Lesson 30
Hyperbolas

NAME:

Start by following the instructions in the Online Lesson.

Objectives

- Write the equation of a hyperbola using the given information.
- Graph a hyperbola using the given information.
- Transform hyperbolas.
- Determine the specific type of conic section (parabola, circle, ellipse, hyperbola).

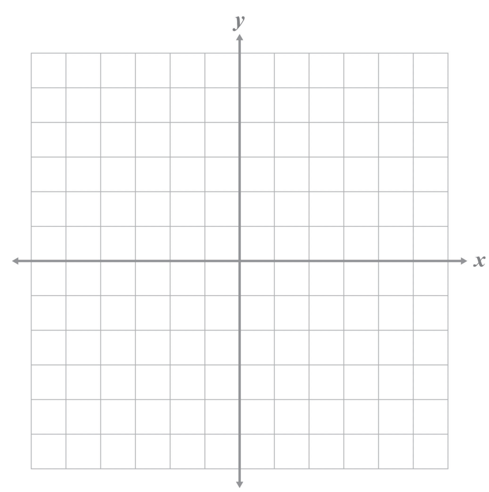
Why?

A hyperbola is a conic section that has applications in engineering and physics, such as if you work as a nuclear scientist on a cooling tower or launch a satellite while tracking the trajectory of other celestial objects. If your career path doesn't take you into engineering or physics, you will still be able to appreciate the mathematics involved in these careers.

Warm Up

Name the point and slope from the given equation in the form $y - y_1 = m(x - x_1)$, then graph.

1) $y - 4 = \frac{3}{2}(x + 1)$



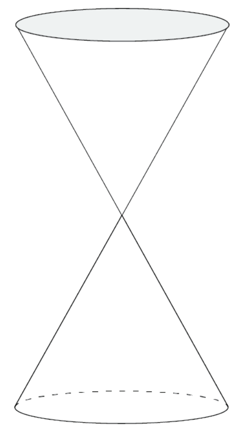
2) $y + 2 = -\frac{2}{5}(x - 3)$

To continue, return to the Online Lesson.

Explore

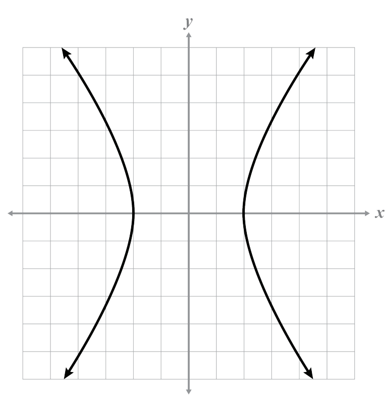
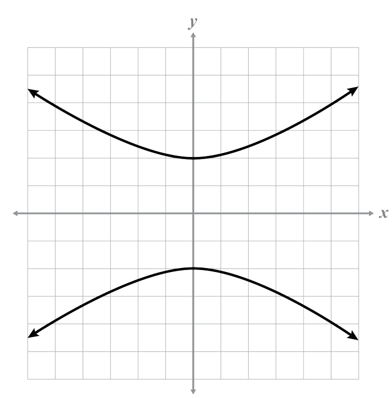
Hyperbolas

Fill in the notes as you watch the video in the Online Lesson.



Hyperbola

- A ____________________ is a conic section that is an open curve with _____________ symmetric u-shaped branches set between asymptotes.
- A hyperbola is called a ___________________________ slice when referring to conic sections.
- The equation of a hyperbola has two general forms to represent either a ____________________ or ____________________ graph.
- The hyperbola is broken into two parts, or branches, that fit between two ______________________________ that meet at the center, (h, k).
- The asymptotes of hyperbolas will be written in point-slope form where ____________.

	Horizontal	Vertical
General Form	$\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$	$\frac{(y-k)^2}{b^2} - \frac{(x-h)^2}{a^2} = 1$
Asymptotes		
Branches		
Transverse Axes		

- The branches of the hyperbola intersect the transverse axis at the __________________.
- The vertices and co-vertices form a ______________________ that can be used to graph the slant asymptotes diagonally across the rectangle and through the center, (h, k).
- The direction that the branches open is determined by the _____________ of the equation.
- Note the _____________________ on your paper with a quick sketch so that you can confirm your final graph matches it.

