

Lesson 33

NAME:

Polynomial Functions and the Leading Coefficient Test



Start by following the instructions in the Online Lesson.

Objectives

- ✓ Determine the type of function (polynomial, power, both, neither) represented by equations or graphs.
- ✓ Describe the end behavior using a graph or equation, including the domain and range.

Why?

You have learned how to write a polynomial equation as a product of its factors and determine the end behavior of parent graphs. Now you will apply this knowledge to determine if you have a polynomial function and what its end behavior will be.



Warm Up

Complete the sentences.

- 1) The end behavior of a graph is a trend in the $f(x)$ values when x approaches _____ infinity ($+\infty$) and when x approaches _____ infinity ($-\infty$).
- 2) When thinking about end behavior you can ask yourself:
 - As x gets _____, what happens to y ?
 - As x gets _____, what happens to y ?



To continue, return to the Online Lesson.

🔍 Explore

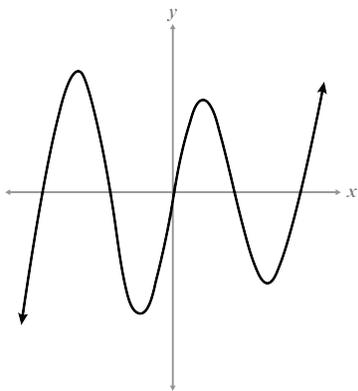
🔍 Polynomial Functions in One Variable

▶ Fill in the notes as you watch the video in the Online Lesson.

Polynomial Functions:

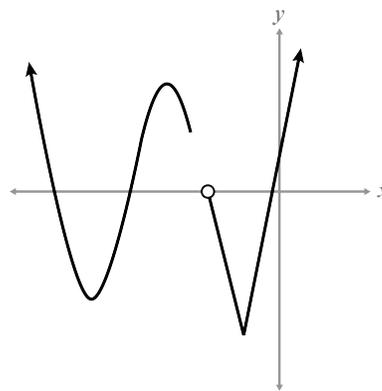
- Are in the form $p(x) = a_n x^n + a_{n-1} x^{n-1} + \dots + a_2 x^2 + a_1 x + a_0$, in which:
 - $a_n \neq$ _____,
 - n is a _____ number, and
 - the coefficients $a_n, a_{n-1}, \dots, a_2, a_1, a_0$ are _____ numbers.
- With graphs that are _____, draw without lifting the pencil or changing direction abruptly.
 - Smooth: no sharp points
 - Continuous: no breaks or gaps

Polynomial Function



This is a graph of a polynomial function because it is _____.

Not a polynomial function



This is not a graph of a polynomial function because it is _____.

Power Functions:

- Are in the form $w(x) = ax^n$, in which a and n are _____ numbers.
- Are monomial functions with a variable base raised to a _____ numerical power.
- May have a numerical _____.
- May have negative or fractional _____ (while a polynomial function cannot).

