Lesson 29
Ellipses

NAME:

Start by logging on to the Digital Toolbox and navigating to the Online Lesson for instructions.

Objectives

- Write the equation of an ellipse using the given information.
- Graph an ellipse using the given information.
- Transform ellipses.

Why?

Ellipses are the next conic section in your study of conics. While ellipses are closed curves like circles, they appear stretched or squished, which introduces a new formula. The formula for ellipses can be used to calculate things like the orbits of planets or design structures like bridges.

Warm Up

Clear the fraction(s) from the equation. Do not solve.

1) $\frac{2}{5}(x-8)=7y$

2) $\frac{4}{3}g+\frac{1}{8}h=-6$

Rewrite the equation so that the value of the constant is equal to one.

3) $5x+3y=9$

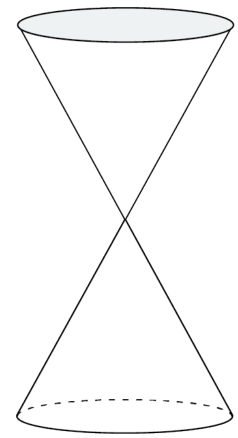
4) $2x-15y=-30$

To continue, return to the Online Lesson.

Explore

Ellipses

Fill in the guided notes as you watch the video in the Online Lesson.


Ellipse

- Ellipses are similar to circles in that they are both ________________ curves. However, while all circles are ellipses, not all ellipses are circles.
- An ellipse is the set of all points in a plane where the _________ of the distances from two fixed points is ____________________.
- The fixed points are called _____________. (One is called a focal point.)
- The _____________________ form of an ellipse is: $\frac{(x-h)^2}{a^2}+\frac{(y-k)^2}{b^2}=1$
- (h, k) represents the ______________ point.
- a, b represent _____________ the length of the axes (where a is horizontal and b is vertical) and must be _________.
- Of the two axes, a or b, one is ________________ (longer) and the other is ________________ (shorter), the combination of which gives the ellipse its squished or stretched shape.
- The value of a and b determines:
  - which of the axes is the _______________ axis, and
  - if the ellipse will be ___________________ or _________________.
- ____________________ are where the ellipse intersects the major axis.
- ____________________ are where the ellipse intersects the minor axis.
- Vertices and co-vertices are sometimes referred to as ______________________ because they determine the values of the domain and range.

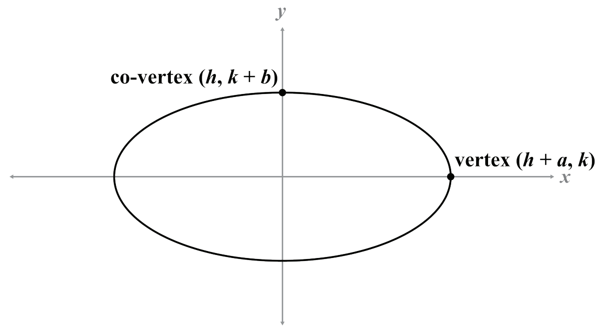
Horizontal Major Axis

$a > b$

Center: (h, k)

Vertices: $(h - a, k)$ and $(h + a, k)$ or ___________

Co-Vertices: $(h, k - b)$ and $(h, k + b)$ or _________



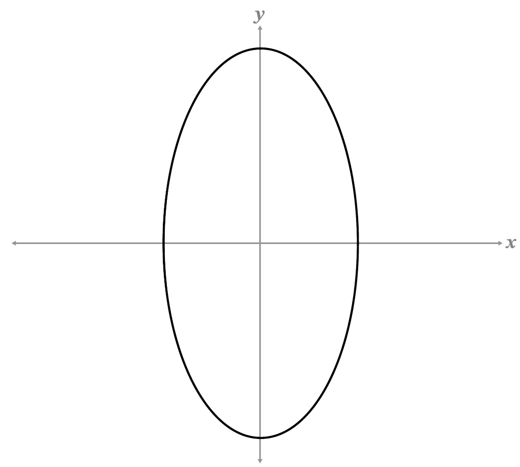
Vertical Major Axis

$a < b$

Center: (h, k)

Vertices: $(h, k - b)$ and $(h, k + b)$ or _________

Co-Vertices: $(h - a, k)$ and $(h + a, k)$ or ___________



- When asked to write the equation of an ellipse from a graph you need the

  _________________, and the values of ________________.

Example 1

Complete the example as you watch the video in the Online Lesson.