Example 1

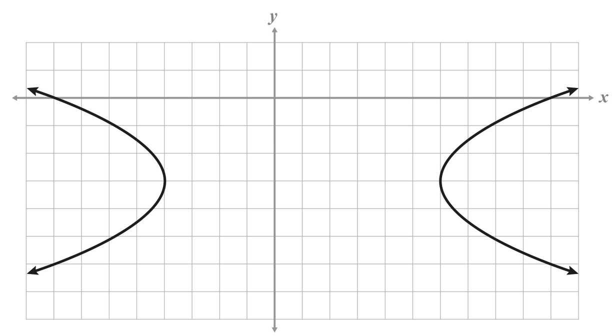
Complete the example as you watch the video in the Online Lesson.

Mark the center, asymptotes, and the vertices and co-vertices on the graph for the equation:

$$\frac{(x-1)^2}{25} - \frac{(y+3)^2}{4} = 1$$

Plan

Determine the center, a, b
Mark the vertices and co-vertices
Mark the asymptotes
Write the equation of the asymptotes



$$m = \pm\frac{b}{a}$$

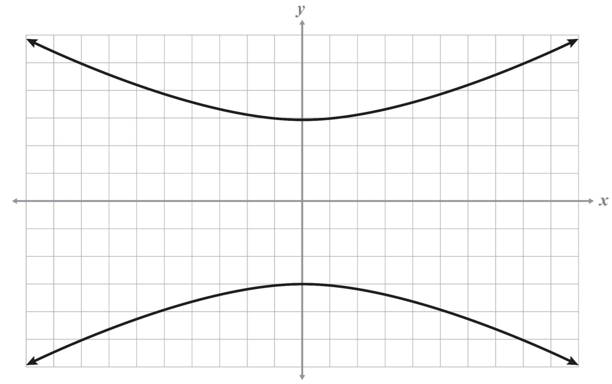
$$y + 3 = \frac{2}{5}(x-1)$$

Example 2

Complete the example as you watch the video in the Online Lesson.

Mark the center, asymptotes, and the vertices and co-vertices on the graph for the equation:

$$\frac{y^2}{9} - \frac{x^2}{36} = 1$$

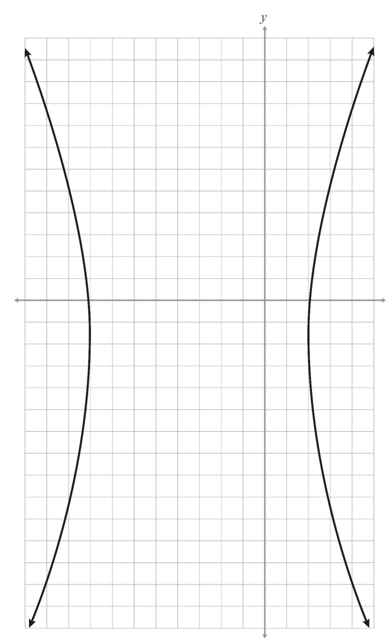


$$m = \pm\frac{b}{a}$$

Checkpoint: Hyperbolas

Mark the center, asymptotes, and the vertices and co-vertices on the graph for the equation:

$$\frac{(x+3)^2}{25} - \frac{(y+2)^2}{121} = 1$$



Write the equation of the asymptotes in point-slope form.

To continue, return to the Online Lesson.

Graphing Hyperbolas

Fill in the notes as you watch the video in the Online Lesson.