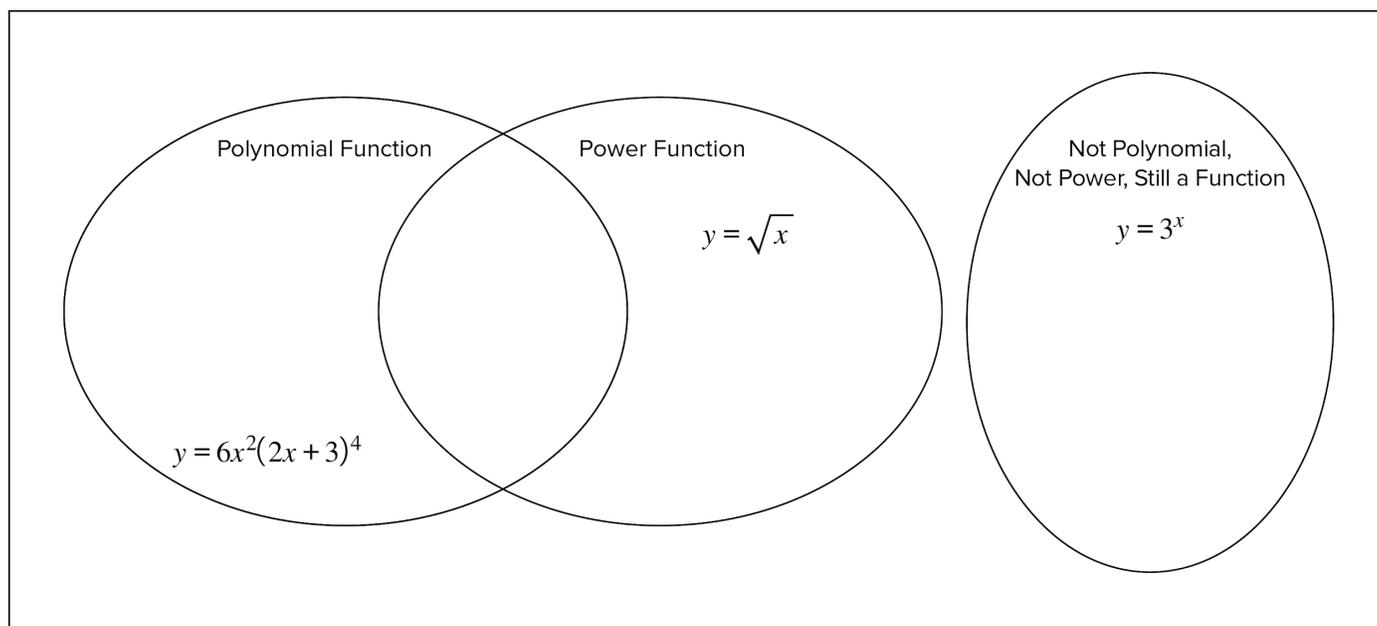
Example 1

▶ Complete the example as you watch the video in the Online Lesson.

Use the Venn diagram to sort the functions. Explain your reasoning.

$$y = x^2 \quad f(x) = x^{-\frac{1}{3}} \quad y = -4x^8 + \sqrt{2} \quad h(x) = 4\pi x^3 - 8x + \pi$$

$$g(x) = x(1+x)^{\frac{1}{2}} \quad y = |x-4| \quad y = \frac{1}{x} \quad k(x) = -\frac{1}{2}x^5$$



Example 2

 Complete the example as you watch the video in the Online Lesson.

Determine if the given functions represent a power and/or polynomial function, or neither.

A) $w(x) = 2^x$

B) $r(x) = \frac{1}{2}(x-8)^2(2x+5)^3$

C) $V(r) = \frac{4}{3}\pi r^3$

D) $y = -2x^3 + 15x^2 - \sqrt{5}$

Checkpoint: Polynomial Functions in One Variable

Determine if the given functions represent a power and/or polynomial function, or neither.

