Lesson 43
Logarithmic Functions

NAME:

Start by navigating to the Online Lesson for instructions.

Objectives

- Write the inverse of an exponential and logarithmic function.
- Graph a logarithmic function.
- Transform a logarithmic function.
- Describe logarithmic functions in words and with an equation.

Why?

Logarithmic functions are the inverse of exponential functions. You will now be able to compare the graphs of exponential and logarithmic functions.

Warm Up

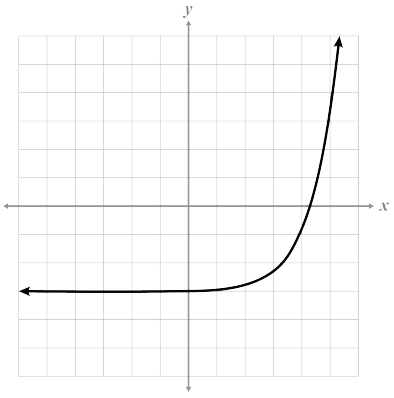
Name the domain and range of the function in interval notation.

1) $y = 2^{x-8} + 5$

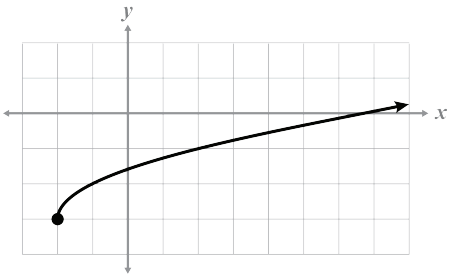
2) $y = 3^{5-x} - 3$

Name the end behavior.

3)



4)



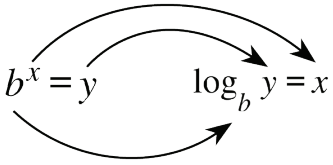
To continue, return to the Online Lesson.

Explore

Logarithmic Functions

Fill in the notes as you watch the video in the Online Lesson.

- A logarithmic function is the _____ of an exponential function for base b (when $b > 0, b \uparrow 1$).



- To find the inverse of an exponential function:
  - **1)** Switch _____ in the equation.
  - **2)** Take $\log_b$ of both sides.
  - **3)** Solve for y.

$y = (b)^x$

$x = (b)^y$

Example 1

Complete the example as you watch the video in the Online Lesson.

Write the inverse of the given function.

A) $f(x) = \left(\frac{1}{3}\right)^x$

B) $r(x) = e^x$

C) $h(x) = \left(\frac{1}{3}\right)^{-x}$

Example 2

Complete the example as you watch the video in the Online Lesson.

Write the inverse of the given function.

$g(x) = 6^x - 4$

Implement

Explain

- Write as $y =$
- Switch x and y
- Take $\log_b$ of both sides
- Solve for y

Checkpoint: Logarithmic Functions

Write the inverse of the given function.

$f(x) = 5^{x+1}$

To continue, return to the Online Lesson.

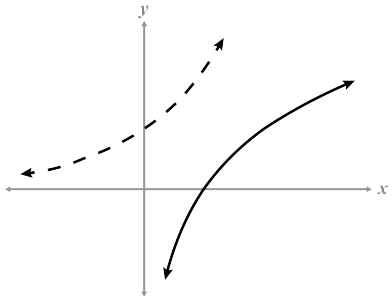
Graphing Logarithmic Functions without Technology

Fill in the notes as you watch the video in the Online Lesson.

- Because the graph of a _____ function, $y = \log_b x$, is the inverse of an _____ function, $y = b^x$:
  - _____
  - The graph is reflected across _____
  - The asymptote is the vertical line _____
  - There is an x-intercept, _____, but no y-intercept
  - The domain is: _____
  - The range is: _____
- A _____ of the exponential function as a dashed graph can help visualize its inverse, the logarithmic function.

Increasing Function

$b > 1$

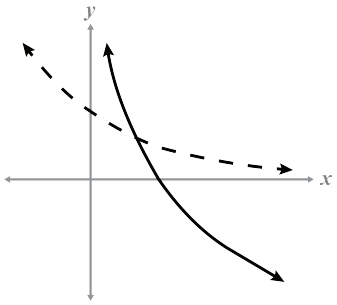


x	y
	1
	0
	−1

End behavior: As _____, $f(x) \to +\infty$ and as _____, $f(x) \to -\infty$

Decreasing Function

$0 < b < 1$



End behavior: As _____, $f(x) \to +\infty$ and as _____, $f(x) \to -\infty$

- Because the value of most logarithms results in an _____ number, it is sometimes necessary to work with the _____ of the function.

