Lesson 32
Composite Functions

NAME:

Start by following the instructions in the Online Lesson.

Objectives

- Write a composition of functions.
- Determine the domain for composite functions.
- Determine if functions are inverses using the composition of functions.
- Decompose a composition of functions.

Why?

Composite functions allow you to compose two functions into one new function. The order in which the functions are composed can result in different functions. In real life, this is very important if you are trying to determine the best price for an item.

Warm Up

Determine the inverse function.

1) $m(x) = -\frac{1}{2}(x - 16) + 10$

2) $g(x) = 4(x + 9)^2 - 11$

To continue, return to the Online Lesson.

Explore

Composition of Functions

Fill in the notes as you watch the video in the Online Lesson.

- A composition of functions occurs when one function is _____ into another function.
- A composition differs from a combination of functions because the _____ property does not hold true for composition of functions.

- The composition of the function f with g is noted as _____.
- For all occurrences of x in $f(x)$, substitute the expression _____ and simplify.
  - $[f \circ g](x) =$ _____
  - "The _____ f _____ with g."
- The domain of the function f composed with g is the set of all x where:
  - x is in the domain of _____
  - $g(x)$ is in the domain of _____
    - If x is not in the domain of $g(x)$, then it is not in the domain of $[f \circ g](x)$.
- The _____ for the inside function is the _____ for the exterior function. Therefore, you simplify from the inside out.

$$f[g(x)]$$

Either brackets or parentheses can be used to note a composition of functions. In this level, brackets are used to distinguish compositions from combinations of functions.

Example 1

Complete the example as you watch the video in the Online Lesson.

Find $[g \circ f](x)$ and $[f \circ g](x)$. State the domain and range.

$f(x) = \{(-25, 17), (-7, 8), (8, 0.5), (15, -3)\}$

$g(x) = \{(-3, -7), (0.5, 0), (8, 15), (17, 33)\}$

$[g \circ f](x) = g[f(x)]$

$[f \circ g](x) = f[g(x)]$

$g[f(-25)] = g(17) = 33$

$g[f(-7)] = g(8)$

$g[f(8)]$

$g[f(15)]$

Recall: A relation is a set of ordered pairs and can represent a discrete function.

Example 2

Complete the example as you watch the video in the Online Lesson.

Find $[f \circ g](x)$, $[f \circ f](x)$, and $[g \circ f](x)$ when $f(x) = \dfrac{3}{x-2}$ and $g(x) = -x + 6$. Determine the domain of the composite function.

Plan

Determine the domain for the interior function
Simplify the composition of functions
Determine the domain for the composition of functions

$[f \circ g](x) = f[g(x)]$

$[f \circ g](x) = \dfrac{3}{(-x+6)-2}$

$\text{Domain}_g: \{x \mid x \in \mathbb{R}\}$

$\text{Domain}_{f \circ g}: \{x \mid x \in \mathbb{R}, x \neq 4\}$

$[f \circ f](x) = f[f(x)]$

$= \dfrac{3}{\dfrac{3}{x-2} - 2}$

$[g \circ f](x) = g[f(x)]$

Checkpoint: Composition of Functions

Determine the domain of $h[g(x)]$ for the functions $g(x) = 3x - 2$ and $h(x) = 3x^2 - 5$.

To continue, return to the Online Lesson.

Evaluating a Composition of Functions

Fill in the notes as you watch the video in the Online Lesson.

- As with combinations, you can evaluate functions using:
  - a _____
  - a _____
  - an _____
- Remember that _____ functions extend infinitely, meaning that a composition's solution may lie _____ the visible graph.
- In these instances, use _____ to algebraically evaluate a composition of functions for given values or expressions.

Example 3

Complete the example as you watch the video in the Online Lesson.

Nancy and Sam want to purchase a $\$1100$ refrigerator from a store that allows them to stack coupons. Nancy found a $\$150$ off coupon, and Sam found a 15% off coupon. What composition of applying the coupons will result in the lowest purchase price?

x: price

Let $f(x) =$

Let $g(x) =$

Example 4

Complete the example as you watch the video in the Online Lesson.

Evaluate: $[b \circ a]\left(\frac{c}{4}\right)$

$a(x) = 3x + 2 \qquad b(x) = x^2 + x$

Example 5

Complete the example as you watch the video in the Online Lesson.