A) $y = x$

B) $A(r) = 9\pi r^2 + 12\pi r + 4\pi$

C) $f(x) = \sqrt[3]{x}$

D) $y = 35x^6 + 14x^5 - x^4 - 2x$

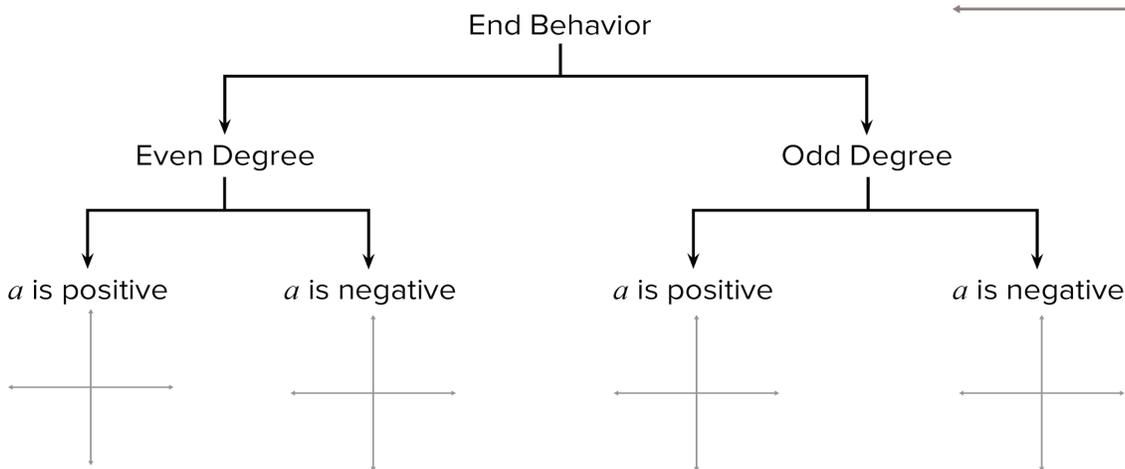
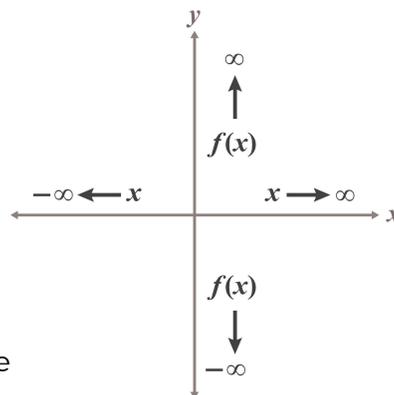


To continue, return to the Online Lesson.

Leading Coefficient Test

Fill in the notes as you watch the video in the Online Lesson.

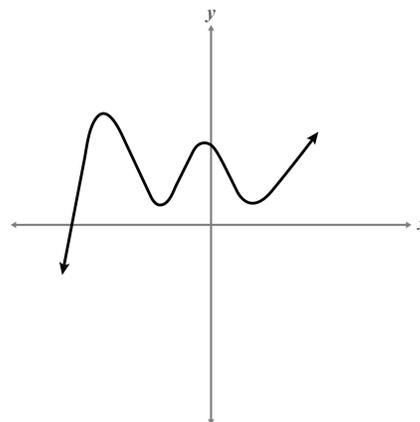
- The leading coefficient test allows you to generalize the _____ of a polynomial function using the leading coefficient a and the degree n .
- Recall the end behavior of a graph:



Example 3

Complete the example as you watch the video in the Online Lesson.

State if the degree of the graph is even or odd and if a is positive or negative. Describe the end behavior of the graph.



Example 4

 Complete the example as you watch the video in the Online Lesson.

Name the degree and leading coefficient. Describe the end behavior of the equation.

$$f(x) = -\frac{1}{4}x^7 + x^5 + \frac{2}{3}$$

Example 5

 Complete the example as you watch the video in the Online Lesson.

Describe the end behavior of the equation.

$$m(x) = 2x^5(x-1)^2(x+3)$$

 Checkpoint: Leading Coefficient Test

Describe the end behavior of the equation. Name the degree and leading coefficient.

$$g(x) = 10x^2(x-12)^3$$

$$y = -7(x + \sqrt{5})^2(x-6)^2$$



To continue, return to the Online Lesson.

 **Practice 1**

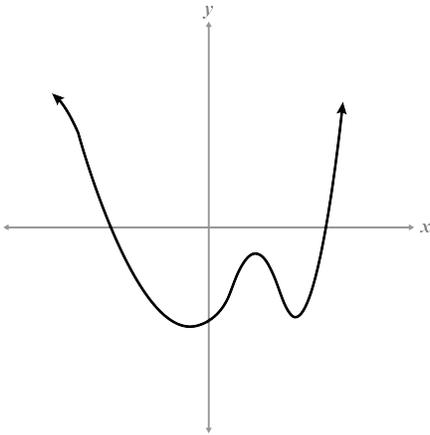
Complete problems on a separate sheet of paper.