- Then you can compare the _____ and the _____ functions.
- When graphing a logarithmic function without technology:
  - _____ it as its inverse.
  - Make a table of _____.
  - Interchange the _____ from the table.
  - Graph the _____.

Example 3

Complete the example as you watch the video in the Online Lesson.

Compare $f(x)$ and $g(x)$.

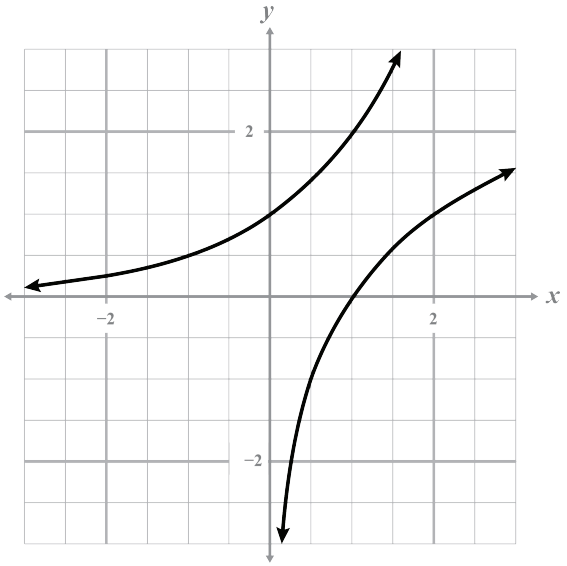
$f(x) = 2^x$ $g(x) = \log_2 x$

x	$f(x) = 2^x$
-2	
-1	
0	
1	
2	

Domain: _____

Range: _____

Asymptote: _____



x	$g(x) = \log_2 x$

Domain: _____

Range: _____

Asymptote: _____

$f(x)$ and $g(x)$ are _____ because their _____ are switched, and the graph of $g(x)$ is _____.

Example 4

Complete the example as you watch the video in the Online Lesson.

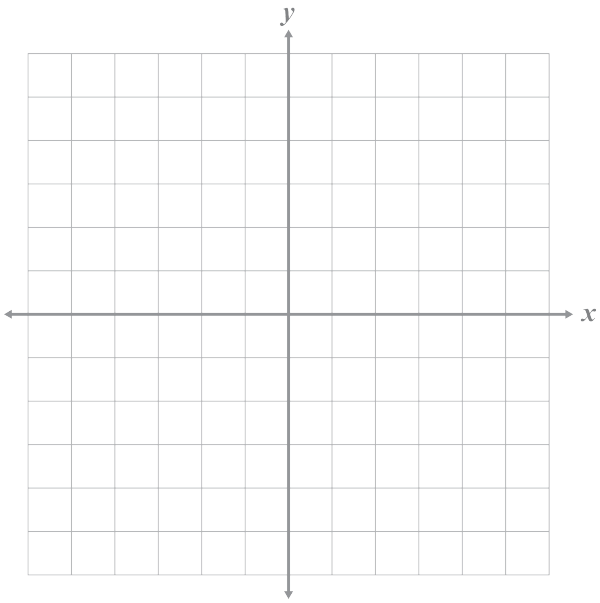
Graph without technology. Describe the end behavior and the domain and range.

$y = \log_5 x$

Plan

Write the inverse
Make a table for the inverse
Switch the points
Graph

x	y^{-1}



Explain

End behavior: As $x \to +\infty$, _____, and as $x \to 0$, _____ .

Domain: _____

Range: _____

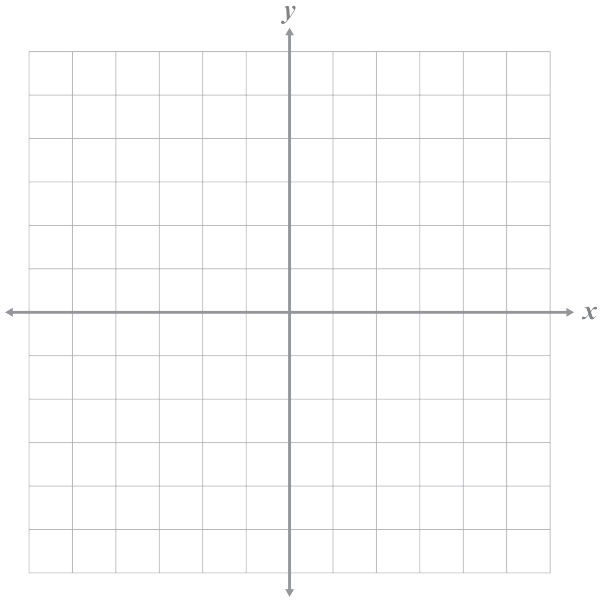
Example 5

Complete the example as you watch the video in the Online Lesson.