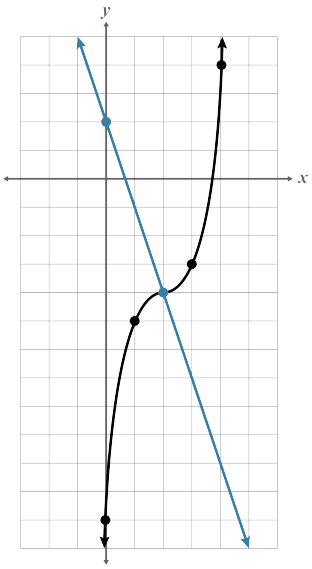
Evaluate. Use the graph or evaluate algebraically.

$j(x) = (x-2)^3 - 4$ $\quad$ $k(x) = -3x + 2$

$[j \circ k](0)$

$k[j(0)]$

$j\left[k\left(\frac{2}{3}\right)\right]$



Checkpoint: Evaluating a Composition of Functions

Use the functions in the previous example to evaluate the composition.

$k[j(-3)]$ $\quad$ $[j \circ k](-a)$

To continue, return to the Online Lesson.

Inverses and a Composition of Functions

Fill in the notes as you watch the video in the Online Lesson.

- A composition of functions can be used to determine algebraically if two functions are _____.
- If $[f \circ g](x) = x$ and $[g \circ f](x) = x$, then the functions are inverses.
- However, if _____ compositions do not _____, then the functions are not inverses.
- Both $[f \circ g](x)$ and $[g \circ f](x)$ must be checked because the composition of functions is _____.

Example 6

Complete the example as you watch the video in the Online Lesson.

Determine which of the given functions are inverses.

$p(x) = \dfrac{\sqrt{x+1}}{2}$ $\quad q(x) = 4\left(x^2 - 1\right), x \geq 0$ $\quad r(x) = 4x^2 - 1, x \geq 0$

$[p \circ q](x)$

Checkpoint: Inverses and a Composition of Functions

Determine if the given functions are inverses.

$f(x) = -\frac{1}{2}(x - 16) + 10$ $\quad$ $g(x) = -2x + 36$

$[f \circ g](x)$ $\quad$ $[g \circ f](x)$

To continue, return to the Online Lesson.

Decomposition of Functions

Fill in the notes as you watch the video in the Online Lesson.

- To mathematically _____ a function, work backward to pull apart a composition to create _____.
- It is possible for there to be _____ to decompose functions.
- If you are unsure that your functions are correct, _____ to see if the result is the given function.

Try to think simply. While a composition can be complicated, the goal of decomposing functions is to think backward, asking yourself, "How did this result come about?"

Example 7

Complete the example as you watch the video in the Online Lesson.

Determine two possible pairs of functions, $p(x)$ and $m(x)$, that form the given composition.

$[p \circ m](x) = w(x)$, when $w(x) = 9\sqrt[3]{x-2} + 6$

First Pair

Second Pair

Checkpoint: Decomposition of Functions

Determine j and k when $[k \circ j](x) = m(x)$.

$$m(x) = \left|\frac{1}{2}x - 6\right| + 2$$

To continue, return to the Online Lesson.

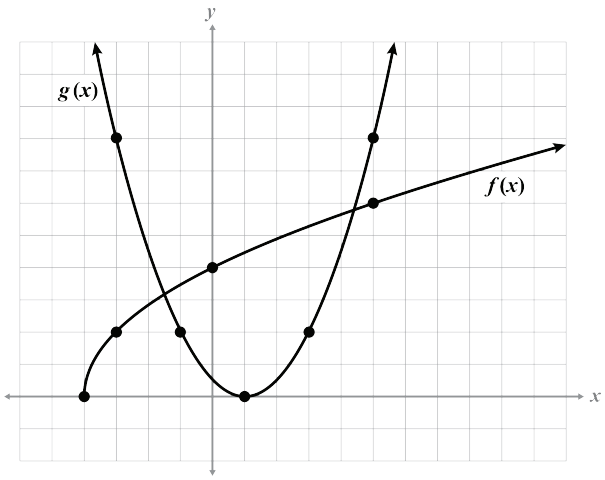
Practice 1

Complete problems on a separate sheet of paper.

For problems 1–3, use the graph and equations.

$f(x) = 2\sqrt{x+4}$

$g(x) = \frac{1}{2}(x-1)^2$



1) $[f \circ f](-4)$

2) $g[f(-4)]$

3) $[f \circ g](-3)$

4) State the domain and range of $[g \circ f](x)$.