- 1) Explain when a polynomial function is also a power function.

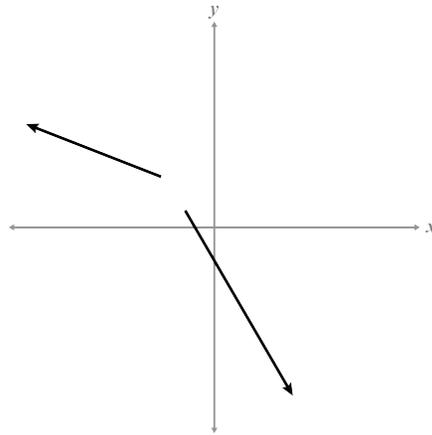
Explain if the graph is a polynomial function.

- If the graph is a polynomial function, describe the end behavior of the graph.
- State if the degree of the graph is even or odd and if a is positive or negative.

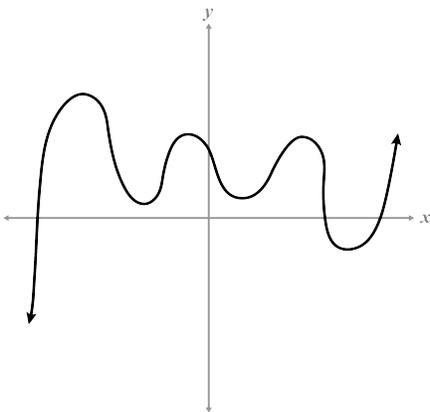
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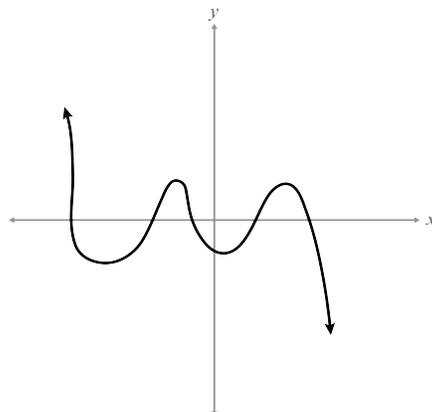
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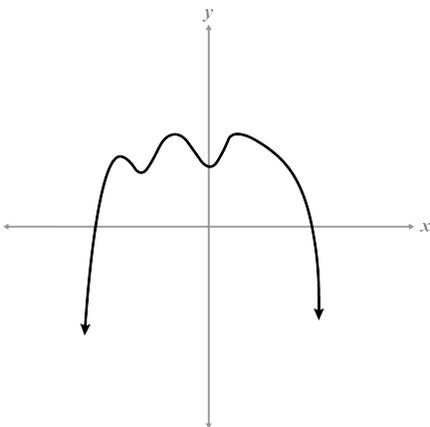
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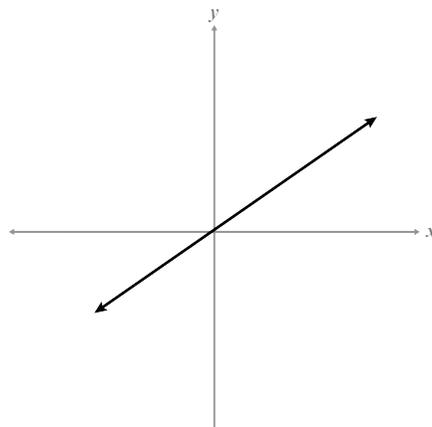
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6)



