Lesson 36
Variation

Start by navigating to the Online Lesson for instructions.

Objectives

- Calculate the constant of variation or missing values given the type of variation.

- Determine the type of variation in words or symbols.

- Write and/or solve the variation equation.

Why?

Many formulas used in real life are a form of variation: direct, inverse, or a combination of them all. With variations, you can answer questions such as: What is my hourly rate? What is the relationship between pressure and volume? How long can I drive on a tank of gas?

☼ Warm Up

Solve.

1) $\dfrac{9}{x} = \dfrac{22.5}{30}$

2) $16 = \dfrac{w\sqrt[3]{64}}{3^2}$

Solve the equations for y.

3) $x = \dfrac{5}{3}(z + y)$

4) $z = \dfrac{k\sqrt{y}}{x}$

To continue, return to the Online Lesson.

🔭 Explore

🔭 Variation

▷ *Fill in the notes as you watch the video in the Online Lesson.*

- The constant of variation is the value _____ that relates _____ directly or inversely proportional

  to _____.

	Direct Variation	Inverse Variation
Equation in terms of y	$y = kx^n, n > 0$	$y = \dfrac{k}{x^n}, n > 0$
Equation in words	y varies _____ as x^n	y varies _____ as x^n
Equation in terms of k		

- The algebraic _____ needed depends on the _____ provided

  in the problem you are solving.

Example 1

▷ *Complete the example as you watch the video in the Online Lesson.*

Fill in the blanks.

A) For the given equation $y = \dfrac{\sqrt{6}}{x^2}$, y varies _____ as x^2, and the constant of variation

is _____.

B) For the given equation $y = \pi x^3$, y varies _____ as _____, and the constant of

variation is _____.

Example 2

▷ *Complete the example as you watch the video in the Online Lesson.*

Write the equation given the words.

A) If y varies directly as x, and $y = 8$ when $x = -4$, write the equation using the value of k.

B) y varies inversely as x^5.

> Remember, when the constant of variation is unknown, the variable k is used. It, k, must be included in the equation even when it is not part of the description in words.

☑ Checkpoint: Variation

Fill in the blanks.

A) For the equation $y = \dfrac{2}{3x}$, y varies _____ as x, and the constant of

variation is _____.

Write the equation symbolically.

B) y varies directly as the square root of x.

🖥 To continue, return to the Online Lesson.

🔭 Solving Variation Problems

▷ *Fill in the notes as you watch the video in the Online Lesson.*

- When solving variation problems, it is important to remember that the _____ of

  variation is not always provided.

- How you solve the problem depends on:

    - _____

    - _____.

Example 3

▷ *Complete the example as you watch the video in the Online Lesson.*

If y varies directly as x^2, and $y = -8$ when $x = 4$, find y when $x = 5.2$.

Implement

Explain

▶ Write the direct variation equation

▶ Isolate k

▶ Solve

Example 4

▷ *Complete the example as you watch the video in the Online Lesson.*

With stringed instruments, the length, l, of a string varies inversely as the frequency, f, of its vibrations. When the length of a string is 40 centimeters, it vibrates at a frequency of 180 cycles per second. Determine the length of a string with a frequency of 300 cycles per second.

The length of the string is

_____ when the frequency

is _____.

☑ **Checkpoint: Solving Variation Problems**

In physics, force is the product of mass and acceleration, $F = ma$. If the mass of an object is 542 kilograms when accelerating at $13 \, \text{m}/\text{s}^2$, what is the acceleration of the same object with a force of 5230.3 Newtons?

To continue, return to the Online Lesson.

⚖ Combined Variation

▷ *Fill in the notes as you watch the video in the Online Lesson.*

- The constant of variation is the value k that relates y _____.

- Combined variation is when one variable depends on two or more others, some

  _____ and some _____, all within a single equation.

- It is important to _____ so that you write equations in

  the correct form.

- _____ variation is a special type of combined variation in which one quantity varies

  directly with the product of two other quantities, _____.

