Lesson 34
Zeros and Multiplicity (of Polynomial Functions)

NAME:

Start by navigating to the Online Lesson for instructions.

Objectives

- State the possible number of real zeros that exist from the graph of a polynomial function.

- Sketch a graph using the roots and leading coefficient also considering the multiplicity (exact *x*-axis values only).

- Determine all roots of a polynomial expression algebraically, expressed as a product of factors or solutions.

- Determine multiplicities in a polynomial function and the number of occurrences.

Why?

You are now ready to connect all of the solving methods for polynomials and how to handle solutions with complex conjugates. This lesson uses your knowledge of synthetic division, complex conjugates, factoring, and the quadratic formula to determine all roots, or *x*-intercepts, of a polynomial function.

☼ Warm Up

Determine the quotient using synthetic division.

1) $\left(3x^4 + x^3 + 5x^2 - x - 8\right) \div \left(x + 1\right)$

2) $\left(x^3 + 1.5x^2 - 4x + 2.5\right) \div \left(x + 3\right)$

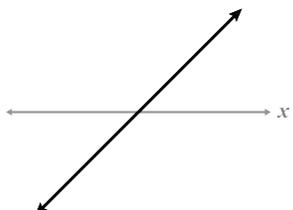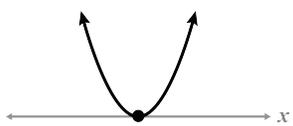**To continue, return to the Online Lesson.**

𝄃𝄃 Explore

𝄃𝄃 Multiplicity

▷ *Fill in the notes as you watch the video in the Online Lesson.*

- Recall that a function's input (*x*-value) that makes the output (*y*-value) equal to zero is called

  _____.

- On a graph, the roots are where the function _____

  the *x*-axis.

- Multiplicity is the _____ a root occurs in a polynomial equation. In

  other words, multiplicity is how many times an _____ makes the equation equal

  to zero.

- And so, multiplicity determines what a graph will look like when it intersects the _____.

  - $(x-h)^2$ is a _____ (or a trinomial square).

  - $(x-h)^3$ is a _____.

- Use the leading coefficient test and multiplicities to create a _____ that estimates

  the _____ of a polynomial.

- When creating a sketch, recall that roots are plotted on the _____.

> Sketches, unlike exact graphs, do not need to include every detail.
> Sketches in this lesson do not include the *y*-axis.

Multiplicity	Description/Implication	Shorthand	Sketch
	cross-through		
	bounce (also called _____)		
	snake		
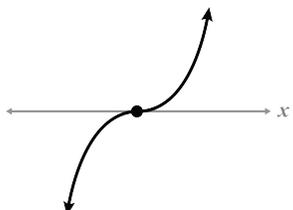
> The descriptions/implications provided in the table are not technical math
> vocabulary; rather, they provide a way to describe multiplicity in simple language.
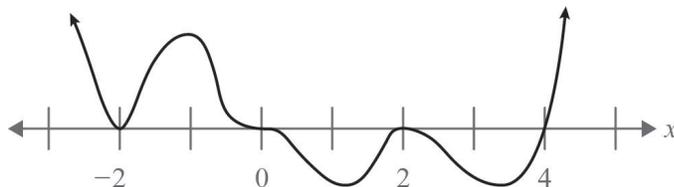
To state the possible multiplicities and the degree given a graph:

1) Perform the _____ (to estimate a, n).

2) Name or estimate the _____.

3) Estimate the _____ at each root.

4) Determine the possible _____.

Example 1

⊳ *Complete the example as you watch the video in the Online Lesson.*

Write the polynomial as a product of factors using the graph.

$$a(x-(-2))^2(x-0)^3(x-2)^2(x-4) = 0$$

Example 2

⊳ *Complete the example as you watch the video in the Online Lesson.*

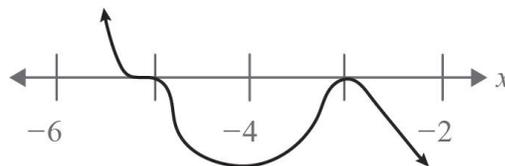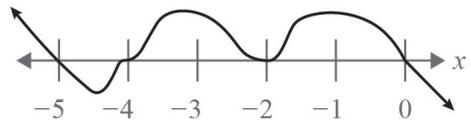**Write the polynomial as a product of factors using the graph. Explain.**

Explain

The end behavior represents a _____ polynomial function.

There is a triple root (_____) at −5 and a double root (_____) at −3.

☑ **Checkpoint: Multiplicity**

Write the polynomial equation given the graph.

📑 **To continue, return to the Online Lesson.**

🔭 Sketching Multiplicities

▷ *Fill in the notes as you watch the video in the Online Lesson.*

- To sketch roots, you need:

    - the polynomial _____

  or

    - the _____ and _____.

