Lesson 15

NAME:

Complex Numbers and the Imaginary Unit

Start by navigating to the Online Lesson for instructions.

Objectives

- Classify complex numbers (real, imaginary, complex).
- Define the imaginary unit as $i^2 = -1$.
- Simplify roots of negative numbers using the imaginary unit.
- Simplify powers of the imaginary unit.

Why?

Complex numbers show the entire number system, Real and Imaginary. This concept helps deepen your understanding when graphing polynomials. But first, you need to learn what complex numbers are and how to work with them. Imaginary numbers are important because they provide ways to find solutions that otherwise would not have a real number solution, which is important in solving problems in engineering and physics.

Warm Up

Simplify.

1) $\left(2x - \sqrt{3}\right)\left(2x + \sqrt{3}\right)$

2) $x^{55} \div x^{11}$

3) $\left(x^8\right)^4$

4) $\sqrt{-4}$

To continue, return to the Online Lesson.

Explore

The Imaginary Unit i

Fill in the notes as you watch the video in the Online Lesson.

- The imaginary unit i is a special number in math that is unique because it does not have a

  ______________________________.

- The imaginary unit i is defined as: ________________________

- The number i is the principal square root of -1, or: ___________________

- Imaginary numbers can be ______________________ using the following guidelines:

Rules for the Imaginary Unit

$i^0 = 1$

$i = \sqrt{-1}$

$i^2 = -1$

$i^3 = -i$

$i^4 = 1$

- When evaluating an expression using the number i, use the ______________________________ to rewrite the base as i^4 raised to a power because $i^4 = 1$ will be easier to work with.
- If there is a ___________________________, it determines if the answer will be i, -1, or $-i$.

Example 1

Complete the example as you watch the video in the Online Lesson.

Evaluate.

A) i^9

$i^8 \cdot i$

$(i^4)^2 \cdot i$

$(1)^2 \cdot i$

B) i^{10}

$i^8 \cdot i^2$

$(i^4)^2 \cdot i^2$

C) i^{11}

$i^8 \cdot i^3$

D) i^{12}

Example 2

Complete the example as you watch the video in the Online Lesson.

Evaluate.

A) i^{39}

$39 \div 4 = 9 \text{ R3}$

$\left(i^4\right)^9 \cdot i^3$

$-i$

B) i^{85}

C) i^{46}

Checkpoint: The Imaginary Unit i

Evaluate.

A) i^{18}

B) i^{43}

To continue, return to the Online Lesson.

Negative Square Roots

Fill in the notes as you watch the video in the Online Lesson.

- The rules to simplify radical expressions are only true when the radicands are ______________________.
- However, because the number i is the principal square root of –1, or $i = \sqrt{-1}$, problems that have a ________________ radicand can now be simplified using the ______________________.
- Therefore, ______ **must** be simplified out of the radicand **first**. Then the remaining radicand can be simplified, if possible.

Example 3

Complete the example as you watch the video in the Online Lesson.

Simplify.

$\sqrt{-16}$

Implement

$\sqrt{-1 \cdot 2^4}$

$i \cdot 2^2$

$4i$

Explain

- Prime factorization
- Simplify

Example 4

Complete the example as you watch the video in the Online Lesson.

Simplify.

$\sqrt{-12}$

Implement

Explain

- Prime factorization
- Simplify

Example 5

Complete the example as you watch the video in the Online Lesson.

Simplify.

$\sqrt{-8} \cdot \sqrt{-6}$

Implement

Explain

- Simplify out $\sqrt{-1}$
- Simplify i and radicands

Checkpoint: Negative Square Roots

Simplify.

A) $\sqrt{-100}$

B) $\sqrt{-33} \cdot \sqrt{3}$

To continue, return to the Online Lesson.

The Complex Number System

Fill in the notes as you watch the video in the Online Lesson.

- The complex number system consists of numbers in the form ________________, where a and b are part of the ________________ number system, or $a \in \mathbb{R}$, $b \in \mathbb{R}$.
- ____ includes the set of real numbers $\mathbb{R}$, and the set of imaginary numbers i.
- The number i can only be combined with ____________________________, meaning other imaginary numbers.
- This means that a real number and an imaginary number can only be written as an __________________________.
  - ______________________________ are written as $a + bi$ where $b = 0$ (or a).
  - ____ are written as $a + bi$ where $a = 0$ (or bi).

Complex Numbers ($\mathbb{C}$)

$a + bi$ $-4 - 9i$ $11 + 2i$

Real Numbers ($\mathbb{R}$)	**Imaginary Numbers (i)**
$a + bi$, where $b = 0$ Includes: Natural $\mathbb{N}$, Whole $\mathbb{W}$, Integer $\mathbb{Z}$, Rational $\mathbb{Q}$, Irrational $\mathbb{I}$ $\sqrt{81}$ 112 -3 $-5 + 0i$	$a + bi$, where $a = 0$ $\sqrt{-1}$ $4i$ $-6i$

Example 6

Complete the example as you watch the video in the Online Lesson.

Classify by all sets to which it belongs: real, pure imaginary, complex. Then add A–D to the Complex Number diagram on the previous page.

A) $15+3i$

B) $-8i$

C) $-48+i^2$

D) $7-10i$

Checkpoint: The Complex Number System

Classify by all sets to which it belongs: real, pure imaginary, complex. Then add to the Complex Number diagram (in the guided notes).