$f(x) = \{(-13.4, 1.56), (8.73, -9.08), (6.54, -6.79), (-23.4, 12.1)\}$

$g(x) = \{(-6.79, 1.2), (1.56, -9.67), (-13.4, 2.43), (12.1, -10.21)\}$

For problems 5–9, use the following functions to determine the given composite function. List all domain restrictions for the composite function.

$k(x) = x^2 - 3x + 2 \qquad j(x) = 2x - 5 \qquad p(x) = \frac{1}{x-2}$

5) $[k \circ j](x)$

6) $[j \circ k](x)$

7) $[p \circ j](x)$

8) $[p \circ k](x)$

9) $[j \circ p](x)$

Determine the composite function.

10) $[g \circ f]\left(\frac{1}{x}\right)$, if $f(x) = 3x$ and $g(x) = x^2$

11) $[h \circ k](2x-1)$, if $h(x) = -x^2$ and $k(x) = 4x + 5$

12) $[b \circ g]\left(\frac{4}{c}\right)$, if $b(x) = \frac{1}{x}$ and $g(x) = \sqrt{x}$

13) Determine f and g such that $[f \circ g](x) = h(x)$ and $h(x) = 2(x-7)^3 + 5$. Select all that apply.

- ☐ $f(x) = x - 7$
  $g(x) = 2x^3 - 5$
- ☐ $f(x) = 2x^3 + 5$
  $g(x) = x - 7$
- ☐ $f(x) = 2x + 5$
  $g(x) = (x-7)^3$
- ☐ $f(x) = (x-7)^3$
  $g(x) = 2x + 5$

14) Determine j and k such that $[j \circ k](x) = m(x)$ and $m(x) = \frac{1}{2}|x-6| + 3$. Select all that apply.

- ☐ $j(x) = \frac{1}{2}x + 3$
  $k(x) = |x - 6|$
- ☐ $j(x) = |x + 3|$
  $k(x) = \frac{1}{2}x - 6$
- ☐ $j(x) = x - 6$
  $k(x) = \frac{1}{2}|x| + 3$
- ☐ $j(x) = \frac{1}{2}|x| + 3$
  $k(x) = x - 6$

15) Determine a and g such that $[a \circ g](x) = z(x)$ and $z(x) = \sqrt[3]{(x-2)^2} - 1$. Select all that apply.

☐ $a(x) = \sqrt[3]{x-1}$
$g(x) = x - 2$

☐ $a(x) = \sqrt[3]{x^2} - 1$
$g(x) = x - 2$

☐ $a(x) = \sqrt[3]{x} - 1$
$g(x) = (x-2)^2$

☐ $a(x) = x - 1$
$g(x) = \sqrt[3]{(x-2)^2}$

16) The regular price of a new gaming system is x dollars. A store allows customers to apply a \$200 off coupon as well as a 15% off coupon. Determine which coupon should be applied first to get the lowest price.

For problems 17–20, determine if $f(x)$ and $g(x)$ are inverses.

17) $f(x) = \sqrt[3]{x-2} + 5 \qquad g(x) = (x-5)^3 + 2$

18) $f(x) = \frac{1}{2}x + 4 \qquad g(x) = 2x - 4$

19) $f(x) = \frac{\sqrt{x-3}}{4} \qquad g(x) = 16x^2 + 3,\ x \geq 0$

20) $f(x) = \sqrt{x-1} + 5 \qquad g(x) = (x-5)^2 + 1,\ x \geq 5$

To continue, return to the Online Lesson.

Mastery Check

Show What You Know

Use the given functions to complete the problem.

$$f(x) = \sqrt{x} \qquad g(x) = \frac{1}{x} + 6 \qquad h(x) = \frac{1}{x - 6}$$

A) Marisol and Nicole were asked to compose h with f. Name the person that has the composition of functions correctly written, then compose the function with the domain restrictions.

Marisol	**Nicole**
$[f \circ h](x) = f[h(x)]$	$[h \circ f](x) = h[f(x)]$

B) Marisol and Nicole believe that g and h are inverses. Show all work to determine if they are correct.

C) Marisol and Nicole's instructor told them $j(x) = x - 6$ and is composed with a new function k and one of the given functions f, g, or h.

Decompose $j(x)$ to determine $k(x)$ and one of the given functions. Show your work to prove you are correct.