- The steps for sketching multiplicities include:

    1)   Perform the _____ (estimate a, n).

    2)   Sketch the _____.

    3)   Plot the _____ on the x-axis.

    4)   Connect the end behavior to the roots using the _____ (CBS).

- Remember, a sketch does not demonstrate the _____ of a graph.

    A sketch gives a general picture of what is happening at key points.

Example 3

▷ *Complete the example as you watch the video in the Online Lesson.*

Sketch the roots. Name the degree.

$a < 0$

$x = -3.5$, multiplicity 1

$x = -1$, multiplicity 3

$x = 1$, multiplicity 1

$\longleftarrow\!\!\!\longrightarrow x$

Example 4

▷ *Complete the example as you watch the video in the Online Lesson.*

Sketch the zeros of the function.

$x^2(x - 2.5)^3(x + 2)(x - 5) = 0$

$\longleftarrow\!\!\!\longrightarrow x$

☑ **Checkpoint: Sketching Multiplicities**

Sketch the roots. Then write the equation.

$a > 0$

$x = -2$, multiplicity 2

$x = 0.5$, multiplicity 1

$x = 3$, multiplicity 3

$\longleftarrow\!\!\!\longrightarrow x$

⬚ **To continue, return to the Online Lesson.**

🔭 Rational Root Theorem

> ▷ *Fill in the notes as you watch the video in the Online Lesson.*

- The Rational Root Theorem (RRT) is used to determine the possible rational roots (zeros,

  x-intercepts) for a polynomial containing _____.

  $$\text{Potential rational roots, } x = \frac{\text{Factors of the } \rule{4cm}{0.4pt}}{\text{Factors of the } \rule{4cm}{0.4pt}}$$

- Use _____ because rational numbers can be positive or negative.

- Recall that a rational number is a number that can be written in the form _____ in which p

  and q are _____.

Example 5

> ▷ *Complete the example as you watch the video in the Online Lesson.*

List all potential rational roots.

$2x^3 - 9x^2 + 7x + 6 = 0$

Factors of the constant: $\text{RRT} =$

Factors of the leading coefficient:

🔭 Solving with the RRT

> ▷ *Fill in the notes as you watch the video in the Online Lesson.*

- Use _____ to:

  - view the graph,

  - determine the rational roots and name any integer roots with RRT, and

  - identify any multiplicities that exist for any root.

■ Then, use one or more of these methods to find the actual roots algebraically:

     ■ _____

     ■ _____

     ■ _____

Example 6

▷ *Complete the example as you watch the video in the Online Lesson.*

Determine all roots of the function.

$$f(x) = x^4 + 8x^3 - 10x^2 - 8x + 9$$

Implement

Explain

▶ Determine possible rational roots with RRT

▶ Find all roots algebraically: synthetic division, factoring, quadratic formula

▶ Check for multiplicities

Example 7

⊳ *Complete the example as you watch the video in the Online Lesson.*

Determine all roots of the function. Then write the equation as a product of its factors using rational numbers.

$9x^3 + 4x = 45x^2 + 20$

$9x^3 - 45x^2 + 4x - 20 = 0$

$$\text{RRT} = \frac{\pm 1, \ \pm 2, \ \pm 4, \ \pm 5, \ \pm 10, \ \pm 20}{\pm 1, \ \pm 3, \ \pm 9}$$

Example 8

⊳ *Complete the example as you watch the video in the Online Lesson.*

Determine all roots of the function. Then write the function as a product of factors using rational numbers.

$g(x) = x^5 - 14x^3 - 16x^2 + 24x + 32$

Example 9

▷ *Complete the example as you watch the video in the Online Lesson.*

Determine all roots of the function.

$h(x) = x^3 - 5x + 4$

☑ **Checkpoint: Solving with the RRT**

Determine all roots of the function. $f(x) = x^3 + 3x^2 - 43x + 15$
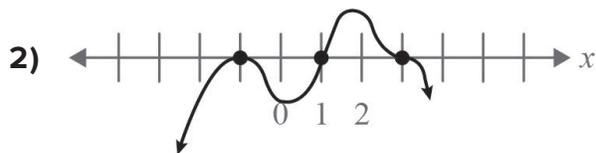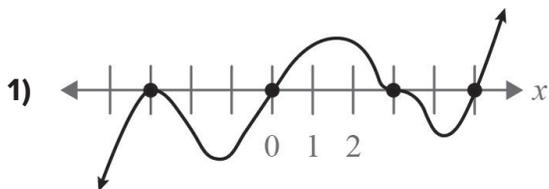
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✎ Practice 1

Complete problems on a separate sheet of paper.

