Lesson 28
Circles

NAME:

Start by navigating to the Online Lesson for instructions.

Objectives

- Write the equation of a circle with the given information (graph, points).
- Graph a circle from the given information.
- Transform circles.

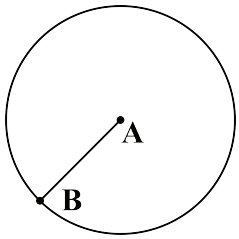
Why?

As you continue to expand your knowledge and dive deeper into quadratic equations and conic sections, this lesson will teach you how to put all of these things together to solve problems involving circles.

Warm Up

Write the word that best describes the image using center, diameter, or radius.

1)



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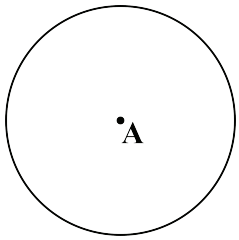
2)



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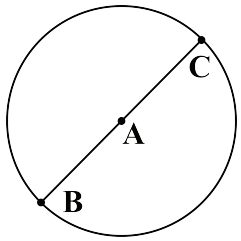
3)



_____

4) Calculate the midpoint M for $G\,(-0.55, 0)$ and $H\,(7.05, 2.5)$.

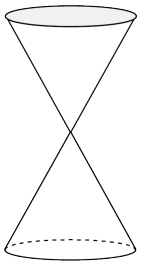
5) Use the distance formula to calculate the length of segment GH and segment GM from problem 4.

To continue, return to the Online Lesson.

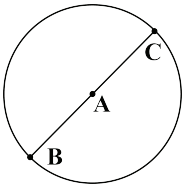
Explore

Circles

Fill in the notes as you watch the video in the Online Lesson.


Circle

- A circle is a conic section that is called a horizontal _____.
- A circle is the set of all points in a plane that are _____ from a fixed point, the center.
- A circle is named using a _____ at its center.
- The _____ is the fixed distance at the center to any point on the circle.



- The _____ of a circle is the line segment (chord) that passes through the center of the circle (point A) and whose endpoints (points B and C) are on the circle.
- The diameter is _____ of the radius.
- The _____ of the equation of a circle, which has a center (h, k) and a radius r, is $(x-h)^2+(y-k)^2=r^2$.
- Points that form the diameter of a circle can be determined by _____ _____ the value of the radius to the center.
- The endpoints of the _____ diameter are $(h+r, k)$ and $(h-r, k)$ and determine the _____ of the circle.
- The endpoints of the _____ diameter are $(h, k+r)$ and $(h, k-r)$ and determine the _____ of the circle.

Because you can determine h, k, and the radius r from the equation, you do not need to write them down unless specifically directed. However, it may be helpful, especially when working with negative values of h and k.

Example 1

Complete the example as you watch the video in the Online Lesson.

Write the equation of circles A, B, and C in standard form.

Plan

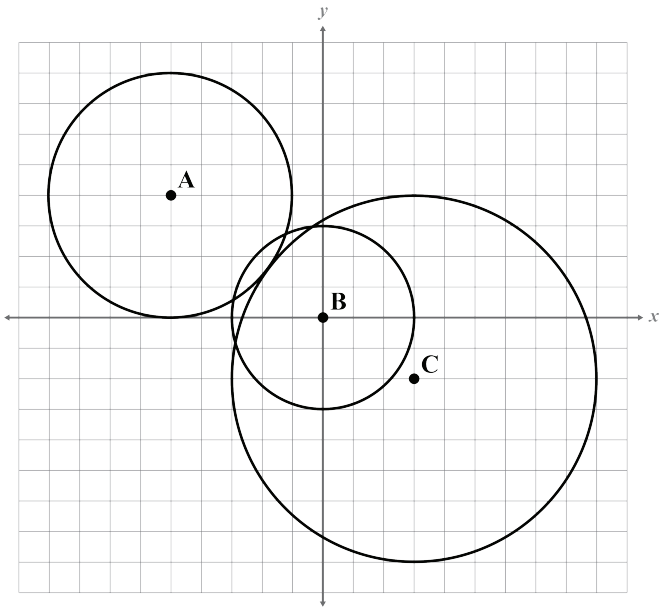
Identify the center and radius
Write the equation