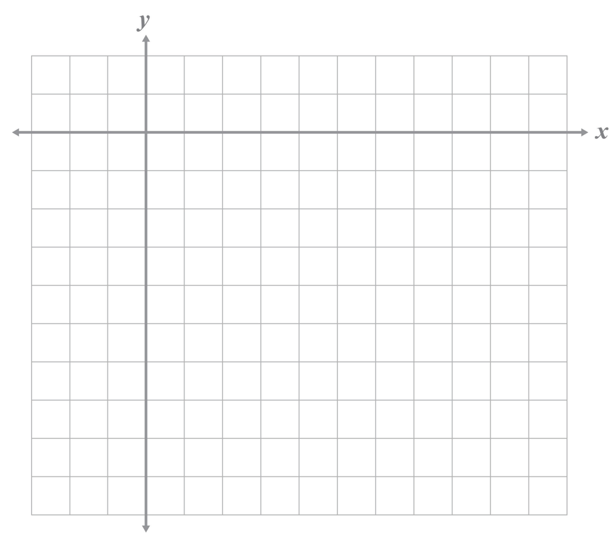
- To graph a hyperbola, follow these guidelines:
  - Write the equation in ________________ form (if needed).
  - Sketch the opening of the ________________ (recommended, but optional).
  - Determine/Mark the ___________.
  - Mark the ____________________________________ as you would for an ellipse.
  - Sketch a dashed __________________ to help graph the asymptotes (recommended, but optional).
  - Graph the ____________________ from the center to the vertex (corner) of the rectangle diagonally (recommended, but optional).
  - Sketch the __________________ in between the asymptotes.

Example 3

Complete the example as you watch the video in the Online Lesson.

Graph.

$4x^2 - 9y^2 - 40x - 72y = 8$

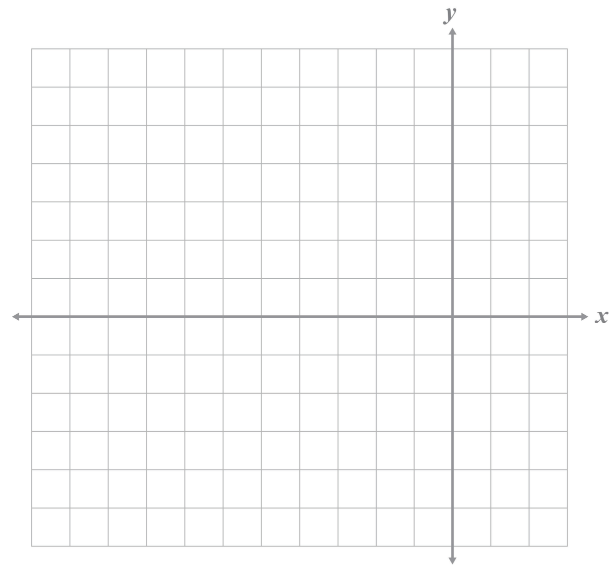


Example 4

Complete the example as you watch the video in the Online Lesson.

Graph the hyperbola with horizontal branches using equations of the asymptotes:

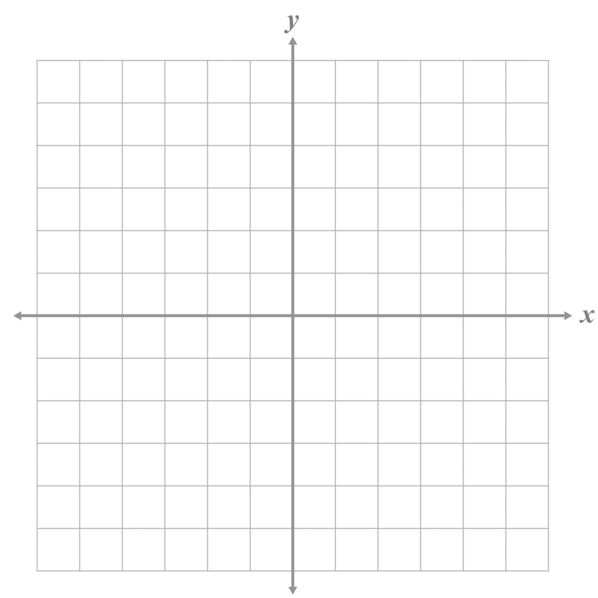
$$y = \pm \frac{5}{4}(x+4)$$



Checkpoint: Graphing Hyperbolas

Graph.

$$x^2 - \frac{y^2}{16} = 1$$



To continue, return to the Online Lesson.

