# Lesson 13
# Solving Radical Equations

NAME:

Start by navigating to the Online Lesson for instructions.

## Objectives

- Solve a radical equation.
- Solve a radical equation written with rational (fractional) exponents.
- Determine if there are any extraneous solutions to a radical equation.

## Why?

Solving equations with radicals or fractional exponents allows you to use expressions and equations to solve problems more efficiently. This practice prepares you for working with many new formulas and applying them to real-life situations in future lessons.

## Warm Up

**Use the Formula Sheet to name the property used to solve the equation.**

$\frac{4}{3}(2x-5)=-12$

**Implement** **Explain**

**1)** $\left(\frac{3}{4}\right)\frac{4}{3}(2x-5)=-12\left(\frac{3}{4}\right)$

**2)** $2x-5=-9$

$+5 \quad +5$

**3)** $\left(\frac{1}{2}\right)2x=-4\left(\frac{1}{2}\right)$

**4)** $x=-2$

**Simplify without using a calculator.**

**5)** $9^{\frac{3}{2}}$

**6)** $(-32)^{\frac{2}{5}}$

**To continue, return to the Online Lesson.**

# Explore

## Solving Radical Equations

*Fill in the notes as you watch the video in the Online Lesson.*

- Solve equations with radicals using _____.
- Recall that radicals can be written as: _____
- To find the inverse of a radical, raise _____ of the equation by the index $d$.
- If you raise both sides of an equation to an **even** power, it is important to check for _____ solutions.
- Raising to an **even** power can produce extraneous solutions because it cannot be undone.
  - If $c = d$, then $c^2 = d^2$ is _____ true. But if $c^2 = d^2$, then _____ $c = d$. For example:

$$x = 5$$
$$x^2 = 25$$
$$\sqrt{x^2} = \sqrt{25}$$
$$x = \pm 5$$
$$-5 \text{ is extraneous}$$

- Raising to an **odd** power does not result in extraneous solutions.
  - If $c = d$, then $c^3 = d^3$, and if $c^3 = d^3$, then $c = d$. For example:

$$x = -5$$
$$x^3 = -125$$
$$\sqrt[3]{x^3} = \sqrt[3]{-125}$$
$$x = -5$$

- Extraneous solutions can occur when:
  - a solved value results in a _____ radicand.
  - there are radicals on _____ of the equation.
- To check for extraneous solutions, use _____.

## Example 1

*Complete the example as you watch the video in the Online Lesson.*

**Solve.**

$\sqrt{3x-5} = \sqrt{8-x}$

**Implement**

$\left(\sqrt{3x-5}\right)^2 = \left(\sqrt{8-x}\right)^2$

**Explain**

- Square both sides
- Isolate $x$
- Using decimal values may be more efficient because a common denominator is not needed. Recommended: Use a calculator to check radical equations

**Check**

$\sqrt{3(3.25)-5} = \sqrt{8-(3.25)}$

$\sqrt{9.75-5} = \sqrt{4.75}$

$\sqrt{4.75} = \sqrt{4.75}$ ✓

## Example 2

*Complete the example as you watch the video in the Online Lesson.*

**Solve.**

$2+\sqrt{x}=\sqrt{x-12}$

**Implement**

**Explain**

- Square both sides
- Distribute on the left side
- Isolate $x$
- The principal square root cannot equal a negative number

## Example 3

*Complete the example as you watch the video in the Online Lesson.*

**Solve.**

$\sqrt{x+8}-\sqrt{x-5}=3$

**Implement**

$\sqrt{x+8}=\sqrt{x-5}+3$

**Explain**

- Rewrite with one radical expression on each side of the equation
- Square both sides
- Isolate the radical to solve for $x$
- Isolate the radical on the right side to solve for the remaining variable in the equation
- Remember to use a simplified fraction

## Checkpoint: Solving Radical Equations

**Solve. Check your work.**

$\sqrt{x+1} = 5 - \sqrt{x}$

**To continue, return to the Online Lesson.**

## Equations with Fractional Exponents

*Fill in the notes as you watch the video in the Online Lesson.*

- Radical equations can be written using _____.
- Use the _____ of a fractional exponent to isolate the variable.
- If the numerator $n$ of a fractional exponent is an even number, then:
- Bases with even powers will only have _____ answers.