Implement

Circle A
$(-5, 4)$, $r = 4$

Circle B

Circle C



Example 2

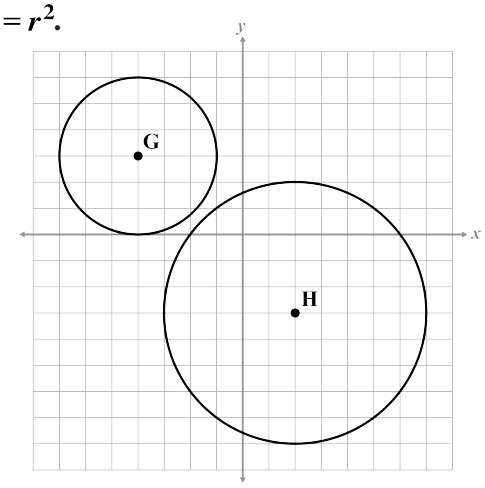
Complete the example as you watch the video in the Online Lesson.

Write the equation of each circle in the form $(x-h)^2+(y-k)^2=r^2$ with a radius of 1.5 units and a center of (7.9, –24).

Remember not all values for the center and radius of a circle are integers.

Checkpoint: Circles

Write the equation of each circle in the form $(x-h)^2+(y-k)^2=r^2$.



To continue, return to the Online Lesson.

Equations of Circles

Fill in the notes as you watch the video in the Online Lesson.

- Methods for finding the **radius** of a circle:
  - On the coordinate plane, _____ its length.
  - Calculate using the _____, if you have the _____ of a circle and a _____ on the circle.
- Methods for finding the **center** of a circle:
  - On the coordinate plane, find the _____ of either the horizontal or vertical diameter.
  - Calculate using the _____ if you have the endpoints of the _____.

Formulas can be found on the Formula Sheet.

For some circles, the center and length of the radius can be determined without using a formula. However, to prove this mathematically, the work will be shown using the midpoint and distance formulas.

Example 3

Complete the example as you watch the video in the Online Lesson.

Determine the equation of the circle with center $C\ (0.8, -1.6)$ and $P\ (1.8, -0.62)$ on the circle. Round the length of the radius to the nearest tenth.

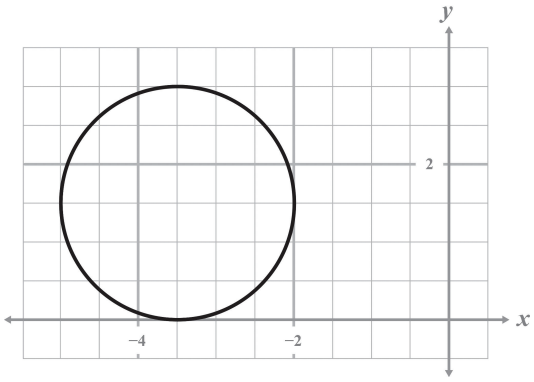
Example 4

Complete the example as you watch the video in the Online Lesson.

Determine the equation of the circle with the endpoints of the diameter being $(-5, 1.5)$ and $(-2, 1.5)$. Name the domain and range of the circle in interval notation.

Plan

Calculate the midpoint (center)
Calculate the distance of the radius
Write the equation



Checkpoint: Equations of Circles

Determine the equation of the circle with endpoints of the diameter being (–5, –7) and (–5, –11).

To continue, return to the Online Lesson.