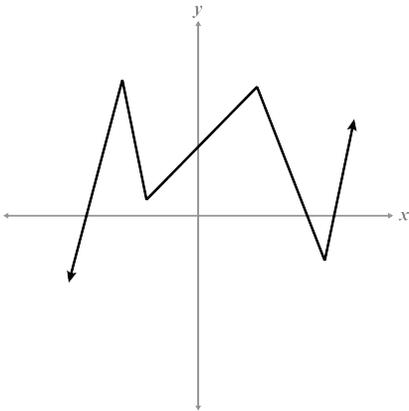
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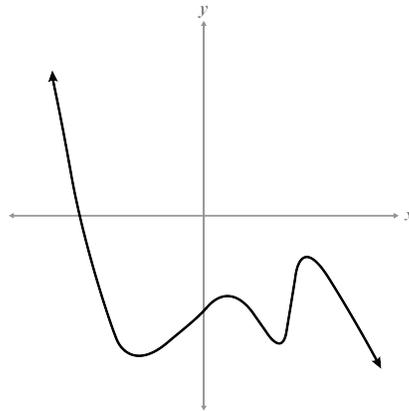
Explain if the graph is a polynomial function.

- If the graph is a polynomial function, describe the end behavior of the graph.
- State if the degree of the graph is even or odd and if a is positive or negative.

8)



9)



Describe the end behavior of the equation.

10) $g(x) = 7x^2 - 2x + 5$

11) $p(x) = -x^3(x + 7)^2(x - 12)$

12) $y = \frac{4}{3}(x - 8)^3$

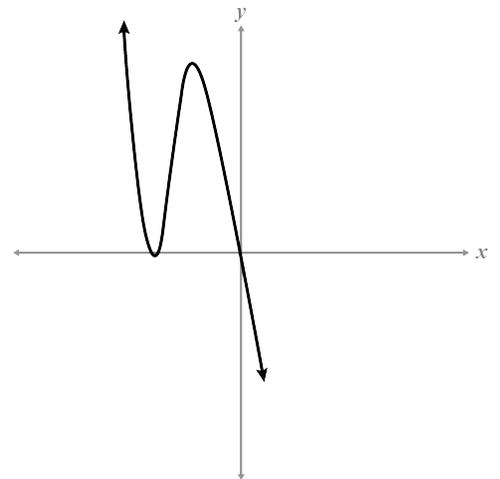
13) $h(x) = -2x(x + 6)^7(x - 2)^2$

14) Draw a Venn diagram to sort the equations into polynomial function, power function, both, or neither.

$r(x) = x^{-\frac{1}{5}}$ $y = 2^x + 1$ $f(x) = -(5x - 1)^3(x + 2)^2$ $m(x) = \frac{5}{8}x - 6$ $A = \pi r^2$

15) Explain if it is possible for the given graph to represent the equation.

$f(x) = \frac{1}{4}x(x - 3)(x + 6)^2$



To continue, return to the Online Lesson.

 **Mastery Check**

 **Show What You Know**

Complete the sentences in parts A, C, and E with sometimes, always, or never.

- A) A polynomial function _____ contains breaks or sharp points. Therefore the graph is _____ smooth and continuous.

- B) Sketch an example and counterexample of a polynomial function.

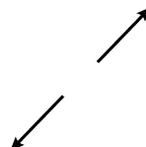
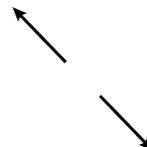
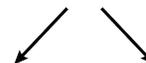
- C) The set of coefficients for a polynomial function $a_n, a_{n-1}, \dots, a_2, a_1, a_0$ are _____ rational numbers.

- D) Explain your response to part C.

- E) The end behavior of polynomial function is _____ determined by the leading coefficient test.

- F) Explain your response to part E.

- G) Determine if the degree of the function is even or odd and if the leading coefficient is positive or negative. Then generalize the end behavior.

Sketch 1 	Sketch 2 	Sketch 3 	Sketch 4 

 **Say What You Know**

In your own words, talk about what you have learned using the objectives for this lesson and your work on this page.