## Example 4

*Complete the example as you watch the video in the Online Lesson.*

**Solve.**

$$\frac{2}{5}(5x-8)^{\frac{1}{3}}+3=5$$

**Implement**

$$\frac{2}{5}(5x-8)^{\frac{1}{3}}=2$$

$$(5x-8)^{\frac{1}{3}}=5$$

$$\left((5x-8)^{\frac{1}{3}}\right)^3=(5)^3$$

**Explain**

- Isolate the expression with the rational exponent
- Raise both sides to the reciprocal of the exponent
- Isolate the variable

## Example 5

*Complete the example as you watch the video in the Online Lesson.*

**Solve.**

$$(2x+5)^{\frac{1}{2}}=x+3$$

## Example 6

*Complete the example as you watch the video in the Online Lesson.*

**Solve.**

$$(3x+1)^{\frac{2}{3}} = 16$$

**Implement**

$$\left((3x+1)^{\frac{2}{3}}\right)^{\frac{3}{2}} = (16)^{\frac{3}{2}}$$

$$(16)^{\frac{3}{2}} = \left(16^{\frac{1}{2}}\right)^3 = 4^3 = 64$$

**Explain**

- $\left(x^{\frac{n}{d}}\right)^{\frac{d}{n}} = |x|$ when $n$ is an even number
- Solve Case 1 and Case 2

## Checkpoint: Equations with Fractional Exponents

**Solve. Check your work.**

$$\frac{5}{3}(4-x)^{\frac{1}{5}} + 5 = 10$$

**To continue, return to the Online Lesson.**

# Practice 1

*Complete problems on a separate sheet of paper.*

**1)** Explain how radicals and rational exponents are related.

**Solve. Check for extraneous solutions.**

**2)** $\sqrt{3x+1}-4=2$

**3)** $5\sqrt{2y-8}=15$

**4)** $(x+6)^{\frac{1}{2}}=x+4$

**5)** $\sqrt{2n-6}+n=3$

**6)** $\sqrt{2z+4}-\sqrt{z}=2$

**7)** $8(3x-1)^{\frac{1}{5}}+2=18$

**8)** $\sqrt{y+7}+5=y$

**9)** $\sqrt{4x+5}=x+2$

**10)** $2(r+3)^{\frac{3}{2}}=54$

**11)** $(6x-5)^{\frac{1}{2}}+3=-2$

**12)** $\sqrt{6x-3}=\sqrt{5x+2}$

**13)** $5(w-3)^{\frac{2}{3}}=45$

**14)** $2\sqrt{a+5}=\sqrt{3a+1}$

**15)** $(n-2)^{\frac{4}{5}}=16$

**16)** The compound interest formula is $F=P(1+r)^n$, where $F$ is the future value, $P$ is the present value, $r$ is the percent interest rate, and $n$ is the number of years. What percent interest rate would you need to end with a future value of \$2,151.86, when you get compound interest on \$2,000 for 18 months? Round to the nearest tenth of a percent.

**To continue, return to the Online Lesson.**

# Mastery Check

## Show What You Know

The ratio of the length to width of a certain rectangle is: $\frac{\sqrt{2x}+5}{\sqrt{2x}+1}$

**A)** Using the given side lengths, determine the perimeter of the rectangle.

**B)** The area of the same rectangle is $5x + 5$ square meters. Solve for $x$.

**C)** Determine the value of the length and width of the rectangle.

## Say What You Know

In your own words, talk about what you have learned using the objectives for this part of the lesson and your work on this page.

**To continue, return to the Online Lesson.**

# Practice 2

*Complete problems on a separate sheet of paper.*

**1)** Explain why the absolute value of $x$ must be taken when $n$ is an even exponent.

**Solve. Check for extraneous solutions.**

**2)** $(4x)^{\frac{1}{3}} = (3x-4)^{\frac{1}{3}}$

**3)** $\sqrt{h^2-9}+3=h$

**4)** $\sqrt{a} = \sqrt{a-8}-2$

**5)** $(2w-5)^{\frac{2}{3}} = 9$

**6)** $\sqrt{x}-\sqrt{x-4}=1$

**7)** $(m+7)^{\frac{1}{2}}-5=m$

**8)** $2(x-2)^{\frac{3}{2}} = 54$

**9)** $(p+3)^{\frac{1}{4}}+5=3$

**10)** $\sqrt{4x+8}=1$

**11)** $6-\sqrt{3u+1}=1$

**12)** $(b-3)^{\frac{4}{5}}-3=13$

**13)** $\sqrt{y+2}=\sqrt{2y-3}$

**14)** $\sqrt{11q+3}=2q$

**15)** $\sqrt{5+x}=\sqrt{x}+1$

**16)** The area of the circular base of a cone is $25\pi$ in$^2$. The height of the cone is 2 times the radius value. Use the formula $A=\pi\left(\frac{3V}{2\pi}\right)^{\frac{2}{3}}$ to find the volume of the cone. Keep your answer in terms of $\pi$.