Graphing Circles on the Coordinate Plane

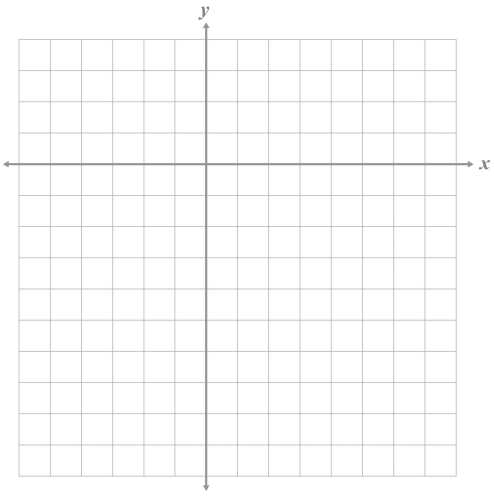
Fill in the notes as you watch the video in the Online Lesson.

- To sketch a circle on a coordinate plane:
  - Graph the _____ of the circle.
  - Use the _____ to determine the horizontal and vertical points of the circle.
  - Sketch a _____ between the points.
- The _____ of the equation of a circle is: $x^2 + y^2 + Dx + Ey + F = 0$
- For this lesson, write all circles in standard form so the values of _____ are known.
- Convert the equation of a circle from general to _____ form by completing the square with _____.

Example 5

Complete the example as you watch the video in the Online Lesson.

Graph and write the equation of a circle with a domain of $\{x \mid x \in \mathbb{R},\ -3 \le x \le 5\}$ and a range of $\{y \mid y \in \mathbb{R},\ -7 \le y \le 1\}$.



Plan

Mark the domain restrictions
Determine the center and radius
Graph the circle
Write the equation

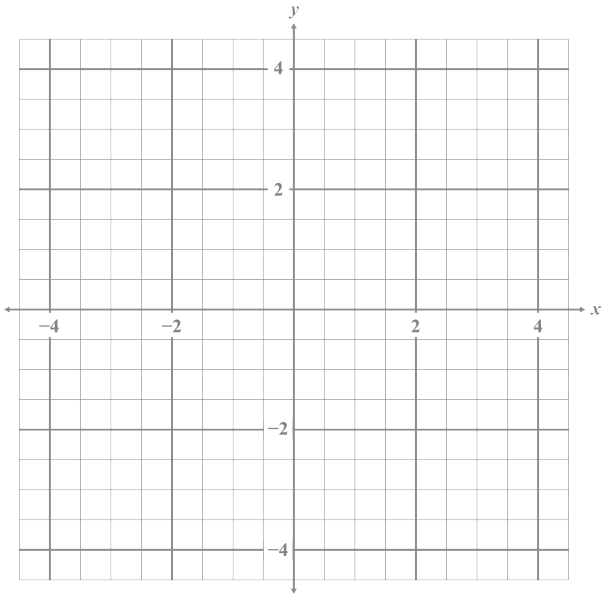
Example 6

Complete the example as you watch the video in the Online Lesson.

Graph: $3x^2 + 3y^2 + 9x - 6y - \frac{9}{4} = 0$

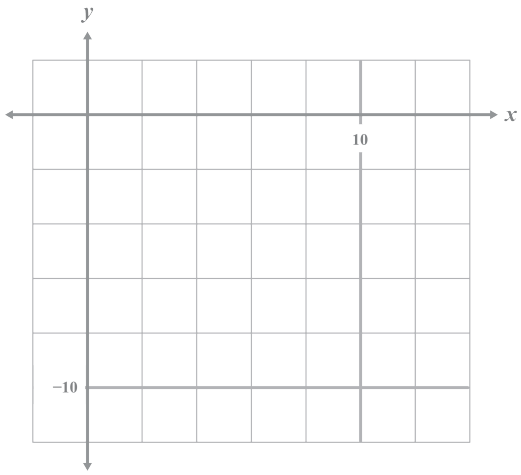
Plan

Write the equation in standard form
(Complete the square for x, then for y)
Name the center and radius
Graph