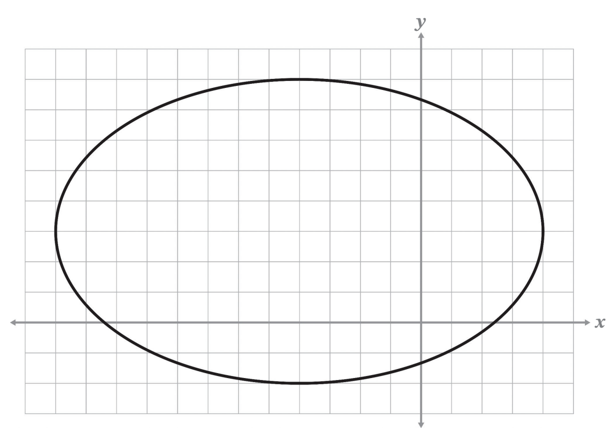
Write the equation of the ellipse on the coordinate plane. Name the length of the major axis and minor axis.

Plan

Determine major axis
Mark on graph: center, vertices, co-vertices
Write equation
Name the length of each axis

Horizontal major axis: $a > b$

Center: $(-4, 3)$

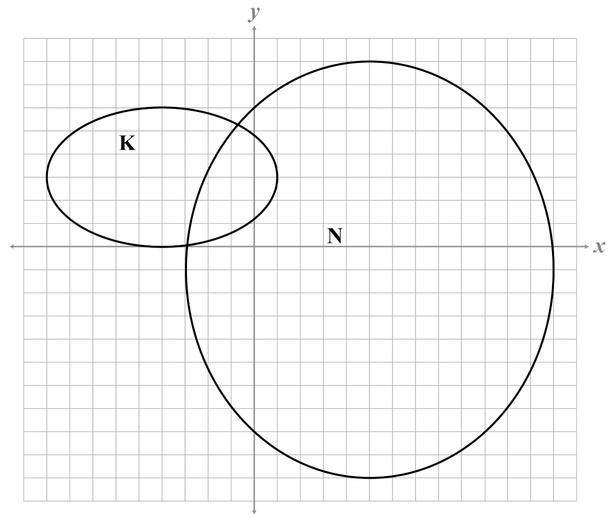


Example 2

Complete the example as you watch the video in the Online Lesson.

Write the equation of ellipse K and ellipse N. Name the co-vertices of ellipse K and the vertices of ellipse N.

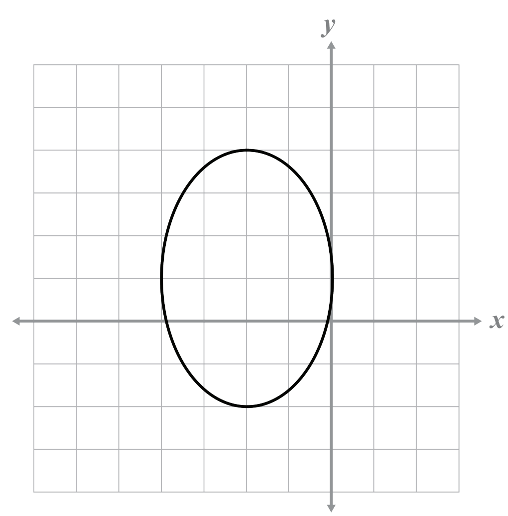
Ellipse K



Ellipse N

Checkpoint: Ellipses

Write the equation of the ellipse. Label the center, vertices, and co-vertices. Name the length of the major and minor axis.



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Graphing and Translating Ellipses

Fill in the guided notes as you watch the video in the Online Lesson.