Graph the inverse of the given equation without technology. Describe the end behavior.

$$q(x) = \left(\frac{5}{4}\right)^{-x}$$

x	$q(x)$
−2	
−1	
0	
1	
2	

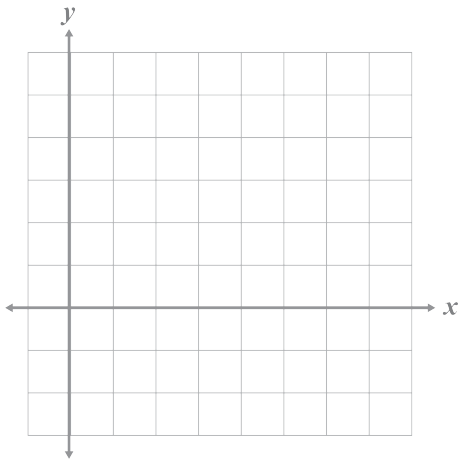


Explain

Checkpoint: Graphing Logarithmic Functions without Technology

Graph the inverse of the given equation without technology.

$$f(x) = \left(\frac{2}{3}\right)^{-x}$$



To continue, return to the Online Lesson.

Transform Logarithmic Functions with Technology

Fill in the notes as you watch the video in the Online Lesson.

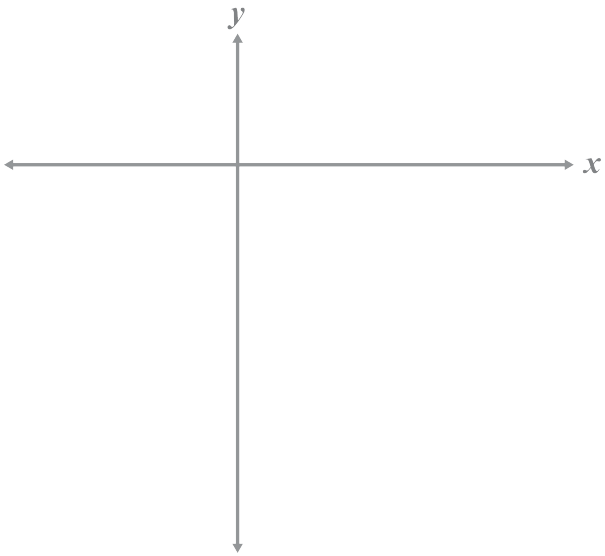
- _____ on logs, $y = a\log_b(x-h)+k$, occur when the values of a, h, and k change, just as they do for other functions.
- Use _____ to transform logarithmic functions because many of the ordered pairs (or points) contain irrational numbers.
- Then, create a sketch including the key details:
  - _____
  - _____
  - _____

Example 6

Complete the example as you watch the video in the Online Lesson.

Write the equation, then sketch the graph.

Transform $y = \ln x$ by shifting the graph left two units and down four units.



Example 7

Complete the example as you watch the video in the Online Lesson.

Describe the transformation of the functions $f(x)$ to $g(x)$.

$f(x) = e^x$ $\quad g(x) = \ln(x - 3) + 2$

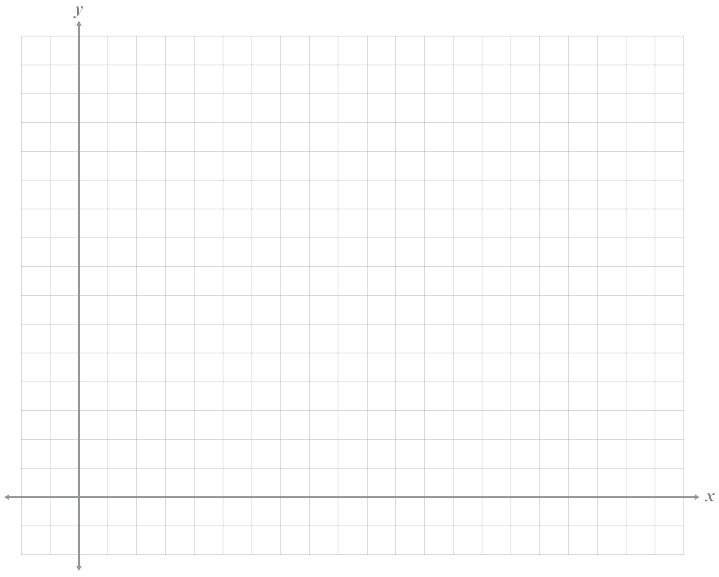
To extend your learning, check out the More to Explore for this lesson.