Write the polynomial as a product of factors from the graph. Explain the multiplicity.

1)

2)

Sketch the roots. Then write the equation.

3) $a < 0$

$x = -2$, multiplicity 2

$x = 0$, multiplicty 1

$x = 5$, multiplicity 1

4) $a > 0$

$x = -3$, multiplicity 1

$x = -1$, mutliplicity 1

$x = 4$, multiplicity 3

Sketch the zeros of the function.

5) $-(x+3)^2(x-3.5)(x-0.5) = 0$

6) $x^3(x+5)^2 = 0$

List all potential rational roots. Do not solve.

7) $h(x) = 4x^5 - 3x^4 + 15x^3 + 2x - 25$

8) $g(x) = 3x^4 - 2x^3 + 12x^2 - 14x - 18$

Determine all roots of the function.

9) $b(x) = x^3 + 2x^2 - 5x - 6$

10) $x^4 - x^3 + 2x^2 - 4x - 8 = 0$

11) $c(x) = x^3 - 4x^2 - 7x + 10$

12) $3x^3 + 11x^2 + 5x - 3 = 0$

Determine all roots of the function. Then write them as a product of factors using rational numbers.

13) $x^4 - 6x^2 - 8x + 24 = 0$

14) $h(x) = 2x^4 + 3x^3 - 11x^2 - 9x + 15$

15) $q(x) = 4x^5 + 12x^4 - 41x^3 - 99x^2 + 10x + 24$
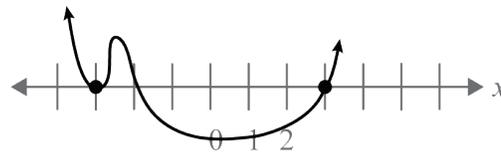
16) $4x^3 - 8x^2 - 3x + 9 = 0$

🖱 **To continue, return to the Online Lesson.**

☑ Mastery Check

✎ Show What You Know

A) A student was given the polynomial function $h(x)$ and asked to sketch the roots. Explain why *only some of the roots* are represented. (Note: This function only contains real roots.)

$h(x) = x^6 - 23x^4 + 171x^2 - 405$

Student sketch:

B) Determine all roots of $h(x)$.

C) Write $h(x)$ as the product of its factors using only rational numbers. Explain the multiplicities.

D) Sketch the complete graph of $h(x)$.
   Name the end behavior.

ᯤ Say What You Know

In your own words, talk about what you have learned using the objectives for this lesson and your work on this page.
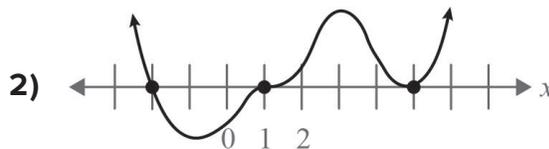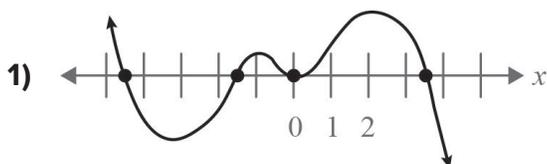
⌨ To continue, return to the Online Lesson.

✎ Practice 2

Complete problems on a separate sheet of paper.

Write the polynomial as a product of factors from the graph. Explain the multiplicity.

1)

2)

Sketch the roots. Then write the equation.

3) $a < 0$

  $x = -5$, multiplicity 1

  $x = -1$, multiplicty 2

  $x = 2$, multiplicity 3

4) $a > 0$

  $x = -2$, multiplicity 3

Sketch the zeros of the function.

5) $x^2(x+2)^2(x-1) = 0$

6) $-(x-3)^2(x+2)^3(x-5) = 0$

List all potential rational roots. Do not solve.

7) $5x^4 - 2x^2 + 7x - 15 = 0$

8) $g(x) = 4x^4 - 6x^3 + 9x^2 - 13x + 18$

Determine all roots of the function.

9) $3x^4 - 10x^3 - 24x^2 - 6x + 5 = 0$

10) $g(x) = 4x^3 - 9x^2 + 6x - 1$

11) $j(x) = x^3 + 4x^2 + x - 6$

12) $f(x) = x^4 - 6x^3 + 7x^2 + 6x - 8$

Determine all roots of the function. Then write them as a product of factors using rational numbers.

13) $2x^3 + 6x^2 + 5x + 2 = 0$

14) $x^4 - 3x^3 - 3x^2 - 75x - 700 = 0$

15) $r(x) = 16x^5 - 32x^4 - 81x + 162$

16) $f(x) = x^3 - 5x^2 + 5x + 3$

🖱 **To continue, return to the Online Lesson.**