Lesson 27
Parabolas

NAME:

Start by navigating to the Online Lesson for instructions.

Objectives

- Write the equation of a parabola by completing the square given an equation or a graph.
- Determine the equation of a parabola given three points (systems of equations).
- Graph the equations of parabolas with a vertical or horizontal axis of symmetry.
- Transform parabolas.

Why?

In Algebra 1, you learned foundational knowledge about parabolas. In Algebra 2, you have added knowledge of quadratic equations, systems of equations, graphing, and parent graphs. Now, it is time to expand your knowledge and dive deeper into quadratic equations and conic sections. This lesson will teach you how to put all of these things together to solve problems involving parabolas.

Warm Up

1) Solve the quadratic equation by completing the square.

$6x^2 + 3x + 6 = 0$

2) Solve the system of equations.

$x - z = -2$

$2x - y + z = 29$

$x + y + 2z = 67$

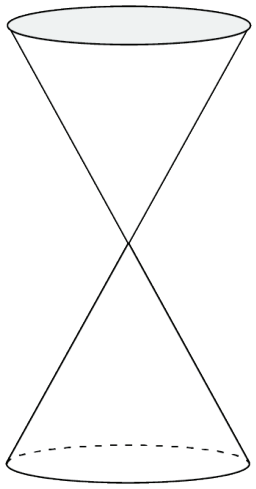
To continue, return to the Online Lesson.

Explore

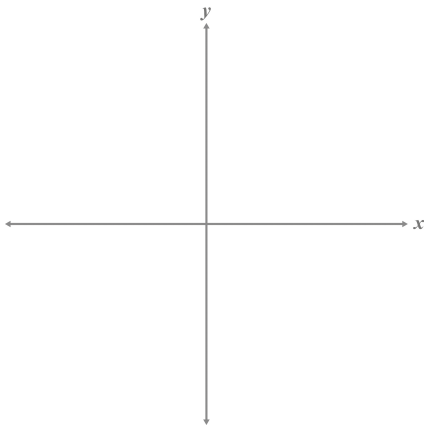
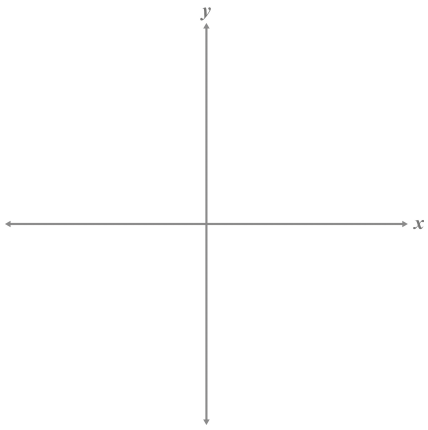
Parabolas

Fill in the notes as you watch the video in the Online Lesson.

- A parabola is a type of _____ that only slices through one cone.
- The _____ equation, either $y = x^2$ or $x = y^2$, determines many things, primarily the _____ of the opening of the parabola.
- You need to identify the direction in which the parabola opens (up/down, right/left) _____ graphing or writing equations in vertex form.
- The variable that is raised to the _____ power will determine the direction of the parabola and whether the equation in standard form will start with x or y.



Parabola

Parabola Parent Graph Form	$y = x^2$	$x = y^2$
Sketch		
Vertex Form		
Direction of Opening	_____ when $a > 0$ _____ when $a < 0$	_____ when $a > 0$ _____ when $a < 0$
Vertex	(h, k)	(h, k)
Axis of Symmetry		
VLT	_____, this parabola _____ a function.	_____, this parabola _____ a function.
HLT	_____, the inverse _____ a function.	_____, the inverse _____ a function.

Example 1

Complete the example as you watch the video in the Online Lesson.

Graph.