Identifying Conics

Fill in the notes as you watch the video in the Online Lesson.

- When given a ______________ of a conic, identify the ____________ to determine if it is a parabola, circle, ellipse, or hyperbola.
- When given an ____________________ of a conic that is not in standard form, complete the square to determine the type of conic.
- Writing equations in standard form (or vertex form for parabolas) is important for identifying:
    - the ______________ of conic section that is given.
    - individual ____________________ of each type of conic.
- Conic sections can be identified when written in the form $Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$ by identifying the distinguishing characteristics of each conic section using the coefficients _______________________.
    - A, B, and C _______________ all equal to zero.
    - A–F are ____________________.
- Using the sample sentence in the table can help you identify conic sections:

When...	**the conic section is a(n) ___________.**
A or C is zero	
$A = C$ and are non-zero	
$A \neq C$ and are non-zero and have the same sign	
A and C are non-zero and have opposite signs	

Example 5

Complete the example as you watch the video in the Online Lesson.

Name the conic sections. Explain.

$P: x^2 + 36y^2 - 72x = 180$ $Q: 2x^2 - 8y^2 + 8x + 32 = 16y$ $R: y^2 - 9x + 18y + 81 = 0$

Plan

Identify A and C
Determine the conic

P	$A = 1, C = 36$ Since A and C are ______________________________, equation P represents ____________.
Q	Since A and C are ______________________________, equation Q represents ____________.
R	

Example 6

Complete the example as you watch the video in the Online Lesson.

Determine the type of conic. Then name the domain and range.

$x^2 + y^2 + 9y + 1.5 = 5x$

$A = 1, C = 1$

Since ____________, and are non-zero, the equation is ____________.

Checkpoint: Identifying Conics

Name the conic section. Explain.

$G: 4x^2 + 20xy + 20 = 2y^2 + 9x$ $H: 5x + 8y = x^2 + y^2$

To continue, return to the Online Lesson.

Practice 1

Complete problems on a separate sheet of paper.

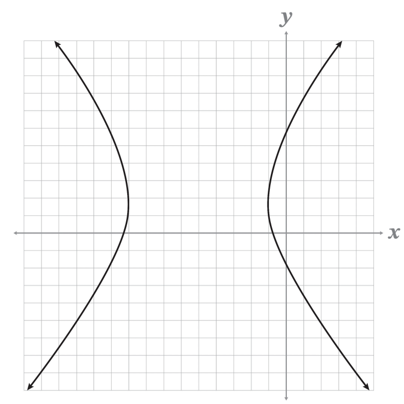
Complete the sentence with *always*, *sometimes*, or *never*.

1) A hyperbola will __________ intersect the larger (longer) axis.

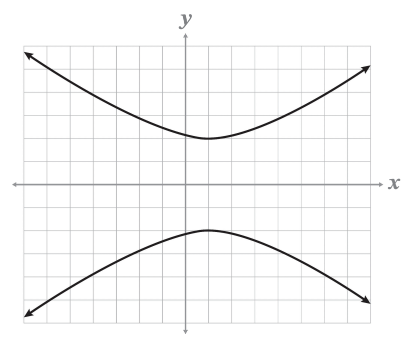
2) Conic sections are __________ functions.

3) A circle is __________ a special form of an ellipse.

4) Mark the center, vertices and co-vertices, and asymptotes on the graph of $\frac{(x+4)^2}{16}-\frac{(y-2)^2}{25}=1$. Then write the equation of the asymptotes.



5) Mark the center, vertices and co-vertices, and asymptotes on the graph of $\frac{y^2}{4}-\frac{(x-1)^2}{9}=1$. Then write the equation of the asymptotes.