 **To continue, return to the Online Lesson.**

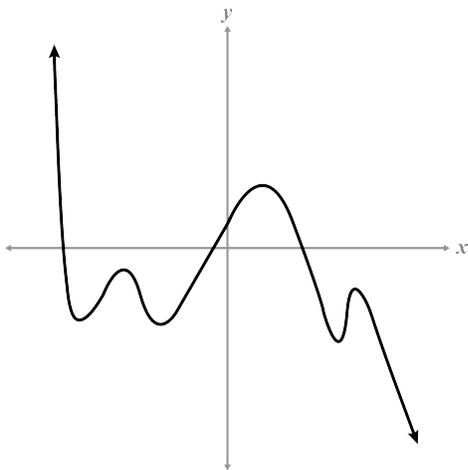
 Practice 2

Complete problems on a separate sheet of paper.

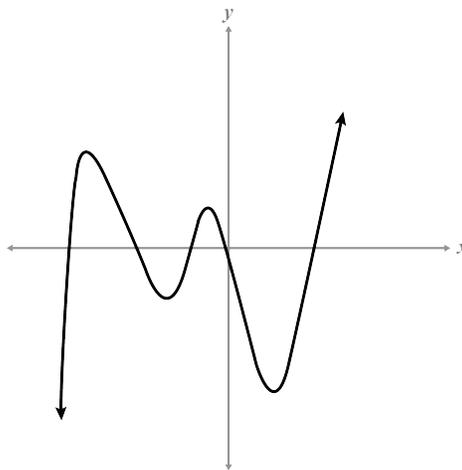
Explain if the graph is a polynomial function.

- If the graph is a polynomial function, describe the end behavior of the graph.
- State if the degree of the graph is even or odd and if a is positive or negative.

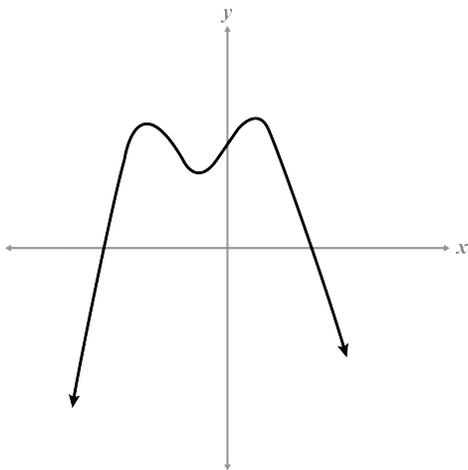
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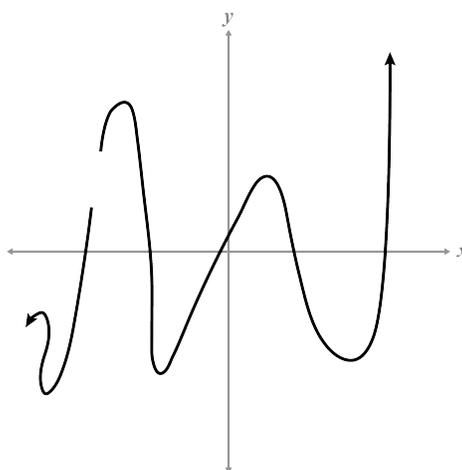
2)



3)



4)



Determine if the given functions represent a power and/or polynomial function, or neither.

5) $f(x) = (3x - 1)^4$

6) $q(n) = 5(n - 8)^{-3} + n$

7) $j(x) = -8x^{\frac{1}{3}}$

8) $y = 3x^5 - 16x^3 + x^2 + x - 14$

Describe the end behavior of the equation.