Example 5
▷ *Complete the example as you watch the video in the Online Lesson.*

Write the equation in terms of g.

d varies inversely as e, and jointly as f, and the sum of g and h.

Example 6

▷ *Complete the example as you watch the video in the Online Lesson.*

The volume of a cone varies jointly as the product of the radius squared and the height. Determine the constant of variation of the cone when the volume is 32π cubic meters, the height is $\frac{2}{3}$ meters, and the radius is 12 meters. Then find the volume of a cone when the lengths of the height and radius are switched.

☑ Checkpoint: Combined Variation

Write the equation algebraically.

F varies jointly with the product of x and y, and inversely with the square of d.

▢ To continue, return to the Online Lesson.

✎ Practice 1

Complete problems on a separate sheet of paper.

Fill in the blanks to make each statement true.

1) In the equation $V = \dfrac{4}{3}\pi r^3$, V varies _____ as r^3, and _____ is the constant

   of variation.

2) In the equation $y = \dfrac{5ab}{c}$, y varies _____ to a and b, and _____ to c, and

   _____ is the constant of variation.

3) y varies directly as the square of x, and inversely as the cube root of 1, and can be modeled by

   the equation _____.

4) y varies inversely as x to the fourth power and can be modeled by the equation

   _____.

Solve.

5) M varies directly to R and inversely to T, when $M = -11$, $R = 22$, and $T = 4$. Find M when
   $R = -3$ and $T = -2$.

6) y varies directly as the cube root of x, when $x = 27$ and $y = 4$. Find y when $x = 64$.

7) z varies jointly as the square of y and the cube of x, when $z = -8, y = 2$, and $x = -3$. Find z when
   $y = 1$ and $x = 2$.

8) a varies inversely as the square of b, when $a = 16$ and $b = 2$. Find a when $b = 7$.

9) y varies directly as x, when $y = 8$ and $x = 0.5$. Find x when $y = -5$.

10) g varies inversely as h, when $g = -9$ and $h = -12$. Find g when $h = -33$.

11) C varies jointly as V and T when $C = 100$, $T = 5$, and $V = 2000$. Find C when $T = 8$ and
    $V = 3200$.

12) f varies directly as g and inversely as the square root of h when $f = 6, g = -4$, and $h = 25$.
    Find f when $g = 10$ and $h = 9$.

13) The amount of money earned (before taxes) in an hourly employee's paycheck is directly
    related to the number of hours they work. If an employee earned $61.13 for 7.5 hours of work,
    how much will they earn for working 20 hours? What is their hourly rate?

Solve.

14) The pressure, P, of a spray can varies inversely to its volume, V. When the volume is 10 cubic inches, the pressure is 14 pounds per square inch. Find the volume when the pressure is 24 pounds per square inch.

15) The owners of Steve's Surfboards determined that the monthly quantity sold, S, varies directly with their advertising budget, A, and inversely with the price, P, of the surfboards. When $75,000 is spent on advertising and the price of each surfboard is $250, the monthly quantity sold is 6,000 surfboards. How much needs to be spent on advertising to increase the monthly quantity sold to 10,000 surfboards?

16) Centrifugal force, F, varies directly with velocity squared, $v2$, and inversely as the radius, r. The body's mass, m, is constant (in this problem, m replaces k). When an object is moving in a circular path with a radius of 5 meters and a velocity of $8\ \mathrm{m}/_\mathrm{s}$, it has a centrifugal force of 32 Newtons. Find the centrifugal force of the object moving in a circular path with a radius of 10 meters and velocity of $6\ \mathrm{m}/_\mathrm{s}$. (The mass unit will be kilograms.)

Write the equation in terms of y.

17) x varies directly to the cube root of z and inversely to y.

18) x varies jointly as z and the sum of y and w.

Write the equation in terms of k.

19) x varies directly as the square of y and inversely with the cube root of z.

20) y varies jointly with x and the difference of c and b, and inversely with the square of z.

To continue, return to the Online Lesson.

