

Lesson 25

The Quadratic Formula

NAME:

 Start by navigating to the Online Lesson for instructions.

Objectives

- ✓ Solve quadratic equations using the quadratic formula.
- ✓ Determine the number of real or complex solutions by solving for the discriminant.

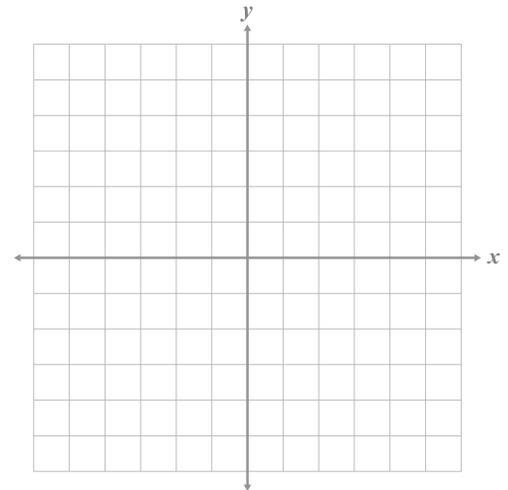
Why?

The quadratic formula can be used to solve any quadratic equation, regardless of the type of roots that result when the equation is in standard form. It can be used to find the x -intercepts (solutions, roots) that are real, imaginary, or complex.

Warm Up

1) Graph.

$$y = 4\left(x - \frac{1}{2}\right)^2 - 4$$



2) Write the equation in standard form.

3) Solve the equation in problem 2 by factoring.

 To continue, return to the Online Lesson.

 Explore The Quadratic Formula

 Fill in the notes as you watch the video in the Online Lesson.

- You already know how to solve a quadratic equation with these methods:
 - _____
 - _____
 - _____

- You can also solve quadratic equations for their roots using the _____
_____.

- The quadratic formula for a quadratic equation in _____ is written as:
$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}, \text{ where } a \neq 0$$

- The quadratic formula is _____ by taking the standard form of a quadratic equation and completing the square to solve for x .

- You can solve _____ quadratic equation in standard form using the quadratic formula.

- If using the quadratic formula to solve a quadratic equation results in a _____
_____, it means you also could have solved by factoring.

Example 1

▶ Complete the example as you watch the video in the Online Lesson.

Solve.

$$4x^2 + 12x + 9 = -5$$

Plan

Write in standard form
Identify a , b , and c
Substitute into formula

Implement

$$4x^2 - 12x + 14 = 0$$

$$a = 4, b = -12, c = 14$$

Explain

- ▶ Standard form
- ▶ Identify a , b , c
- ▶ Write the formula
- ▶ Substitute a , b , and c
- ▶ Simplify the right side

Example 2

 Complete the example as you watch the video in the Online Lesson.

Solve.

$$5x - 3 = 2x^2$$

 Checkpoint: The Quadratic Formula

Solve.

$$x^2 + 29 = 4x$$



To continue, return to the Online Lesson.

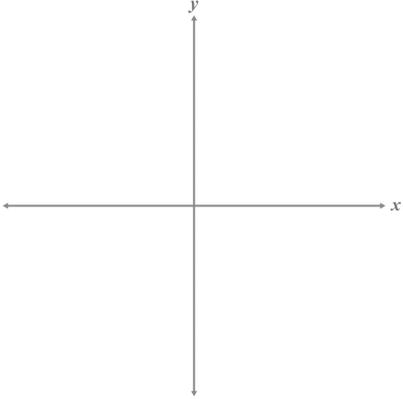
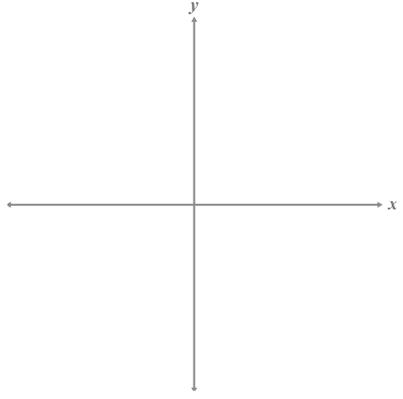
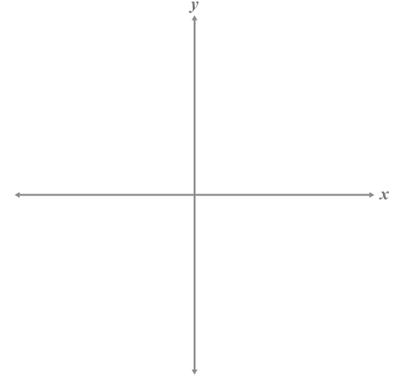
The Discriminant

 Fill in the notes as you watch the video in the Online Lesson.