9) $q(x) = 5(x - 8)^7 + x$

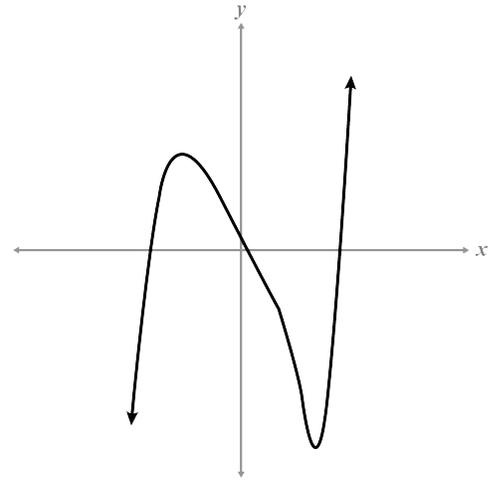
10) $f(x) = -x^4 - 3x^3 + 2x^2 + 8x - 1$

11) $h(x) = -18x^3(x - 1)(x + 7)$

12) $b(x) = x^2(x + 7)^3(x - 6)$

13) Explain if it is possible for the given graph to represent the equation.

$$j(x) = x^5 - 8x^4 + 5x^3 - 12x + 4$$



14) Explain what is needed to determine the end behavior of a polynomial function from an equation.

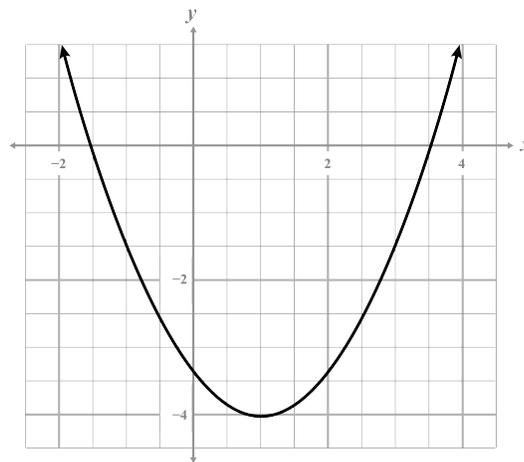


To continue, return to the Online Lesson.

Targeted Review

Complete items on a separate sheet of paper.

- Evaluate: $[j \circ k](-2)$
 $j(x) = (x-2)^3 - 4$ $k(x) = -3x + 2$
- Find $f(-2)$ when $f(x) = ax^2 - 5$ and $f(3) = -23$.
- Determine if the graph has a minimum or maximum point. Name the point.
- Name the x -intercepts from the graph in the previous problem.



- Solve: $3n^3 + 10n = 17n^2$
- Solve under the set of complex numbers: $x^4 = 21x^2 + 100$
- Use the quadratic formula to solve: $5x^2 - 7x + 6 = 0$
- Reflect $f(x) = \sqrt{x+2} - 3$ over the x -axis, and then translate the function five units right, and two units up.

Multiple Choice

- _____ 9) Simplify the polynomial expression: $(x-3)^2 - 2(x-1)^2$
- $-x^2 + 7$
 - $-x^2 - 8x + 10$
 - $3x^2 - 10x + 11$
 - $-x^2 - 2x + 7$

____ 10) Find $[f \circ g](x)$ when $f(x) = 3x^2 - x - 1$ and $g(x) = -x + 6$.

- A) x
- B) $3x^2 - 35x + 101$
- C) $-3x^2 + 8$
- D) $3x^2 + x + 101$

____ 11) Evaluate $(g - f)(4)$ when $f(x) = 3x^2 - 5$, $g(x) = x - 2$.

- A) -41
- B) 41
- C) $\frac{2}{43}$
- D) 45

12) Select all that apply.
A polynomial expression **cannot** contain:

- fractional exponents
- variables inside absolute value bars
- negative coefficients
- variables in the denominator

Problem	1	2	3	4	5	6	7	8	9	10	11	12
Origin	L32	L31	L27	L27	L23	L25	L25	L18	L03	L32	L31	L03

L = Lesson in this level, A1 = Algebra 1: Principles of Secondary Mathematics



To continue, return to the Online Lesson.