☑ Mastery Check

✍ Show What You Know

Use the equation to complete the problem.

$$x = \frac{kw \sqrt[3]{z}}{y^3}$$

A) Two students were given an equation and asked to describe the variation in words. Explain the *errors* and how to correct the misunderstanding.

Cruz's Description

x varies directly with the product of k, w, and the cube root of z, and inversely as y-cubed.

Terri's Description

x varies jointly as w- and z-cubed while being divided by y to the third power.

B) Correctly describe the equation in words.

C) Write the equation with the value of the constant of variation when $w = 7$, $x = 10$, $y = 2$, and $z = 125$.

ᏟᏟᏟ Say What You Know

In your own words, talk about what you have learned using the objectives for this lesson and your work on this page.

▭ To continue, return to the Online Lesson.

✎ Practice 2

Complete problems on a separate sheet of paper.

Fill in the blanks to make each statement true.

1) In the equation $y = 4x^2z^3$, y varies _____ with x^2 and z^3, and _____ is the

constant of variation.

2) In the equation $a = \dfrac{v_f - v_i}{t}$, a varies _____ as the _____ of v_f

and v_i, and _____ with t.

3) z varies directly as the fourth root of x, and inversely as y raised to the fifth power, and can be

modeled by the equation _____.

4) y varies directly as the sum of a and b, and inversely as the cube of c, and can be modeled by

the equation _____.

Solve.

5) y varies directly with the cube of x, when $y = 54$ and $x = 3$. Find y when $x = -5$.

6) y varies jointly with x and the square root of z and inversely as b, when $y = 4$, $x = 7$, $z = 16$, and $b = 0.75$. Find y when $x = -28$, $z = 25$, and $b = 2$.

7) y varies inversely to x when $y = 4$ and $x = 0.125$. Find x when $y = 9$.

8) x varies directly to the cube of y, when $x = 16$ and $y = -2$. Find x when $y = \dfrac{4}{5}$.

9) y varies jointly with x and z, when $y = 14$, $x = -\dfrac{2}{3}$, and $z = 28$. Find y when $x = \dfrac{2}{3}$ and $z = \dfrac{1}{4}$.

10) z varies jointly with a and b and inversely with the square root of c, when $z = -1$, $a = 4$, $b = 5$, and $c = 9$. Find c when $z = -2$, $a = 1$, and $b = 4$.

11) a varies directly with the square of b and inversely as the cube of c, when $a = 4$, $b = 2$, and $c = -1$. Find a when $b = -3$ and $c = 2$.

12) y varies directly as x, when $y = -6$ and $x = 13$. Find x when $y = \dfrac{5}{2}$.

13) The total charge to purchase, p, gasoline varies directly to the number of gallons of gasoline, g, purchased. Susan purchased 13 gallons of gasoline for $45.12. Determine the total number of gallons Susan could purchase for $25.

Solve.

14) The volumetric flow rate of water, W, varies inversely as time, t. At 2 seconds, the volume flow rate is 15 cubic meters/second. What is the flow rate at 7 seconds?

15) Kinetic energy, KE, varies jointly as the mass, m, and the square of velocity, v. When the mass of an object is 15 kilograms, the velocity is 4 meters per second and has a kinetic energy of 240 Newtons. Determine the velocity of an object when the kinetic energy is 315 Newtons with a mass of 12 kilograms.

16) When an object slides along a surface, kinetic friction opposes the object's motion. Kinetic friction, Ff, varies directly to the normal force of the object, FN. If the force of kinetic friction is 0.35 Newtons when the normal force is 0.22 Newtons, determine the force of kinetic friction when the normal force is 0.4 Newtons.

Write the equation, and then solve for y.

17) x varies directly to the sum of z and y, and inversely to the square of a.

18) x varies inversely to the joint variation of z and y.

Write the equation, and then solve for k.

19) y varies directly as the sum of x and z, and inversely as the square root of b.

20) y varies jointly with x and z, and inversely as the difference of a and b.

To continue, return to the Online Lesson.