6) Graph the hyperbola: $\frac{x^2}{9}-\frac{(y-3)^2}{25}=1$

7) Graph the hyperbola: $16x^2-64x-36y^2+72y=116$

8) Graph: $\frac{y^2}{49}-\frac{x^2}{81}=1$

9) Graph the horizontal hyperbola with asymptotes: $y-6=\pm\frac{6}{7}(x+1)$

10) Graph the hyperbola with vertices at $(-8, 0)$ and $(8, 0)$ and co-vertices at $(0, 3)$ and $(0, -3)$.

11) Mike was asked to write the hyperbola $9x^2-36x-16y^2-96y=252$ in standard form. Verify or find the error in his solution. If there is an error, determine the correct equation.

$$9x^2-36x-16y^2-96y=252$$

$$9\left(x^2-4x\right)-16\left(y^2-6y\right)=252$$

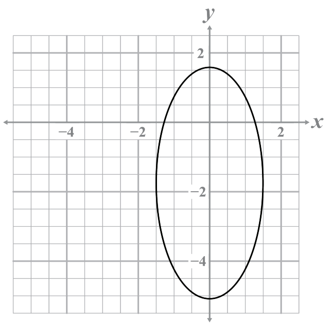
$$9\left(x^2-4x+\boxed{\left(-\frac{4}{2}\right)^2}\right)-16\left(y^2+6y+\boxed{\left(\frac{6}{2}\right)^2}\right)=252+9\boxed{\left(-\frac{4}{2}\right)^2}+16\boxed{\left(\frac{6}{2}\right)^2}$$

$$9(x-2)^2-16(y-3)^2=432$$

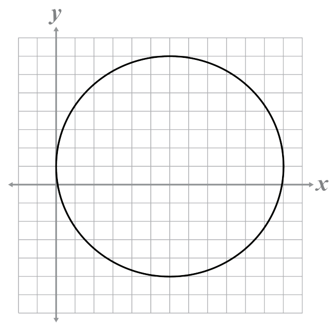
$$\frac{(x-2)^2}{48}-\frac{(y-3)^2}{27}=1$$

Match the graph to the equation.

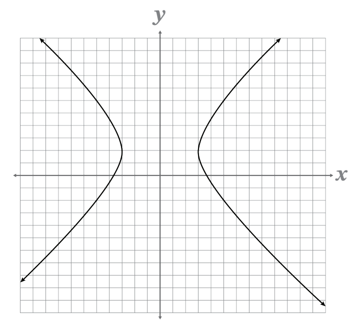
A)



B)



C)



12) $x^2 - 12x + y^2 - 2y = -1$

13) $x^2 - y^2 + 4y = 13$

14) $6x^2 + y^2 + 4y = 9$

Name the conic section. Explain.

15) $8y + 2 - x^2 = x - 1$

16) $3y^2 - 6y = 3x^2 + 15$

17) $4x^2 = -9y^2 + 36y$

18) $x = 2y^2 + 4$

To continue, return to the Online Lesson.

Mastery Check

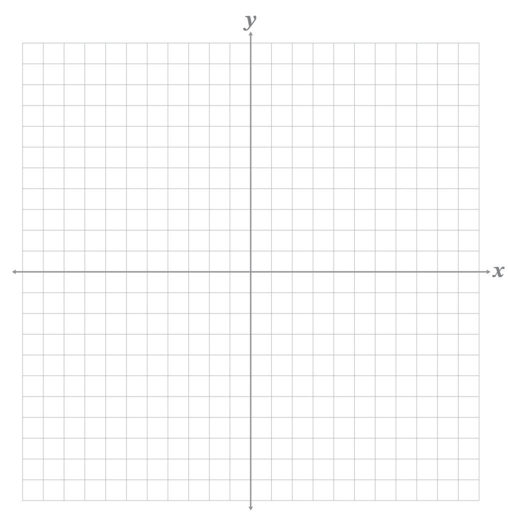
Show What You Know

A) Determine which of the given equations is a hyperbola. Explain.

$J:\ 4x^2+2y^2=32x+32y-2y^2$ $K:\ y^2=4x^2+32x+100$ $L: x^2+2y^2=x^2+64y+32x$

B) Write the equation of a hyperbola from part A in standard form. Name the center and equation for the asymptotes.