☑ Checkpoint: Graphing Circles on the Coordinate Plane

Write and graph the equation of a circle: $x^2 + y^2 - 20x + 10y = -116$

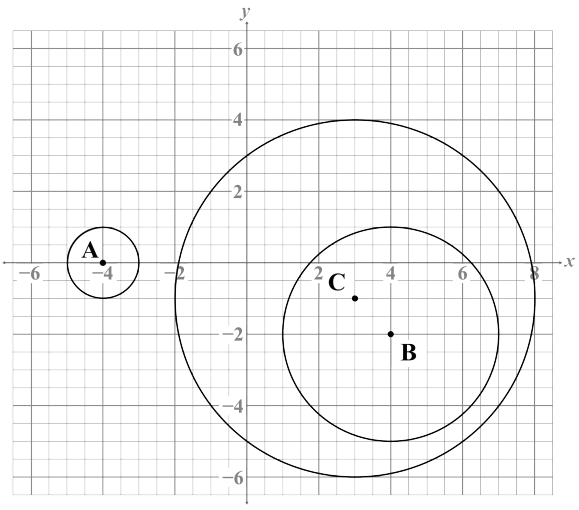


To continue, return to the Online Lesson.

Practice 1

Complete problems on a separate sheet of paper.

For problems 1–3, use the given graph.



1) Find the equation of circle A.

2) Find the equation of circle B.

3) Find the equation of circle C.

4) Write the equation of the circle in the form $(x-h)^2+(y-k)^2=r^2$ with a center of $(5.43, -3.1)$ and a radius of 2.3 units.

5) Write the equation of a circle with center $\left(-2\sqrt{5}, \sqrt{6}\right)$ and radius of $\sqrt{13}$ units.

6) Name the domain and range of the circle in set builder notation given the equation: $(x+12)^2+(y-14)^2=100$

7) Name the domain and range of the circle in set builder notation given the equation: $(x-4.2)^2+(y-8.5)^2=1.21$

8) A circle has a center at $(-4, 6)$ and an endpoint at $(1, -6)$. Find the equation of the circle and graph.

9) Determine the equation of the circle when the endpoints of the diameter are $(-5, 2)$ and $(7, 6)$. Graph.

10) Graph and find the equation of the circle with the domain of $\{x \mid x \in \mathbb{R},\ -3 \le x \le 7\}$ and the range of $\{y \mid y \in \mathbb{R},\ -7 \le x \le 3\}$.

11) Graph and find the equation of the circle with the domain of $\{x \mid x \in \mathbb{R},\ 4 \le x \le 9\}$ and the range is $\{y \mid y \in \mathbb{R},\ -1 \le y \le 4\}$.

12) Write the equation of a circle in standard form: $2x^2 - 4x + 2y^2 + 8y = 8$

13) Write the equation of the circle in standard form: $x^2 + y^2 - 11x - 12y = -3$

14) The current transmission tower is centered at $(-6, 3)$ and has a range of 4 miles. The transmission company wants to build a new tower at $(1, 5)$ to increase the signal to include the nearby town. Determine how much further the new transmission tower will reach.

15) Graph: $x^2 + (y - 2)^2 = 25$

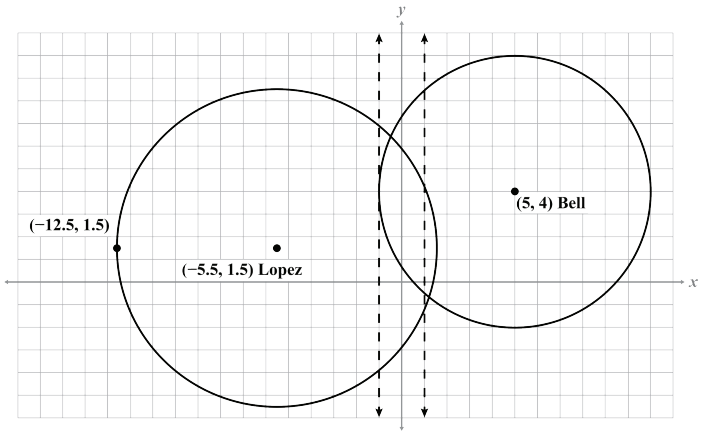
16) Graph: $(x - 4)^2 + (y - 6)^2 = 1$

To continue, return to the Online Lesson.