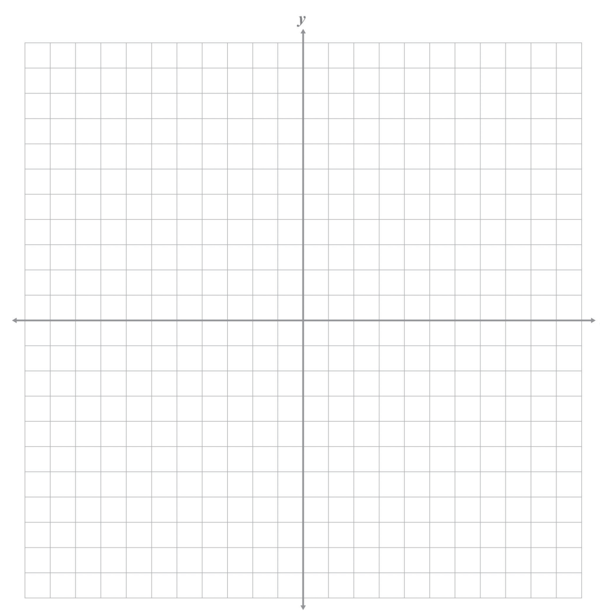
- To graph an ellipse on the coordinate plane, mark these items:
  - ____________________
  - ____________________
  - ____________________
- Optionally, you can sketch vertical and horizontal ____________________________ to help make your graph more accurate.
  - A tangent line is a line that touches a curve at ____.
  - When drawing tangent lines for ellipses, use ___________________ lines.
- An ellipse is transformed by _________________________ it horizontally or vertically.
  - Horizontal translations move the center _________________________.
  - Vertical translations move the center _________________________.

Example 3

Complete the example as you watch the video in the Online Lesson.

Graph. Describe the similarities and differences between the graphs.

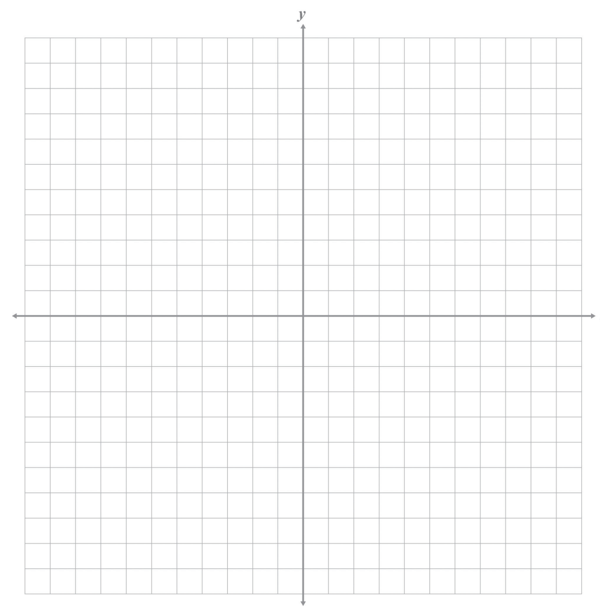
$P: \frac{x^2}{49} + \frac{y^2}{49} = 1 \quad Q: \frac{x^2}{81} + \frac{y^2}{49} = 1$



Example 4

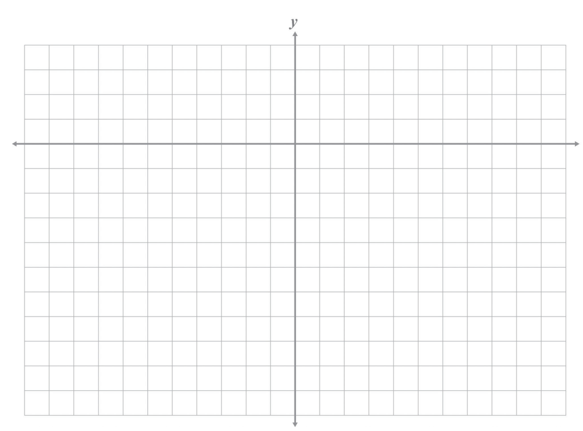
Complete the example as you watch the video in the Online Lesson.