Checkpoint: Transform Logarithmic Functions with Technology

Describe the transformation of the functions $f(x)$ to $g(x)$. Graph $g(x)$.

$f(x) = \log_2 x$

$g(x) = -\log_2(x - 2) + 4$



To continue, return to the Online Lesson.

Practice 1

Complete problems on a separate sheet of paper.

Write the inverse of the given function.

1) $f(x) = 13^x$

2) $v(x) = 2^{-x}$

3) $p(x) = 3^x - 4$

4) $f(x) = e^{x-2}$

5) $g(x) = 7^x - 6$

6) $h(x) = 5^{-x} + 2$

Graph the inverse without technology. Describe the end behavior.

7) $f(x) = 3^x$

8) $g(x) = \left(\frac{1}{2}\right)^{-x}$

Graph without technology.

9) $h(x) = \log_4 x$

10) $f(x) = \log x$

Describe the transformation from $f(x)$ to $g(x)$.

11) $f(x) = 5^x$

$g(x) = \log_5(x-3) + 1$

12) $f(x) = \log_2 x$

$g(x) = 4 \log_2 x + 3$

13) $f(x) = \log_3 x$

$g(x) = -\log_3(x+2)$

Graph using technology. Name the domain and range.

14) $y = \ln(x+4) - 2$

15) $y = 3 + \ln x$

16) $y = -\log_3(x+5) - 3$

To continue, return to the Online Lesson.

Mastery Check

Show What You Know

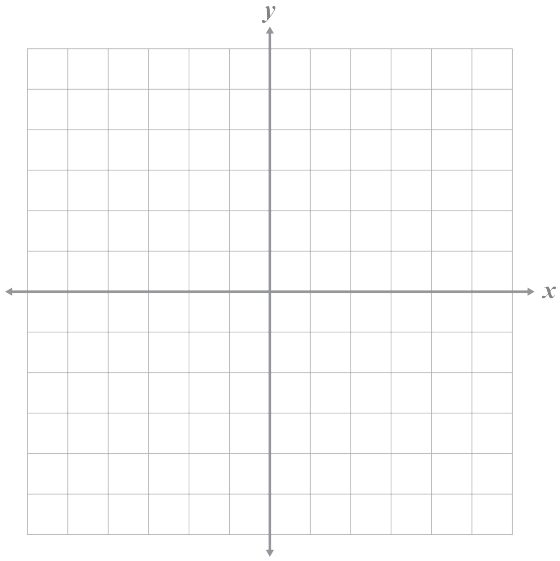
Complete the problem using $f(x)$ and $h(x)$.

$f(x) = 4^x$ $\qquad$ $h(x) = -\log_4(x) - 1$

A) Write a new function, $g(x)$, that is the inverse of $f(x)$. Then, shift the graph up three units and right two units.

B) Describe the transformation from $h(x)$ to $f(x)$.

C) Graph $h(x)$.



Say What You Know

In your own words, talk about what you have learned using the objectives for this lesson and your work on this page.

To continue, return to the Online Lesson.

Practice 2

Complete problems on a separate sheet of paper.

Write the inverse of the given function.

1) $f(x) = \left(\frac{3}{8}\right)^{-x}$

2) $f(x) = e^x + 3$

3) $f(x) = 4^{x+1}$

4) $f(x) = 6^x$

5) $r(x) = e^{x-7}$

6) $q(x) = 11^x + 9$

Graph the inverse without technology. Name the domain and range.

7) $y = \left(\frac{1}{4}\right)^x$

8) $y = \left(\frac{5}{3}\right)^{-x}$

Graph without technology. Name the end behavior.

9) $y = \log_7 x$

10) $y = \log_{\frac{1}{2}} x$

Describe the transformation from $f(x)$ to $g(x)$.