Mastery Check

Show What You Know

The Lopez and Bell families are next door neighbors and both have dogs. They have electric fences that need to be recalibrated so they are one foot behind the property line (y-axis). The center of each circle is the location of the electric fence beacon and cannot be changed.



A) Write the equations of the circles to represent the current range of the electric fences for both families.

B) Write and graph the new equation for the Bell family that will stay within the property.

C) The Lopez family has the new equation in general form. Write the equation in standard form and graph. $x^2 + y^2 + 11x - 3y + 12.25 = 0$

Say What You Know

In your own words, talk about what you have learned using the objectives for this lesson and your work on this page.

To continue, return to the Online Lesson.

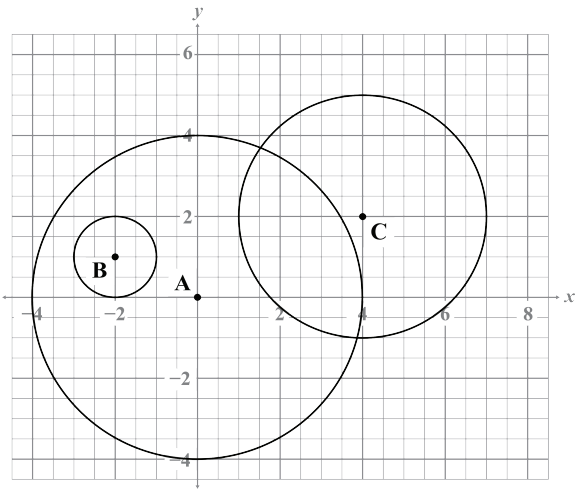
Practice 2

Complete problems on a separate sheet of paper.

1) Find the equation of circle A.

2) Find the equation of circle B.

3) Find the equation of circle C.



4) Write the equation of the circle in the form $(x-h)^2+(y-k)^2=r^2$ with a center of $(10.45, -23.4)$ and a radius of 12.3 units

5) Write the equation of the circle in the form $(x-h)^2+(y-k)^2=r^2$ with a center of $\left(-\sqrt{2}, 4\sqrt{3}\right)$ and a radius of $\sqrt{10}$.

6) Name the domain and range of the circle given the equation: $(x+2)^2+(y-4)^2=196$

7) Name the domain and range of the circle given the equation: $(x-1.32)^2+(y-4.13)^2=5.76$

8) Determine the equation of the circle when the endpoints of the diameter are $(4, 5)$ and $(-9, 3)$.

9) Determine the equation of the circle with the center at $(-5, 2)$ and an endpoint $(2, 1)$.

10) Find the equation of the circle with the domain $\{x \mid x \in \mathbb{R},\ 2 \le x \le 8\}$ and the range $\{y \mid y \in \mathbb{R},\ -2 \le y \le 4\}$.

11) Find the equation of the circle with the domain $\{x \mid x \in \mathbb{R},\ -2 \le x \le 6\}$ and the range $\{y \mid y \in \mathbb{R},\ -4.5 \le y \le 3.5\}$.

12) Write the equation of the circle in standard form: $\frac{2}{3}x^2+3x+\frac{2}{3}y^2-7y=1$

13) Write the equation of a circle in standard form: $5x^2 - 20x + 5y^2 + 10x = 2$

14) A food delivery company centered at $(7, 5)$ will deliver within a radius of 6 units. Determine if a house located at $(2, 6)$ is within that radius.

15) Graph: $(x - 1)^2 + (y - 3)^2 = 4$

16) Graph: $(x + 5)^2 + y^2 = 9$

To continue, return to the Online Lesson.