A) $26i$

B) $3+7i$

To continue, return to the Online Lesson.

Practice 1

Complete problems on a separate sheet of paper.

Evaluate.

1) i^{29} **2)** i^{75} **3)** i^{16}

4) i^{50} **5)** i^{120} **6)** i^{101}

7) i^{38} **8)** i^{67}

Simplify.

9) $\sqrt{-18}$ **10)** $\sqrt{-36} \cdot \sqrt{-16}$ **11)** $\sqrt{-54}$

12) $\sqrt{-32}$ **13)** $\sqrt{-144}$ **14)** $3\sqrt{-10} \cdot 7\sqrt{-35}$

15) $2\sqrt{-81} \cdot \sqrt{-24}$ **16)** $\sqrt{-98}$

Simplify. Then classify by all sets to which it belongs: real, pure imaginary, complex.

17) $\sqrt{12}$ **18)** $\sqrt{16} + \sqrt{-8}$ **19)** $\sqrt{-121}$

20) $5 + \sqrt{49}$ **21)** $3i$ **22)** $\sqrt{-25}$

23) $\sqrt{-49} \cdot \sqrt{-4}$ **24)** $13 - \sqrt{-9}$

To continue, return to the Online Lesson.

Mastery Check

Show What You Know

Complete the statement with always, sometimes, or never.
If your answer is sometimes or never, provide an example to back up your choice.

A) The imaginary unit i raised to a whole number power is ____________________ a pure imaginary number.

B) Complex numbers __________________ include the sets of real and imaginary numbers.

C) The square root of a negative number is _______________ a real number.

D) Complex numbers are ____________________ classified as real, complex numbers.

Say What You Know

In your own words, talk about what you have learned using the objectives for this part of the lesson and your work on this page.

To continue, return to the Online Lesson.

Practice 2

Complete problems on a separate sheet of paper.

Evaluate.

1) i^{2}

2) i^{78}

3) i^{15}

4) i^{10}

5) i^{61}

6) i^{99}

Simplify.

7) $3\sqrt{-5}\cdot 2\sqrt{-4}$

8) $6\sqrt{-40}$

9) $\sqrt{-12}\cdot\sqrt{-3}$

10) $\sqrt{-196}$

11) $20\sqrt{-7}\cdot\sqrt{-14}$

12) $\sqrt{-64}$

Simplify. Then classify by all sets to which it belongs: real, pure imaginary, complex.

13) $\sqrt{-4}$

14) $\sqrt{-49}\cdot\sqrt{-100}$

15) $-5\sqrt{50}\cdot\sqrt{-8}$

16) $\sqrt{-9}\cdot\sqrt{-81}$

17) $\sqrt{121}-\sqrt{-20}$

18) $\sqrt{-45}$

To continue, return to the Online Lesson.

Targeted Review

Complete problems on a separate sheet of paper.

Simplify.

1) $\sqrt{35x} \cdot \sqrt{15x^3}$

2) $\left(10+\sqrt{7}\right)-\left(8+12\sqrt{7}\right)$

3) $\dfrac{2+\sqrt{5}}{10+\sqrt{6}}$

Solve.

4) $\sqrt[3]{x+3}=3$

5) $\sqrt{x}+5 \geq 7$

6) $\sqrt{2x-3}<7$

Graph.

7) $y=\dfrac{4}{3}x-2$

8) $y=-\dfrac{1}{x-3}-2$

Multiple Choice

_____ 9) Simplify. $\sqrt[3]{48x^8y^3}$

A) $2x^2y\sqrt[3]{6x^2}$

B) $2x^2y\sqrt[3]{3x^2}$

C) $4x^4y^2\sqrt[3]{3y}$

D) $16x^2y\sqrt[3]{x^2}$

_____ 10) Solve. $\sqrt{3x-1}-6=-2$

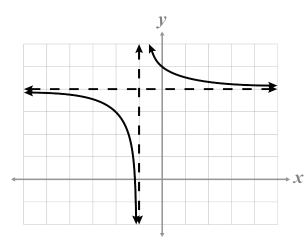
A) $\dfrac{1}{3}$

B) 5

C) $\dfrac{17}{3}$

D) no solution

_____ 11) Name the range using the graph of the rational function.



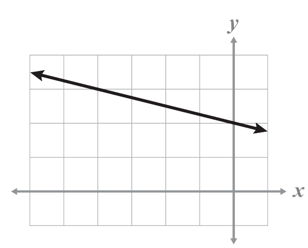
A) $\{y \mid y \in \mathbb{R},\ y \neq -1\}$

B) $\{y \mid y \in \mathbb{R},\ y \neq 0\}$

C) $\{y \mid y \in \mathbb{R},\ y \neq 4\}$

D) $\{y \mid y \in \mathbb{R},\ y \neq 5\}$

_____ 12) Determine the equation that best represents the linear function.



A) $y=\dfrac{1}{4}x+2$

B) $y=-\dfrac{1}{4}x+2$

C) $y=-\dfrac{1}{5}x+2$

D) $y=\dfrac{1}{5}x+2$

Problem	1	2	3	4	5	6	7	8	9	10	11	12
Origin	L11	L12	L12	L13	L14	L14	A1	L10	L11	L13	L10	A1

L = Lesson in this level, A1 = Algebra 1: Principles of Secondary Mathematics, FD = Foundational Knowledge

To continue, return to the Online Lesson.