C) Graph your equation in part B.



Say What You Know

In your own words, talk about what you have learned using the objectives for this lesson and your work on this page.

To continue, return to the Online Lesson.

Practice 2

Complete problems on a separate sheet of paper.

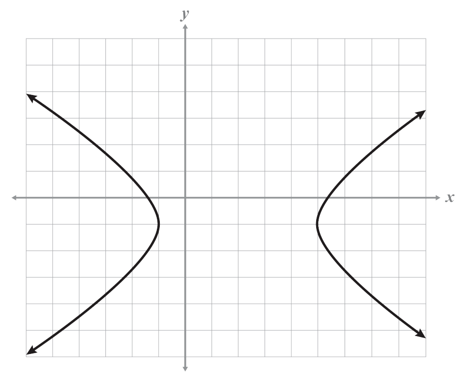
Complete the sentence with *always*, *sometimes*, or *never*.

1) An ellipse is __________ a function.

2) The center of any conic section is __________ (h, k).

3) A hyperbola in the form $\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$ will __________ have horizontal branches.

4) Roy graphed the following hyperbola: $\frac{(x+2)^2}{9} - \frac{(y-1)^2}{4} = 1$.
Find the error in his solution.



5) Graph the hyperbola. Mark the center, vertices and co-vertices, and asymptotes of $(x+1)^2 - \frac{y^2}{25} = 1$. Then write the equation of the asymptotes.

6) Graph the hyperbola. Mark the center, vertices and co-vertices, and asymptotes of $\frac{(y-3)^2}{25} - \frac{x^2}{4} = 1$. Then write the equation of the asymptotes.

7) Graph the horizontal hyperbola when the equation of the asymptotes is $y = \pm\frac{2}{3}(x-6)$. Then write the equation of the hyperbola.

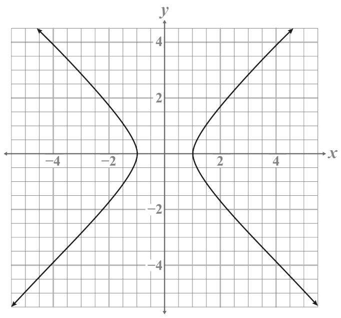
8) Write the equation and graph the hyperbola $6y^2 - 36y - 6x^2 + 12x = 102$.

9) Find an equation of the vertical hyperbola when the equation of the asymptotes is $y + 2 = \pm(x+1)$.

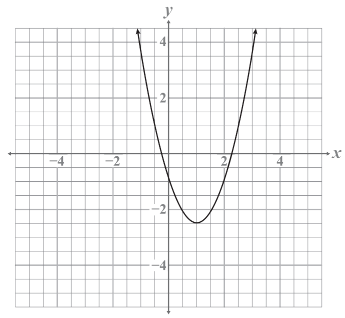
10) Write the equation of the hyperbola $8x^2 - 8x - 8y^2 - 32y = 158$.

Match the graph to the equation.

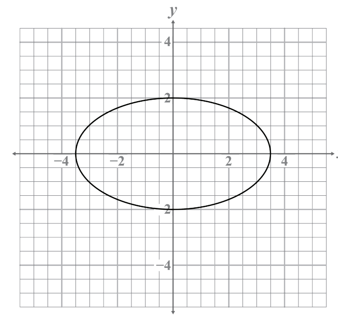
A)



B)



C)



11) $2y = 3x^2 - 6x - 2$

12) $x^2 - y^2 = 1$

13) $x^2 + 3y^2 = 12$

Name the conic section. Explain.

14) $\frac{2(x+4)^2}{15} - 4y = 1$

15) $\frac{6y^2}{13} - x^2 = 4$

16) $x^2 + 6y^2 - 14y = 7y^2 - 10$

17) $-4x^2 - 8x + 9 = 4y^2 - 10$

18) $32x - 16x^2 + 14y - 7y^2 = 12$

To continue, return to the Online Lesson.