Write the equation of an ellipse with a vertical major axis length of 10 units and a minor axis of 8 units that is translated right 5 units and down 3 units from the origin, then graph.



Checkpoint: Graphing and Translating Ellipses

Write the equation of an ellipse with a horizontal major axis of 12 units, a minor axis of 2 units, and a center at (–1, –3). Then graph.



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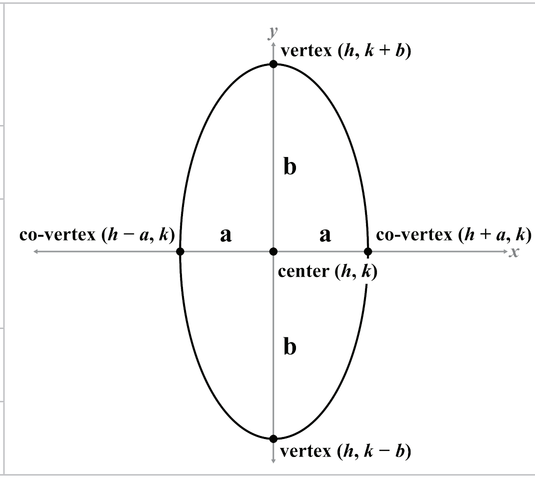
Equations of Ellipses

Fill in the guided notes as you watch the video in the Online Lesson.

- Recall what you already know about ellipses:

	$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$	
	Center point	
	Half the length of the axes, ____ horizontal and ____ vertical	
	Where ellipse intersects major axis	
	Where ellipse intersects minor axis	



- If you are given the vertices and co-vertices, you must also determine the distance of the major and minor axes to calculate the ______________________________.
- The ________________ of the major or minor axis is the center of the ellipse.
- The ________________ (vertices and co-vertices) determine the domain and range.
  - ________________ is between $h - a$ and $h + a$.
  - ________________ is between $k - b$ and $k + b$.
- For an ellipse equation that is not in standard form, rewrite it using this method:
  - ______________________________ for both x and y.
  - Then divide all terms by the value of the ____________________ so that the equation equals 1.

Example 5

Complete the example as you watch the video in the Online Lesson.

Write the equation in standard form. Name the domain and range in set builder notation.

$9x^2 + 4y^2 + 18x - 16y = 11$

Example 6

Complete the example as you watch the video in the Online Lesson.

Write the equation of an ellipse with endpoints: $(-17, 0), (8, \sqrt{13}), (8, -\sqrt{13}), (33, 0)$

Horizontal $(-17, 0), (33, 0)$

$d = \sqrt{(-17-33)^2 + (0-0)^2}$

$d = 50$

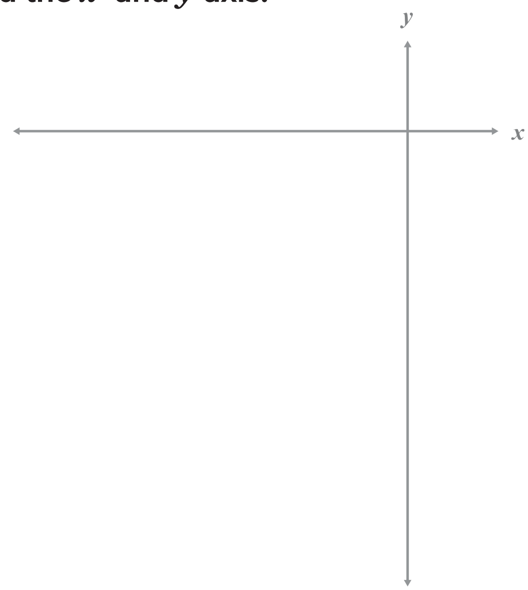
$a = \frac{50}{2} = 25$

$a^2 = 625$

Example 7

Complete the example as you watch the video in the Online Lesson.

Write the equation of an ellipse tangent to $x = -4$, $y = -5$, and the x- and y-axis.



The directions in the last example do not require you to graph. However, sketching the graph is helpful for determining the information needed to write the equation.