$x = (y-3)^2 - 4$

Plan

Steps to graph parabolas in vertex form

1) Identify a, h, k

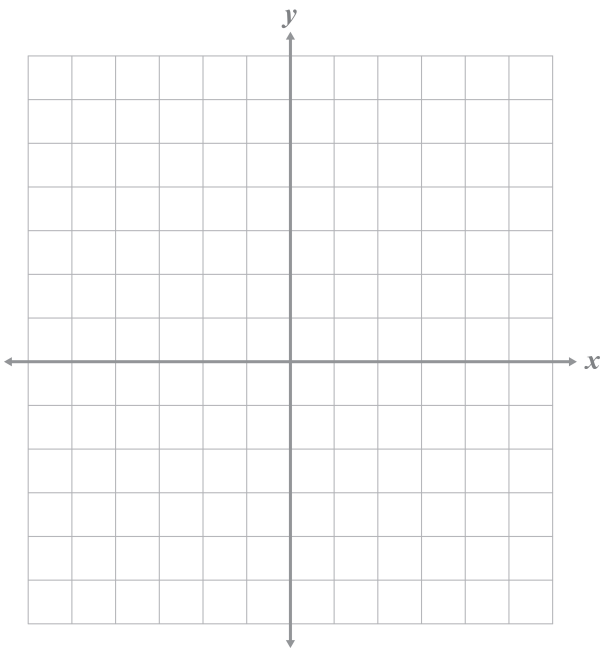
2) Determine the direction of the opening

3) Plot the vertex (h, k) and axis of symmetry (AoS is optional)

4) Plot symmetric points using the value of a (or use a table of values)

Implement

x	y
	0
	1
	2
	3
	4
	5
	6

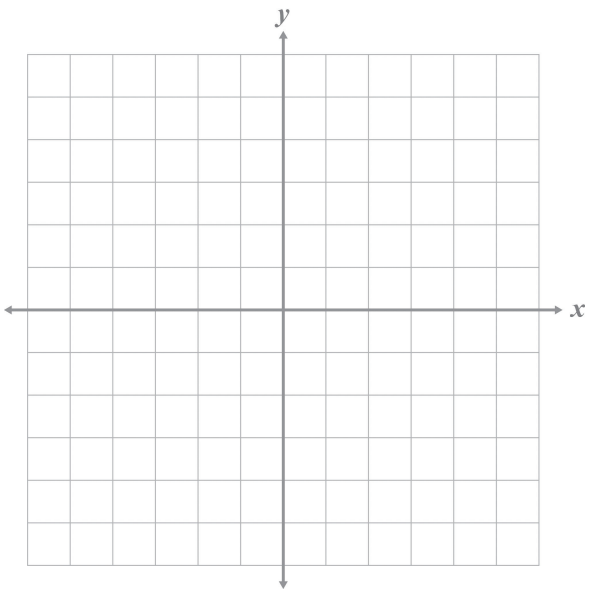


☑ Checkpoint: Parabolas

A) How can you determine the direction of the parabola from a given equation? Explain.

B) Graph.

$x = -2(y+1)^2$



To continue, return to the Online Lesson.

Writing Quadratic Equations in Vertex Form to Graph

Fill in the notes as you watch the video in the Online Lesson.

- It is more efficient to graph quadratic equations in _____ form than in standard form.
- For quadratic equations that are not in vertex form, use the _____ method to rewrite the equation in vertex form.

Quadratic Equation	Rewrite Using Completing the Square
$y = ax^2 + bx + c$ or $x = ay^2 + by + c$	**1)** Write the equation in standard form
$ax^2 + bx + c = 0$ or _____	**2)** Set the equation equal to zero
$y = a(x-h)^2 + k$ or _____	**3)** Complete the square to write in vertex form

- _____ from the expression rather than dividing all terms by the leading coefficient a.
- You need to know the value of a to determine if the graph is _____ and/or _____ and the _____ the parabola opens.
- To maintain equality, you must do the same thing to _____ of the equation.

Example 2

Complete the example as you watch the video in the Online Lesson.

Write the quadratic equation in vertex form. Name the vertex and the axis of symmetry. Then graph.

