check with $x = 6$.

9) $f(x) = 2\sqrt{x} - 3$ for $\{x|x \geq 0\}$, check with $x = 9$.

10) $f(x) = 8(x+1)^3$, check with $x = -2$.

Find the inverse of $f(x)$ algebraically.

Name the domain and range for the given function as well as its inverse.

11) $f(x) = \frac{1}{x-4} + 1$, check with $x = 8$.

12) $f(x) = (x+1)^2 + 3$, $\{x|x \geq -1\}$, check with $x = 5$.



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 **Mastery Check** **Show What You Know**

Two groups of students (Red and Blue) want to determine if the functions $f(x)$ and $g(x)$ are inverses of one another. Both groups decided to use $x = 7$ and stated that $f(x)$ and $g(x)$ were inverses.

Red Group

$$f(x) = \frac{5}{3}(x-4) + 2$$

$$f(7) = \frac{5}{3}(7-4) + 2$$

$$f(7) = \frac{5}{3}(3) + 2$$

$$f(7) = 5 + 2$$

$$f(7) = 7$$

Blue Group

$$g(x) = \frac{3}{5}\sqrt{x-7} + 7$$

$$g(7) = \frac{3}{5}\sqrt{7-7} + 7$$

$$g(7) = \frac{3}{5}\sqrt{0} + 7$$

$$g(7) = 0 + 7$$

$$g(7) = 7$$

The teacher told both groups to look more closely at the problem because $f(x)$ and $g(x)$ are not inverses.

- A)** The Red group decided to switch the domain and range. Show the work that the Red group would have completed.

B) The Blue group decided to use a calculator when $x = -2$ and $x = 10$. Show the work that the Blue group would have completed.

C) Using parts A and B, explain why $f(x)$ and $g(x)$ are not inverses of one another.

 **Say What You Know**

In your own words, talk about what you have learned using the objectives for this lesson and your work on this page.



To continue, return to the Online Lesson.

 **Practice 2**

Complete problems on a separate sheet of paper.

- 1) Explain how you can check to determine if the equation you found is the inverse.

Given relation R , create a table of R and a mapping of R^{-1} . Explain whether the relation and its inverse are functions.

2) $R = \{(-3, 12), (-6, 7), (1, 3), (-12, 12)\}$

3) $R = \{(3, 4), (9, 2), (-6, 1), (4, 3), (2, -1)\}$

- 4) The same class from Practice 1 was asked to list their favorite sport. The results were: $R = \{(Maddie, softball), (Hope, volleyball), (Stetson, soccer), (Austin, baseball), (Natalee, volleyball)\}$.

Verify that the given functions are inverses of one another using $f(3)$.

5) $f(x) = -(x-2)^3 + 2$ $g(x) = \sqrt[3]{-x+2} + 2$

6) $f(x) = x^2 + 3$ $g(x) = \sqrt{x+3}$

7) $f(x) = 2x + 5$ $g(x) = \frac{x-5}{2}$

Find the inverse of $f(x)$ algebraically.

8) $f(x) = \frac{2}{3}x + 1$, check with $x = 6$.

9) $f(x) = \sqrt{x+4}$ for $x \geq -4$, check with $x = 5$.

10) $h(x) = \frac{1}{x+2} - 3$, $x \neq -2$, check with $x = 2$.

Find the inverse of $f(x)$ algebraically.

Name the domain and range for the given function as well as its inverse.

11) $f(x) = \frac{2}{5}x + 7$, check with $x = 5$.

12) $g(x) = \frac{1}{4}x^2 + 3$, for $\{x|x \geq 0\}$, check with $x = 3$.



To continue, return to the Online Lesson.

Targeted Review

Complete items on a separate sheet of paper.

Simplify.

1) $(x + \sqrt{5})(2x - \sqrt{3}) + \sqrt{15}$

2) $(i^{36})(i^{17})$

3) $(2 + 3i)(7 - i^3)$

4) $\frac{y}{3 + \sqrt{x}}$

5) Describe the end behavior for the function $m(x) = \sqrt{x - 8} - 2$

Solve. Graph solutions on a number line.

6) $\sqrt{x + 5} \geq \sqrt{3 - x}$

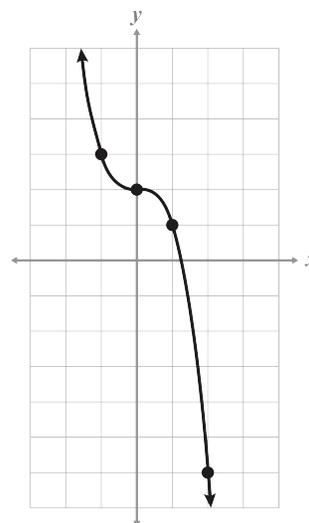
7) $x > \sqrt{2x + 3}$

8) Name the domain and range of the absolute value parent function in set builder notation.

Multiple Choice

_____ 9) Which type of equation would best represent the given graph?

- A) absolute value
- B) cubic
- C) square root
- D) quadratic

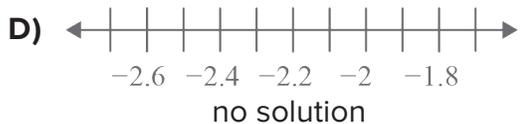
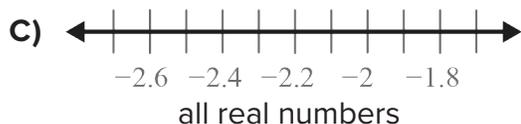
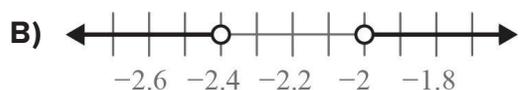
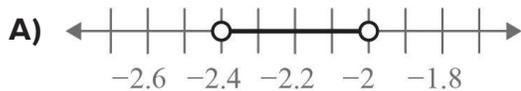


_____ 10) Choose the best description of the transformation of the quadratic parent function for the equation $q(x) = 2(x + 8)^2 - 11$.

- A) The graph is vertically stretched by a factor of 2, moved 8 spaces left, and 11 spaces down from the parent graph.
- B) The graph is vertically compressed by a factor of 2, moved 8 spaces right, and 11 spaces down from the parent graph.
- C) The graph is vertically compressed by a factor of 2, moved 8 spaces left, and 11 spaces down from the parent graph.
- D) The graph is vertically stretched by a factor of 2, moved 8 spaces right, and 11 spaces down from the parent graph.

- ____ 11) Find the perimeter of a rectangle with a length of $6x + 7$ units and a width of $4x - 1$ units.
- A) $10x + 6$ units
 - B) $20x + 12$ units
 - C) $24x^2 + 22x - 7$ square units
 - D) $20x + 12$ square units

____ 12) Solve: $-|5x + 11| > 1$



Problem	1	2	3	4	5	6	7	8	9	10	11	12
Origin	L12	L15	L16	L12	L18	L14	L14	L17	L17	L18	A1	A1

L = Lesson in this level, A1 = Algebra 1: Principles of Secondary Mathematics, FD = Foundational Knowledge

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