**To continue, return to the Online Lesson.**

# Targeted Review

*Complete problems on a separate sheet of paper.*

**Simplify. Write answers in radical form.**

**1)** $\sqrt[4]{162x^8y^{14}}$

**2)** $\frac{5\sqrt{3}}{2}\left(\sqrt{\frac{3}{5}}\right)$

**Simplify and name the numbers using all classifications for the set of real numbers.**

$\{\mathbb{N}, \mathbb{W}, \mathbb{Z}, \mathbb{Q}, \mathbb{I}, \mathbb{R}\}$

**3)** $\sqrt[3]{27} - 3$

**4)** $\frac{1}{2^{-1}}$

**5)** $\sqrt{12}$

**6)** $\frac{5-8}{8-2}$

**7)** Solve. $\frac{x-11}{x^2-2} = \frac{6}{x}$

**8)** Determine the value of $Q$ to make the polynomial identity true.
$(3x - Q)^2 = 9x^2 - 16(3x - 4)$

**Multiple Choice**

**____ 9)** Simplify. $\frac{5}{\sqrt{6}-4}$

**A)** $-\sqrt{3}+2$

**B)** $\frac{-\sqrt{30}+20}{10}$

**C)** $-\frac{\sqrt{6}+4}{2}$

**D)** $\frac{\sqrt{6}+4}{2}$

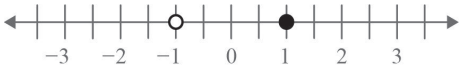
**____ 10)** Determine the number line that best matches the solution to the compound inequality.
$-3x + 1 > 4$ OR $3x + 1 \geq 4$

**A)**


**B)**

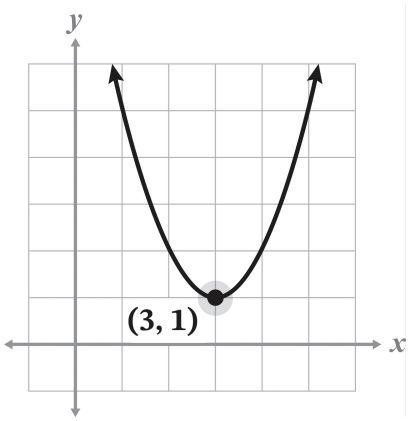

**C)**


**D)**

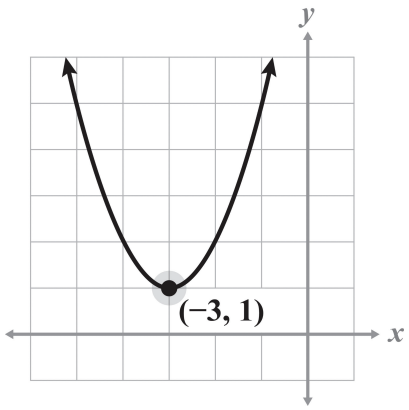


**____ 11)** Select the graph of the quadratic function $g(x) = (x-3)^2 + 1$.

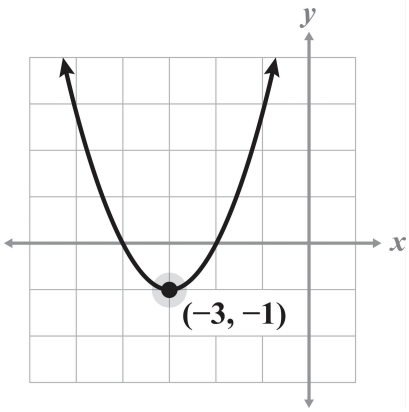
**A)**



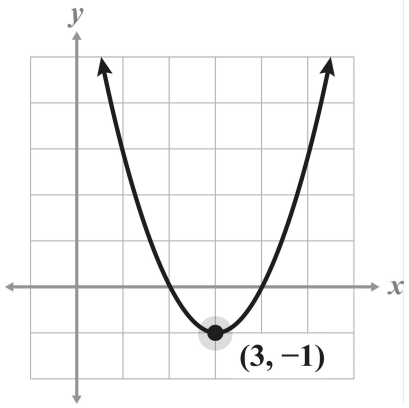

**B)**




**C)**




**D)**




**12)** Select the expressions that form conjugates.

- ☐ $(3\sqrt{2})$
- ☐ $(3\sqrt{2}+1)$
- ☐ $(3\sqrt{2}-1)$
- ☐ $\pm 1$

| **Problem** | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **Origin** | L11 | L12 | A1 | A1 | A1 | A1 | L9 | L4 | L12 | A1 | A1 | L12 |

*L = Lesson in this level, A1 = Algebra 1: Principles of Secondary Mathematics, FD = Foundational Knowledge*

**To continue, return to the Online Lesson.**