$x - 19 + 12y = 2y^2$

Plan

Isolate x because y is the squared term
Complete the square
Write equation in vertex form
Identify vertex, axis of symmetry
Graph

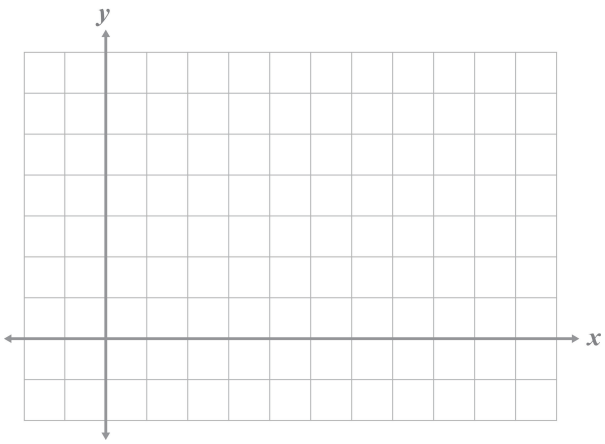
Implement

$$x = 2y^2 - 12y + 19$$

$$2y^2 - 12y + 19 = 0$$

$$2(y^2 - 6y) = -19$$

$$2(y^2 - 6y + \square) = -19 + 2(\square)$$



Explain

- Isolate x, then set equation equal to zero
- Complete the square
- Recall you must do the *same thing* to both sides of the equation

The vertex of the parabola is _____.

The axis of symmetry is _____.

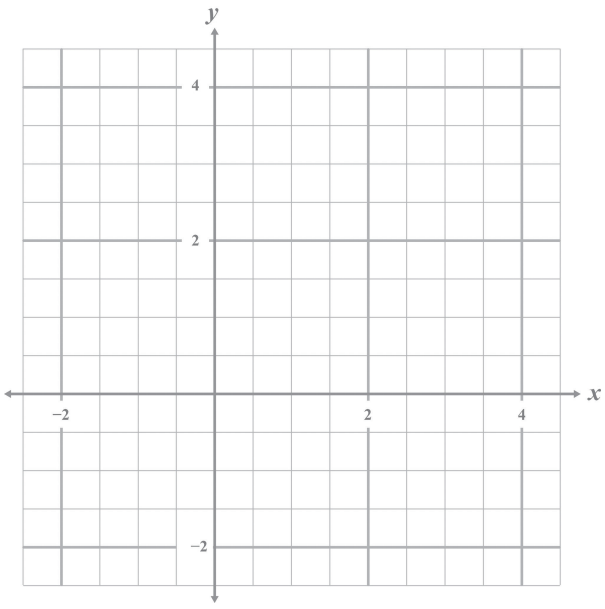
Example 3

Complete the example as you watch the video in the Online Lesson.

Write the quadratic equation in vertex form. Name the vertex and the axis of symmetry.

$y + 3x = x^2 + 2$

$y = x^2 - 3x + 2$

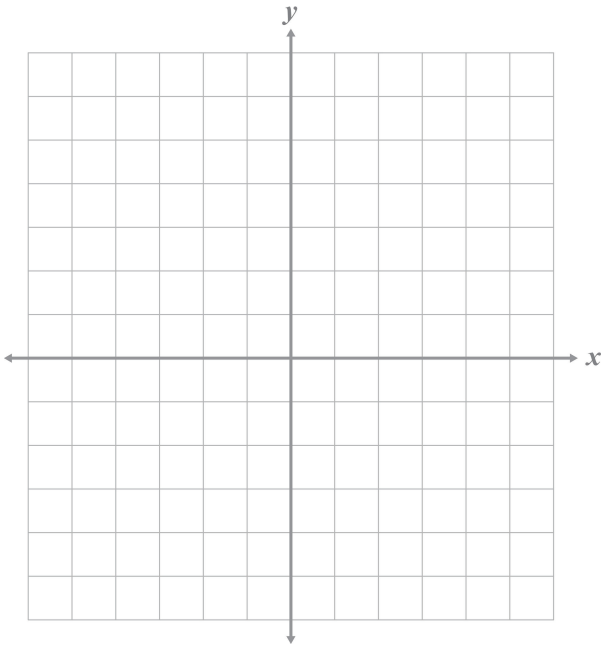


Example 4

Complete the example as you watch the video in the Online Lesson.

Graph.

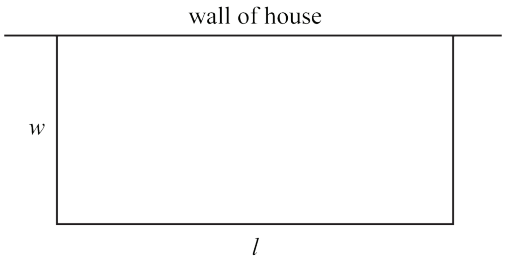
$0.5y^2 + x = -3y - 0.5$



Example 5

Complete the example as you watch the video in the Online Lesson.