Checkpoint: Equations of Ellipses

Determine the equation of an ellipse with endpoints $(-7, 5)$, $(13, 5)$, $(3, 17)$, $(3, -7)$. Name the domain and range in interval notation.

To continue, return to the Online Lesson.

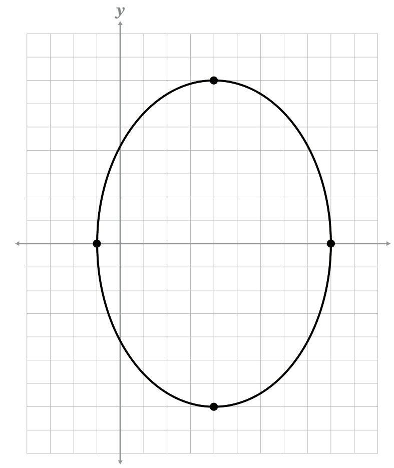
Practice 1

Complete problems on a separate sheet of paper.

Write the equation of an ellipse in standard form that satisfies each set of conditions.

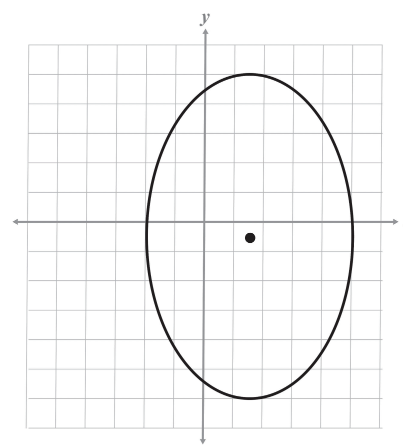
1) The center of an ellipse is $(-6.3, 19.2)$ and has a major horizontal radius of $a = \sqrt{61}$ units and a minor axis length of $b = 8$ units.

2) Write the equation of the ellipse from the graph and label the center, vertices, and co-vertices.

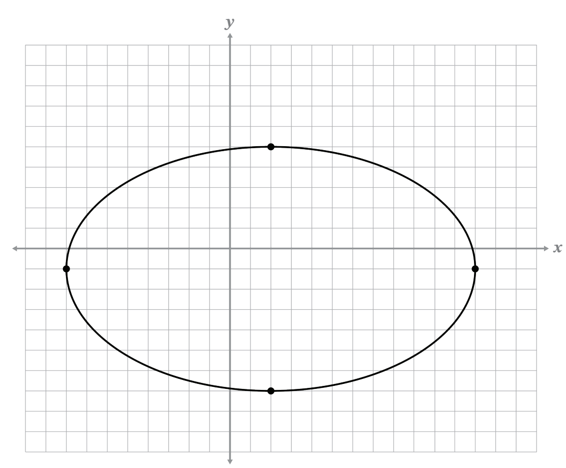


3) Find the equation of the ellipse with vertices: $(-10, 5), (0, 5), (-5, 16), (-5, -6)$

4) Write the equation of the ellipse tangent to: $x = 5, x = -2,$ $y = 5, y = -6$.



5) Write the equation of the ellipse when the given graph is translated 3 right and 1 down.



6) Write the equation of the ellipse when it is translated 2 to the left and 6 down and the major axis and minor axis are doubled.

$x^2 + \frac{y^2}{4} = 1$

7) The National Statuary Hall in The U.S. Capitol building is an elliptical room measuring 46 feet by 97 feet. With a center at the origin, write the equation to represent the footprint of the room.

8) A new arena is being built for music and sports performances. The footprint of the building will be elliptical with the length of the axes 288 meters and 432 meters. Write the equation of a model of the elliptical arena centered around the origin that will be 144th the size of the original.

Write the equation in standard form.

9) $4x^2 - 24x + 16y^2 = 28$

10) $x^2 + 9y^2 + 4x - 18y = 23$

11) Explain the relationship between a circle and an ellipse.

12) What points on an ellipse determine the domain and range?

Graph.

13) Graph: $\frac{(x-2)^2}{9} + \frac{(y+2)^2}{4} = 1$

Label the center, vertices, and co-vertices.

