Lesson 42
Natural Logs

NAME:

Start by navigating to the Online Lesson for instructions.

Objectives

- Write equivalent expressions using natural logarithms and the number e.
- Use the properties of logarithms to simplify or evaluate expressions with natural logarithms and the number e.
- Solve natural logarithm equations.

Why?

The mathematician Leonhard Euler discovered the natural logarithm in the 18th century. With this discovery, mathematicians and scientists have been able to describe irrational numbers more accurately using e and $\ln x$ rather than lengthy decimal values.

Warm Up

Simplify.

1) $\left(e^2\right)^x$

2) $e^x \cdot e^x$

Solve.

3) $\log_3(x-3)+\log_3(x+3)=3$

To continue, return to the Online Lesson.

Explore

Simplify Natural Logarithms

Fill in the notes as you watch the video in the Online Lesson.

- _____ is defined as: $e = \lim_{n \to \infty} \left(1 + \frac{1}{n}\right)^n$

Above is the exact mathematical definition of e. For Algebra 2, you need to know the approximate value of the number e. Knowledge of limits is not required for this level.

- This irrational number _____ has an approximate value of _____, which is used when working with natural logarithms.
- The natural log is written either of these ways:
  - _____
  - _____

Be very careful not to confuse the letter "l" with the number "1" throughout this lesson. Likely, any "l" followed by "n" refers to the letter and therefore to the natural log.

- With ln:
  - The letter "l" stands for _____.
  - The letter "n" stands for _____.

$\log_e$ replaces ln only when the base e is written.

The base of e is understood when using the notation for natural logs.

Properties of Logs

For all rules of logs, the variables a, b, and c are positive real numbers, n and x are real numbers, and $a \uparrow 1$.

Properties of Logs	Logarithm Rule(s)	Natural Logarithm
Foundational Properties	If $a^0 = 1$, then $\log_a 1 = 0$	
	If $a^n = a^n$, then $\log_a(a^n) = n$	
	If $a^1 = a$, then $\log_a(a) = 1$	
Quotient Rule	$\log_a \frac{b}{c} = \log_a b - \log_a c$	
Product Rule	$\log_a bc = \log_a b + \log_a c$	
Power Rule	$\log_a b^n = n \cdot \log_a b$	

Example 1

Complete the example as you watch the video in the Online Lesson.

Evaluate the expression.

A) $e^{\ln 5}$

B) $\ln\sqrt{e}$

C) $e^{\ln 3 + \ln 5}$

Checkpoint: Simplify Natural Logarithms

Evaluate.

A) $(\ln e^3)^2$

B) $e^{\ln 8 - \ln 5}$

To continue, return to the Online Lesson.

Rewrite using Natural Logarithms

Fill in the notes as you watch the video in the Online Lesson.

- To expand and contract natural logarithmic expressions and equations, all terms must be in the _____.
- If a natural logarithmic expression contains a number, rewrite it using a _____ _____ because $x = \ln\left(e^x\right)$.
- Once all terms are written in terms of $\ln$ (or as a natural log), expand and contract with the _____.

Remember to use your Formula Sheet as you work through the problems in this lesson.

Example 2

Complete the example as you watch the video in the Online Lesson.

A) Write the equation as a natural log.
$e^3 = 20.0855$

B) Write the equation in exponential form.
$\ln 0.2725 = -1.3$

Example 3

Complete the example as you watch the video in the Online Lesson.

Write as a single natural logarithm.
$\frac{1}{2}\ln 35 - 4$

Checkpoint: Rewrite using Natural Logarithms

Write as a single natural logarithm.

$\ln x - (4\ln y + 3\ln z)$

To continue, return to the Online Lesson.

Solve Natural Logarithms

Fill in the notes as you watch the video in the Online Lesson.

The Equality Rule

When a is a positive number not equal to one, $\log_a x = \log_a y$ if and only if $x = y$.