The Dowell family has a new puppy and is fencing in a portion of their yard. They want to maximize the area by using all of the 100 feet of fencing they purchased. The fence will be used on three sides, and the fourth side will be the wall of the house. What are the dimensions and area of the fenced in space?



Plan

Identify key information and formulas
Determine the vertex

Four sides: $P = 2l + 2w$
Three sides: $P = l + 2w$

Implement

Explain

☑ Checkpoint: Writing Quadratic Equations in Vertex Form to Graph

Write the quadratic equation in vertex form. Name the vertex and axis of symmetry.

$x = 4y^2 - 8y - 5$

To continue, return to the Online Lesson.

Writing Quadratic Functions Given Three Points

Fill in the notes as you watch the video in the Online Lesson.

- You can write the equation for a quadratic function if you have _____ on a parabola.
- However, because you do not know if any of the points is the _____, you cannot write the equation in vertex form.
- Instead, you can use a method that combines:
  - solving a _____ with three variables and
  - working with the _____ of a quadratic equation, using either $y = ax^2 + bx + c$ or $x = ay^2 + by + c$.
- To confirm that your equation is correct, you can _____ each point into the equation, or utilize technology to graph the equation and confirm that all three points are on the parabola.

You need to eliminate at least one variable in two out of the three equations to begin the process of solving. Then solve for one of the remaining variables and use repeated substitution to find all of the variables and check your work.

Example 6

Complete the example as you watch the video in the Online Lesson.

Write the equation of a parabola in $x = ay^2 + by + c$ form using the points D (14, 1), E (7, –2.5), F (10, 0.5).

Implement

$D\,(14,\,1)$

$a(1)^2 + b(1) + c = 14$

$D\colon\ a + b + c = 14$

Implement

$x = ay^2 + by + c \ \Leftrightarrow\ ay^2 + by + c = x$

Explain

- Eliminate c from the first two equations $(D + -E)$
- Eliminate c from the last two equations $(E + -F)$
- Eliminate b, solve for a
- Solve for b, solve for c
- Write the equation

You can use the More to Explore activity to check your solution using technology.

Example 7

Complete the example as you watch the video in the Online Lesson.

Write the equation of the parabola in $y = ax^2 + bx + c$ form using the points $(1, 5.5)$, $(-2, 19)$, $(3, 1.5)$.

Checkpoint: Writing Quadratic Functions Given Three Points

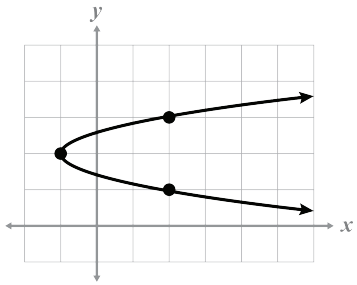
Verify the equation in Example 7 is correct using the given points. Explain.

To continue, return to the Online Lesson.

Practice 1

Complete problems on a separate sheet of paper.

1) Given the graph, determine a, h, and k. Then write the equation.



2) Write a verbal description of the parabola given the equation: $y = -\frac{2}{5}(x+5)^2 + 3$

Write the equation of the parabola in vertex form. Name the axis of symmetry and vertex.

3) $x = y^2 + 8y - 1$

4) $3x^2 + y = 6x - 2$

5) $y = \frac{1}{3}x^2 + 2x - 9$

6) $x = 2y^2 + 6y - 7$

Write the equation of the parabola in vertex form. Then graph.

7) $2x^2 - y + 4 = 8x$

8) $x = y^2 + 10y - 5$

9) $x + y^2 = 2y$

10) $3x^2 + y = 12x$

Write the equation of the parabola given three points.

11) $(4, 0), (2, 2), (-4, -1)$ when $x = ay^2 + by + c$.

12) $(-2, 10), (2, -2), (3, 5)$ when $y = ax^2 + bx + c$.

13) $(-1, 17), (1, -3), (3, 25)$ when $y = ax^2 + bx + c$.