14) Graph: $\frac{x^2}{36} + \frac{y^2}{64} = 1$

Label the center, vertices, and co-vertices.

15) The major vertical axis of an ellipse is 13 units with co-vertices (2, 7) and (7, 7). Determine the equation, then graph.

16) Graph the ellipse with the center (3, 2), with a major horizontal axis length of 24 units, and a minor axis length of 18 units. Label the center, vertices, and co-vertices.

17) Graph $\frac{(x-2.5)^2}{49} + \frac{(y-3.5)}{9} = 1$ if it was translated 4 units to the left and 3 units down. Label the center, vertices, and co-vertices.

18) Graph: $\frac{x^2}{12} + \frac{y^2}{30} = 1$

Label the center, vertices, and co-vertices.

To continue, return to the Online Lesson.

Mastery Check

Show What You Know

Using the values in the given set *only once*, write the equation of an ellipse that meets the following requirements:

- A vertical major axis
- A center in Quadrant 3
- $h < k$
- The values of a^2 and b^2 are perfect squares

$$\{-3, -2, -1, 0, 1, 2, 3, 4, 5, 6\}$$

$$\frac{(x - \square)^2}{\square\square} + \frac{(y - \square)^2}{\square\square} = 1$$

Say What You Know

In your own words, talk about what you have learned using the objectives for this lesson and your work on this page.

To continue, return to the Online Lesson.

Practice 2

Complete problems on a separate sheet of paper.

1) Write the equation of the ellipse with a center $(0, 0)$, a horizontal major axis length of 5, and a minor axis length of 3.

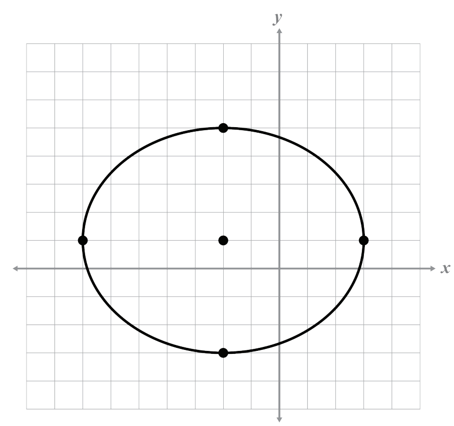
2) Write the equation of the ellipse with a center $(-23.12, 98.54)$, when $a = 4\sqrt{5}$ and $b = 2\sqrt{3}$.

3) Write the equation of the ellipse with vertices $(6, -4)$ and $(-10, -4)$, and co-vertices $(-2, -1)$ and $(-2, -7)$.

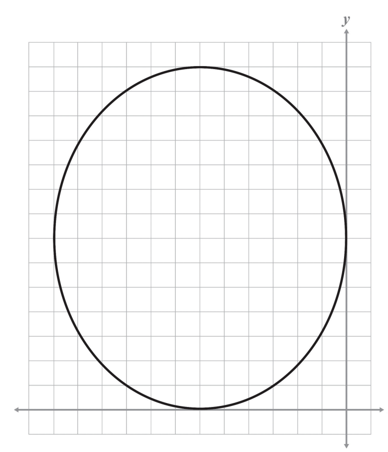
4) Write the equation of the ellipse when it is translated 8 units to the left, 4 units up, and the major axis is increased by 2.

$$\frac{x^2}{9} + \frac{(y-3)^2}{64} = 1$$

5) Write the equation that translates the given graph 3 units right, 2 units down, and increases the minor *radius* by 3.



6) Write the equation of the ellipse tangent to:
$x = -12, x = 0, y = 14, y = 0$



7) A domed building with an elliptical room measures 23 feet wide and 97 feet long. Write the equation of the footprint of the room centered at the origin.

8) The blueprint for an elliptical stadium shows the horizontal major axis with a length of 4 units and the minor axis length of 2 units. The building will be 20 times the scale of the blueprint. Write the equation of the constructed stadium centered at the origin.

Write the equation in standard form.

9) $100x^2 - 200x + 225y^2 - 450y = 22{,}175$

10) $3x^2 + 12x + 8y^2 - 48y = 12$

11) Explain how to determine if the ellipse is horizontal or vertical from an equation.