- When the quadratic equation is written in standard form, $ax^2 + bx + c = 0$, the values of a , b , and c are used in the _____.
- The discriminant is _____, which is the expression in the quadratic formula that is under the square root symbol.
- The discriminant is used to determine the _____ and _____ a quadratic equation will have.
- The discriminant:
 - _____ tell you the exact values for the roots.
 - determines whether you will have 0, 1, or 2 _____ roots.
 - can help determine:
 - if the _____ of a quadratic equation intersects the _____ as you expect.

OR

- what _____ of answer to expect when using the quadratic formula.

Discriminant Value	Roots	Graph
$b^2 - 4ac > 0$ If $b^2 - 4ac > 0$ is a perfect square If $b^2 - 4ac > 0$ is not a perfect square	_____ roots, or two x -intercepts _____ roots _____ roots	
$b^2 - 4ac = 0$	_____ root, or one x -intercept Also referred to as a _____ root because the equation in factored form will be $(x - h)^2$	
$b^2 - 4ac < 0$	_____ roots, or zero x -intercepts _____ roots	

- Remember _____ are part of the complex number system. However, when describing the type of roots using the discriminant, complex refers to values in the form $a \pm bi$, where $b \neq 0$.
- If the discriminant is a _____, then the roots of the equation can be found using your preferred method of solving a quadratic equation.

Example 3

▶ Complete the example as you watch the video in the Online Lesson.

Determine the type of roots to the quadratic equation using the discriminant. Explain what the discriminant tells you about the roots.

$$3x^2 + 8x - 4 = x^2 - 1$$

Plan

Write equation in standard form

Identify a , b , and c

Write the discriminant formula

Substitute values into the formula

Simplify

Implement**Explain**

Because the discriminant is _____
_____, but not a perfect square,
there are _____ roots.

Example 4

▶ Complete the example as you watch the video in the Online Lesson.

Determine the type of roots to the quadratic equation using the discriminant. Explain.

$$x^2 + 8 = 5x$$

Implement**Explain**

Because the discriminant is _____,
there are _____ roots; however, there
are _____ roots.

Checkpoint: The Discriminant

Determine the type of roots to the quadratic equation using the discriminant. Explain.

$$32x^2 = -8x$$



To continue, return to the Online Lesson.

Applications of the Quadratic Formula

 Fill in the notes as you watch the video in the Online Lesson.

- There are many instances in the real world of objects following a _____ path: fireworks, fountains, throwing or dropping an object.



- The general equation for projectile (parabolic) motion path is: $h = \frac{1}{2}at^2 + vt + s$
 - _____ = final height
 - _____ = time
 - _____ = initial velocity
 - _____ = starting height
 - _____ = acceleration due to gravity

- Depending on the _____ of the parabola and the _____ used, a = acceleration due to gravity will be one of these options:
 - 32 feet per second squared (32 ft/s^2)
 - 9.8 meters per second squared (9.8 m/s^2)
- Remember, creating a _____ of a word problem can help you determine what is happening in the problem.

Example 5

🎥 *Complete the example as you watch the video in the Online Lesson.*

A football player caught a ball kicked down field. The catch was made 3 feet above the ground. If the ball was kicked with an initial velocity of 75 feet per second, how much time passed before the ball was caught? (Hint: The ball was kicked from the ground.)

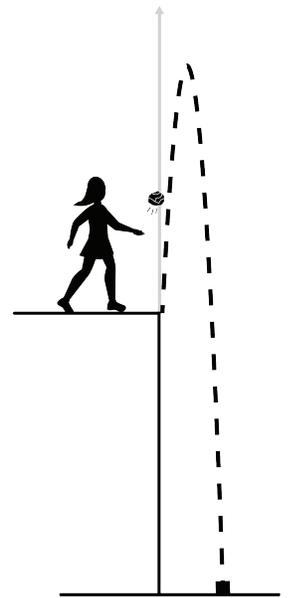
Example 6

▶ Complete the example as you watch the video in the Online Lesson.

In an egg drop experiment, the goal is to cushion an egg so that when it is dropped from a height of 50 feet, it does not crack. The science teacher starts by having the students calculate the drop time of the control group (an uncushioned egg). Determine the time a control group egg takes to hit the ground.

 Checkpoint: Applications of the Quadratic Formula

A stone is tossed off a cliff and into a crevasse from a starting height of 20 meters and an initial velocity of 15 meters per second. How long will it take the stone to hit the floor of the crevasse?



To continue, return to the Online Lesson.

 **Practice 1**

Complete problems on a separate sheet of paper.

Solve. Write all answers in simplest form.