14) $(-6, -4), (-10, -2), (15, 3)$ when $x = ay^2 + by + c$.

15) Describe how you know a parabola will open left/right rather than up/down.

16) Natalee sets a volleyball to her hitter. The hitter wants to time her approach to hit the ball at its maximum height. The ball's route can be modeled by the equation: $h(t) = -16t^2 + 16t + 7$. What is the maximum height the ball will reach and how long after the ball is set will it reach that height?

17) The perimeter of a rectangular fence is 40 meters. What are the dimensions and the maximum area?

18) The Hawkins family purchased 180 feet of fence to enclose a rectangular area of their yard. They plan on using an existing wall for one side of the rectangular enclosure. Find the dimensions and maximum area of the enclosure.

To continue, return to the Online Lesson.

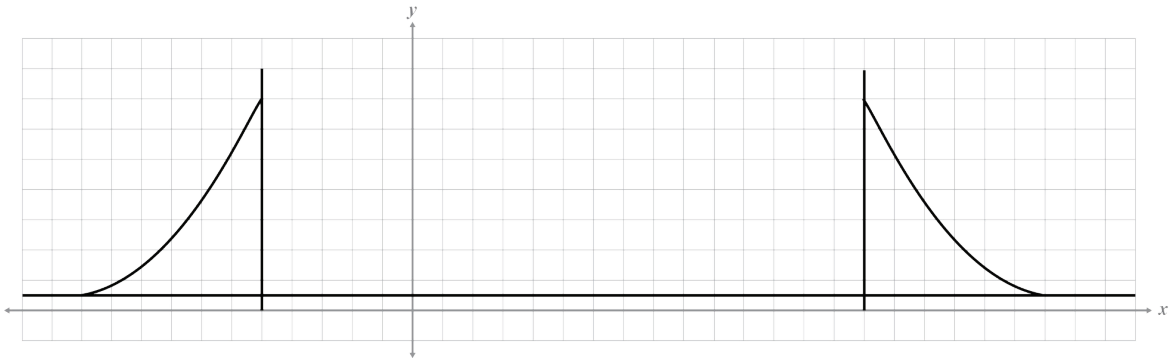
Mastery Check

Show What You Know

A) A scale model of a self-anchored suspension bridge was created on a coordinate plane so that the equation of the main eyebar chain could be determined. Determine the parabolic equation for the eyebar chain from the three points $(0, 2.5)$, $(-5, 7)$, $(12, 3.94)$.

B) Rewrite the eyebar chain equation from part A in vertex form.

C) Graph the eyebar chain using the information from parts A and B.



Say What You Know

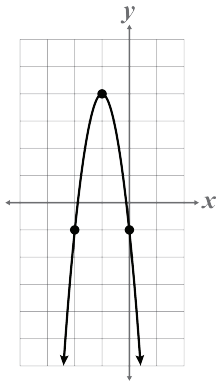
In your own words, talk about what you have learned using the objectives for this lesson and your work on this page.

To continue, return to the Online Lesson.

Practice 2

Complete problems on a separate sheet of paper.

1) Given the graph, determine a, h, and k. Then write the equation.



2) Write a verbal description of the parabola given the equation: $x = \frac{1}{2}(y-7)^2 - 2$

Write the equation of the parabola in vertex form. Then graph.

3) $y - 4x^2 = 4x + 3$

4) $x = -2y^2 + 4y - 1$

5) $x^2 - y + 5x = 1$

6) $y^2 - \frac{1}{2}x + 2 = 6y$

7) $y = \frac{1}{2}x^2 + x - 1$

8) $3y^2 - x + 6y = 2$

9) $x = -\frac{1}{3}y^2 - 2y$

10) $5x^2 + y = -5x$

Write the equation of the parabola given three points.

11) $(1, 9.5), (0, 8), (4, 20)$ when $y = ax^2 + bx + c$.

12) $(-2, 0), (5, -2), (-7, 10)$ when $x = ay^2 + by + c$.

13) $(3, -30), (1, 4), (-1, -2)$ when $y = ax^2 + bx + c$.

14) $(-18, -3), (-1, -1), (-15, 3)$ when $x = ay^2 + by + c$.