12) Explain how to determine the length of the major and minor axis given the equation of the ellipse.

Graph. Label the center, vertices, and co-vertices.

13) $\frac{(x-2)^2}{32} + \frac{(y-3)^2}{12} = 1$

14) $\frac{x^2}{12.25} + \frac{(y-4)^2}{16} = 1$

15) $\frac{(x+5)^2}{25} + \frac{(y-2)^2}{25} = 1$

16) $\frac{x^2}{64} + \frac{y^2}{81} = 1$

17) $\frac{(x+4)^2}{10} + \frac{y^2}{24} = 1$

18) The center of an ellipse is translated 5 units right and 6 units down from $(8, 4)$. The horizontal major axis is 17 units and the minor axis is 11 units.

To continue, return to the Online Lesson.

Targeted Review

Complete problems on a separate sheet of paper.

Write the equation in standard form.

1) Write the equation $x = 2y^2 - 2y + 2$ in the form $x = a(y-k)^2 + h$.

2) Graph problem 1.

3) Write the equation $x^2 + y^2 + 4x + y - 4\frac{3}{4} = 0$ in the form $(x-h)^2 + (y-k)^2 = r^2$.

4) Graph problem 3.

5) Find the distance between the vertices in Quadrant 2 and Quadrant 4 from the graph in problem 3.

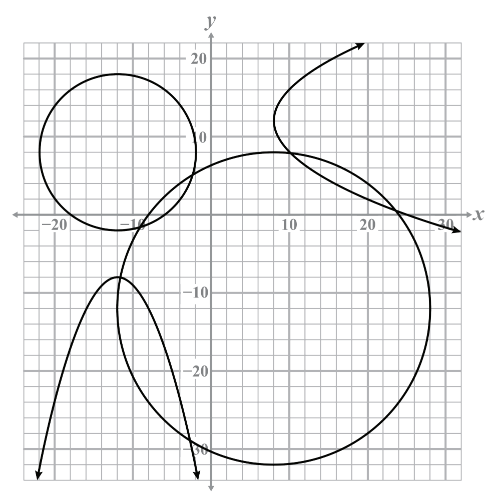
6) Complete the square over the set of complete numbers. $x^2 = 22 - 6x$

7) Solve $y = 2x^2 - 2x + 2$ using the quadratic formula.

8) Solve $3x^3 - 15x - 20 = 4x^2$ by factoring under the set of complex numbers.

Multiple Choice

Use the graph to answer problems 9–10.



_____ **9)** Determine the equation of the circle with a center in the fourth quadrant.

A) $(x-8)^2 + (y+12)^2 = 20^2$

B) $(x+8)^2 + (y-12)^2 = 20^2$

C) $(x-8)^2 + (y+12)^2 = 10^2$

D) $(x+8)^2 + (y-12)^2 = 10^2$

_____ **10)** Write the equation of the parabola reflected over the x-axis compared to the parent graph $y = x^2$.

A) $x = -\frac{1}{8}(y-12)^2 + 8$

B) $y = -\frac{1}{4}(x+12)^2 - 8$

C) $x = \frac{1}{8}(y-12)^2 + 8$

D) $y = \frac{1}{4}(x+12)^2 - 8$

_____ **11)** Determine a possible polynomial equation with rational coefficients that represents the roots $2i,\ \pm\sqrt{6}$.

A) $x^3 - 2ix^2 - 6x + 12i$

B) $x^4 - 2x^2 - 24$

C) $x^4 - 10x^2 + 24$

D) $x^4 - 2x^2 + 24$

12) The process of completing the square can be used to write the equations for a _____ in standard form.

☐ circle

☐ ellipse

☐ parabola

☐ rational

Problem	1	2	3	4	5	6	7	8	9	10	11	12
Origin	L27	L27	L28	L28	L26	L24	L25	L23	L28	L26	L23	L28

L = Lesson in this level, A1 = Algebra 1: Principles of Secondary Mathematics

To continue, return to the Online Lesson.