11) $f(x) = \log_7 x$

$g(x) = \log_7 (x + 4) - 2$

12) $f(x) = 2^x$

$g(x) = \log_2 x + 3$

13) $f(x) = \ln x$

$g(x) = -\ln (x + 1) - 5$

Graph with technology.

14) $y = -\log_5 (x - 3) + 1$

15) $y = 3 \ln x - 6$

16) $y = -\ln (x + 2)$

To continue, return to the Online Lesson.

Targeted Review

Complete items on a separate sheet of paper.

1) Write the equation of the exponential function $g(x)$ that translates $f(x)$ up 3 spaces and left 5 spaces.

$f(x) = 4^{x+2} - 6$

2) Solve: $\left(\frac{1}{27}\right)^{x-2} = 9^{(-x+6)}$

3) Write in exponential form. Do not solve.

A) $\log_{216} 36$

B) $\ln 7.5$

4) Evaluate: $\log_8 16$

5) Solve with common logs: $17^x = 254$

6) Solve: $2\ln x + \ln 36 = \ln 18 + \ln 8$

7) Express as a single logarithm.

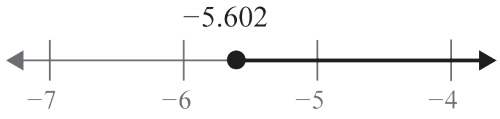
$\log_5 x - 2\left(\log_5 2 + \log_5 y\right)$

8) Expand: $\log_x\left(\frac{(y-2)^3}{8}\right)$

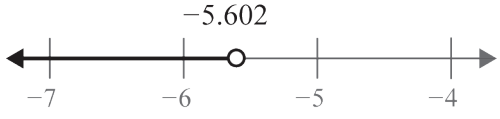
Multiple Choice

____ 9) Solve: $4^{x+5} > 1$

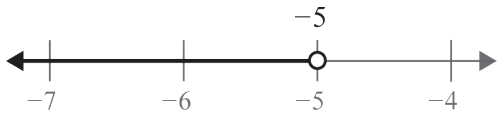
A)



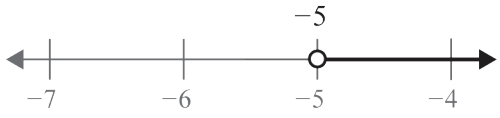
B)



C)

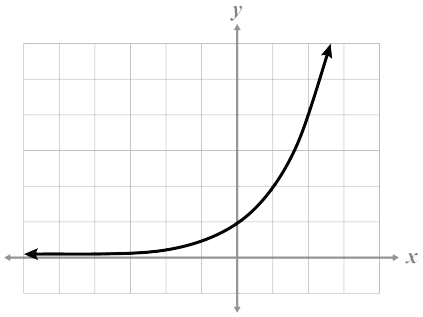


D)



Multiple Choice

_____ **10)** Name the end behavior of the given function.



A) As $x \to +\infty, f(x) \to +\infty$, and as $x \to 0, f(x) \to -\infty$

B) As $x \to +\infty, f(x) \to -\infty$, and as $x \to 0, f(x) \to +\infty$

C) As $x \to +\infty, f(x) \to +\infty$, and as $x \to -\infty, f(x) \to 0$

D) As $x \to +\infty, f(x) \to -\infty$, and as $x \to -\infty, f(x) \to 0$

_____ **11)** Write $\log_5 2$ as a common log.

A) $\dfrac{\log 2}{\log 5}$

B) $\dfrac{\log 5}{\log 2}$

C) $\dfrac{\log_2 5}{\log_2 2}$

D) $\dfrac{\log_5 2}{\log_5 5}$

_____ **12)** Write $3\ln y + \ln 2 - \ln(y-5)$ as a single logarithm.

A) $\ln\left(\dfrac{2y^3}{y-5}\right)$

B) $\ln\left(\dfrac{y^3+2}{y-5}\right)$

C) $\ln\left(\dfrac{6y}{\frac{y}{5}}\right)$

D) $\ln\left(\dfrac{2y^3}{\frac{y}{5}}\right)$

Problem	1	2	3	4	5	6	7	8	9	10	11	12
Origin	L37	L38	L39, 42	L39	L41	L42	L40	L40	L38	L37	L41	42

L = Lesson in this level, A1 = Algebra 1: Principles of Secondary Mathematics

To continue, return to the Online Lesson.