- The Equality Rule is true for natural logs because _____ replaces _____.
- Use the Equality Rule to solve _____ log equations the same way you did for _____ log equations.

Example 4

Complete the example as you watch the video in the Online Lesson.

Solve. Write the answer as a natural logarithm and as a number to four decimal places.

$300^{4x} = 10$

Example 5

Complete the example as you watch the video in the Online Lesson.

Solve. Round to the ten-thousandth.

$\ln 317 - \ln x = 5$

Checkpoint: Solve Natural Logarithms

Solve using natural logarithms.

$7^{0.3x} = 712$

To continue, return to the Online Lesson.

Practice 1

Complete problems on a separate sheet of paper.

Evaluate the expression.

1) $(\ln e^2)^6$

2) $e^{(\ln 2 - \ln 3)}$

3) $\ln e^3 + \ln e^2$

4) $e^{(\ln 5 + \ln 6)}$

Write as a single natural logarithm.

5) $3\ln x + 2\ln y - 4(\ln a + \ln b)$

6) $\frac{1}{3}\ln 12 - \ln 7$

Expand.

7) $\ln \frac{xy^4}{z^2}$

8) $\ln \frac{\sqrt[4]{x^3}}{yz^5}$

Solve. Write the answer as a natural logarithm and as a number to four decimal places.

9) $6^{\frac{x}{4}+2} = 51$

10) $17^{x+1} = 34$

11) $7^{5x+4} = 56$

12) $2^x = 83$

Solve. Round to the ten-thousandth.

13) $\frac{1}{2} = \ln(4x - 3)$

14) $\ln(5x)^{\frac{2}{3}} = 8$

15) $\ln(x+1) - \ln 2 = 3$

16) $\frac{1}{3}\ln(x+6) = \ln 2$

To continue, return to the Online Lesson.

Mastery Check

Show What You Know

A) Two groups of students solved the same equation and found the same incorrect answer. They argued that because they found the same result, both groups must be correct.

Find and correct the error that each group made.

Green Group

$$2e^{5x} - 4 = 6$$

$$\frac{2e^{5x}}{2} = \frac{10}{2}$$

$$e^{5x} = 5$$

$$\frac{\ln 5}{5} = \frac{5x}{5}$$

$$\ln 1 = x$$

$$x = 0$$

Purple Group

$$2e^{5x} - 4 = 6$$

$$\frac{2e^{5x}}{2} = \frac{10}{2}$$

$$\frac{e^{5x}}{5} = \frac{5}{5}$$

$$e^{x} = 1$$

$$\ln 1 = x$$

$$x = 0$$

B) Explain the error that each group made and how to correct their misunderstanding.

Say What You Know

In your own words, talk about what you have learned using the objectives for this lesson and your work on this page.

To continue, return to the Online Lesson.

Practice 2

Complete problems on a separate sheet of paper.

Evaluate the expression.

1) $e^{(\ln 3 + \ln 9)}$

2) $\ln e^5 - \ln e^2$

3) $e^{(\ln 8 - \ln 9)}$

4) $(\ln e^5)^2$

Write as a single natural logarithm.

5) $\frac{1}{3}\ln x + \frac{2}{3}\ln y - \ln 7$

6) $3\ln 2 + \ln 6 - \ln 5$

Expand.

7) $\ln \sqrt[5]{x^2y^3}$

8) $\ln \frac{2y(x+3)^2}{z}$

Solve. Write the answer as a natural logarithm and as a number to four decimal places.

9) $215^{x+4} = 37$

10) $71^{0.3x} = 19$

11) $5^x = 12$

12) $9^{\frac{2}{3}x-1} = 15$

Solve. Round to the ten-thousandth.

13) $\ln(2x+1) - \ln x = 3$

14) $\ln(2x-5) = 6$

15) $7^{x-3} = 21$

16) $\ln 2 - \ln(x-1) = 3$

To continue, return to the Online Lesson.