15) Jacob was holding a ball 2.5 feet above the ground and tossing it into the air. The height of the ball relative to the ground as a function of time could be represented by the equation: $h(t) = -16t^2 + 32t + 2.5$ where t is time in seconds from when the ball leaves his hands and h is the height in feet the ball is above the ground. Find the maximum height of the ball and the time it occurred.

16) The Donovans were building a rectangular chicken yard. They have 150 feet of fence. Find the dimensions of the yard and the maximum area.

To continue, return to the Online Lesson.

Targeted Review

Complete items on a separate sheet of paper.

1) Solve: $-(x+8)^2 = 24$

2) Solve: $7x^3 = -56x$

3) Write the polynomial equation with integer coefficients given the roots: $x = \sqrt{3},\ \pm 2i$

4) Calculate the discriminant for the equation $y = -5x^2 - x - 0.2$. Explain what this tells you about the roots.

Problems 5–7 work together.

5) Graph.
$y = -2(x-1)^2 + 1$
$y = 2x - 5$

6) Using your graph from problem 5, calculate the midpoint of the line segment inside the parabola using the points of intersection. Use the intersection points to calculate the midpoint of the line segment inside the parabola.

7) Using your work for problems 5 and 6, calculate the distance between the vertex and midpoint of the line segment. Write the answer in radical form.

8) Solve $8x^2 - 5x = 0$ using the quadratic formula.

Multiple Choice

_____ **9)** Determine the value of b to form a perfect square trinomial: $x^2 + bx + \frac{4}{25}$

A) $-\frac{4}{5}$ **B)** $\frac{4}{5}$

C) $\frac{2}{5}$ **D)** $\frac{4}{625}$

_____ **10)** Right triangle ABC is graphed on the coordinate plane. Determine the length of the hypotenuse, AC, when A (–7, 10), B (8, 4), and C (4, –6).

A) $3\sqrt{29}$ **B)** $2\sqrt{29}$

C) $\sqrt{377}$ **D)** 5

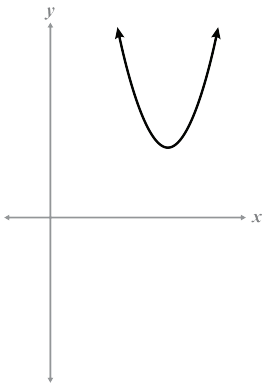
_____ **11)** Determine the polynomial equation given the roots: $3i, 5i$.

A) $x^2 + 2ix + 15 = 0$ **B)** $x^4 + 34x^2 + 225 = 0$

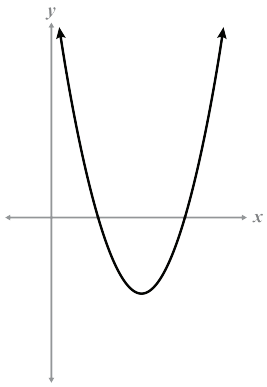
C) $x^3 + 5ix^2 + 9x + 45i = 0$ **D)** $x^4 - 34x^2 + 225 = 0$

_____ **12)** Select the graph that represents a discriminant value of zero.

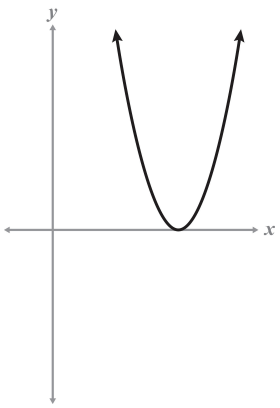
A)



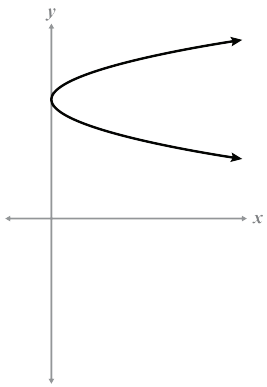
B)



C)



D)



Problem	1	2	3	4	5	6	7	8	9	10	11	12
Origin	L23	L23	L23	L25	L18	L26	L26	L25	L24	L26	L23	L25

L = Lesson in this level, A1 = Algebra 1: Principles of Secondary Mathematics

To continue, return to the Online Lesson.