Say What You Know

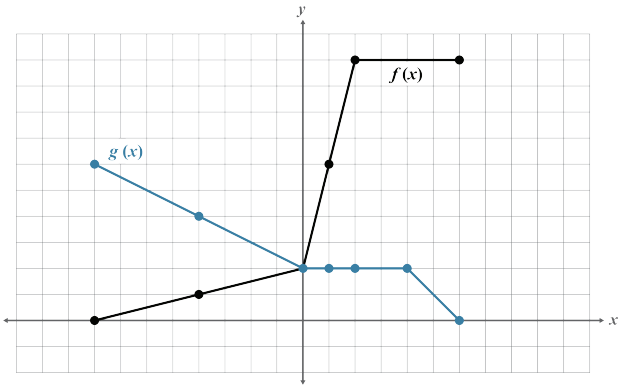
In your own words, talk about what you have learned using the objectives for this lesson and your work on this page.

To continue, return to the Online Lesson.

Practice 2

Complete problems on a separate sheet of paper.

For problems 1–5, solve with the graph and algebraically.



1) $[f \circ g](0)$

2) $[g \circ f](-8)$

3) $[f \circ g](2)$

4) $[g \circ f](-4)$

5) $[f \circ g](-4)$

For problems 6–10, use the following functions to determine the given composite function. List all domain restrictions for the composite function.

$f(x) = 3x^2 - 5 \qquad g(x) = x - 2 \qquad h(x) = \dfrac{1}{2x+1}$

6) $[g \circ g](x)$

7) $[g \circ h](x)$

8) $[g \circ f](x)$

9) $[h \circ f](x)$

10) $[f \circ f](x)$

Determine the composite function.

11) $[v \circ z](3n^2)$, if $v(x) = \dfrac{1}{4}x^2$, $x \geq 0$ and $z(x) = 2\sqrt{x}$

12) $[r \circ m](5a)$, if $r(x) = 2\sqrt[3]{x} + 4$ and $m(x) = (x-2)^3$

13) $[g \circ h](7y)$, if $g(x) = 8x + 2$ and $h(x) = \dfrac{1}{x}$

14) Determine f and g such that $[f \circ g](x) = h(x)$ and $h(x) = \sqrt{5x^2 - 7}$. Select all that apply.

- ☐ $f(x) = \sqrt{x}$
  $g(x) = 5x^2 - 7$
- ☐ $f(x) = \sqrt{x - 7}$
  $g(x) = 5x^2$
- ☐ $f(x) = x$
  $g(x) = \sqrt{5x^2 - 7}$
- ☐ $f(x) = \sqrt{5x - 7}$
  $g(x) = x^2$

15) Determine p and r such that $[p \circ r](x) = w(x)$ and $w(x) = \dfrac{10x - 17}{3}$. Select all that apply.

- ☐ $p(x) = 10x - 17$
  $r(x) = \dfrac{x}{3}$
- ☐ $p(x) = \dfrac{x}{3}$
  $r(x) = 10x - 17$
- ☐ $p(x) = \dfrac{x}{3} - 17$
  $r(x) = 10x$
- ☐ $p(x) = \dfrac{x - 17}{3}$
  $r(x) = 10x$

16) Determine q and c such that $[q \circ c](x) = h(x)$ and $h(x) = x^2 - 11$. Select all that apply.

☐ $q(x) = \frac{x}{3} - 2$
$c(x) = 3x^2 - 27$

☐ $q(x) = x - 2$
$c(x) = x^2 - 9$

☐ $q(x) = x^2$
$c(x) = x - 11$

☐ $q(x) = x - 11$
$c(x) = x^2$

Determine if $f(x)$ and $g(x)$ are inverses.

17) Russell received a 20% off coupon in the mail for a new generator. The store was also having a sale that allowed for 10% off all items. The store determined it was best to use the sale first, then the coupon. Russell asked to use the coupon first, then the sale. Which combination saves Russell the most money?

18) $f(x) = 8(x + 14)^3 - 18$ $\quad g(x) = \frac{\sqrt[3]{x + 18}}{2} - 14$

19) $f(x) = \frac{3}{x - 4}$ $\quad g(x) = \frac{3}{x} + 4$

20) $f(x) = \frac{5}{8}x - 7$ $\quad g(x) = \frac{8}{5}x + 7$

To continue, return to the Online Lesson.