1) $5x^2 - 11x + 6 = 0$

2) $4x^2 + 8x - 2 = x^2 - 2x$

3) $x^2 + 6x = 2$

4) $x^2 + 3x + 7 = 0$

5) $9x^2 + 4 = 0$

6) $-2x^2 = -11x + 12$

7) $7x^2 - 10x + 2 = 0$

8) $x^2 - 3x - 2 = -6$

Determine the type of roots to the quadratic equation using the discriminant. Explain.

9) $5x^2 + x + 1 = 0$

10) $5x^2 - 7 = 3x^2 + 3x$

11) $6x^2 + 13x + 2 = 0$

12) $x^2 + 6x + 12 = 0$

13) $3x^2 + 10x = 2$

14) $x^2 + 4x + 4 = 0$

15) $4x^2 - 7x = 3x^2 - 4$

16) $16x^2 + 8x + 1 = 0$

- 17) Cole threw a ball at an initial velocity of 35 feet per second from a starting height of 6 feet. Determine how long the ball will be in the air before it hits the ground. Round your answer to the nearest hundredth.
- 18) Grant dropped a rock off a 20 meter cliff at an initial velocity of 0 meters per second. Find the time it takes to hit the ground in meters per second.
- 19) The referee tossed a coin prior to the game. It was flipped at a velocity of 12 feet per second at a height of 5.75 feet. How much time passed until the coin hit the ground?
- 20) Two monkeys were tossing a banana back and forth. The first monkey threw the banana at a height of 2.5 feet and the other caught it at 2 feet. The initial velocity was 25 feet per second. How long was the banana in the air for one toss?



To continue, return to the Online Lesson.

 **Mastery Check** **Show What You Know**

Use $\{-5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5\}$ *only once* when completing parts A and B.

A) Write a quadratic equation that has two complex roots. Then find the roots.

$$\boxed{} x^2 + \boxed{} x + \boxed{} = 0$$

B) Write a quadratic equation that has two real, irrational roots. Then find the roots. Remember that any value that is used in part A cannot be used again in part B.

$$\boxed{} x^2 + \boxed{} x + \boxed{} = 0$$

C) Explain how you know what type of solutions a quadratic equation will have.

 **Say What You Know**

In your own words, talk about what you have learned using the objectives for this lesson and your work on this page.



To continue, return to the Online Lesson.

 **Practice 2**

Complete problems on a separate sheet of paper.

Solve. Write all answers in simplest form.

1) $x^2 - 3x + 9 = 0$

2) $5x^2 + 2x = 3x^2 + 11$

3) $4x^2 - 12x + 9 = 0$

4) $-x^2 - 2 = 5x + 7$

5) $x^2 + 6x + 13 = 0$

6) $7x^2 + 8x = 5x^2 + 6x$

7) $-3x^2 + x + 1 = 0$

8) $4x^2 + 25 = 0$

Determine the type of roots to the quadratic equation using the discriminant. Explain.

9) $6x^2 - 2 = 3x$

10) $x^2 + x + 5 = 0$

11) $x^2 - 2x = -1$

12) $x^2 + 5x + 6 = 0$

13) $3x^2 + 8x = 4x^2 - 2x + 3$

14) $6x^2 + 15 = 0$

15) $7x^2 + 13x - 2 = 0$

16) $10x^2 - 3x = 9x^2 + 5$

- 17) A rock is pushed off a 45 meter cliff at an initial velocity of 0 meters per second. Find the time it takes to hit the ground in meters per second.
- 18) A ball is thrown with an initial velocity of 42 feet per second from a starting height of 9 feet. How long until the ball is 2 feet from the ground? Round your answer to the nearest hundredth.
- 19) Elena jumped from the high dive into the local pool. She left the springboard at a velocity of 10 feet per second at a height of 15 feet. How long did it take until she touched the water? Round your answer to the nearest hundredth.
- 20) Kristin and Russell were playing catch in the back yard. The ball traveled at a velocity of 65 feet per second at a height of 5.5 feet. If Russell catches the ball 5 feet off the ground, how long after Kristin threw the ball did Russell catch it?



To continue, return to the Online Lesson.

Targeted Review

Complete items on a separate sheet of paper.

- Determine any missing roots, then write a possible polynomial equation.
 $\pm\sqrt{6}, -2$
- Determine the missing value that will make the expression a perfect trinomial square.
 $x^2 + bx + \frac{49}{25}$
- Graph. (piecewise function)

$$y = \begin{cases} -\frac{1}{3}x + 2 & \text{when } -6 < x < 0 \\ \sqrt{x} & \text{when } 0 \leq x < 4 \\ -(x-5)^2 + 4 & \text{when } 4 \leq x \leq 7 \end{cases}$$
- Graph: $y = 2(x-1)^2 - 3$
- Solve $2x^2 - 4x = 1$ by completing the square.

Match both the equation in vertex form and the domain and range to the function name below.

A) $y = a(x-h)^3 + k$	B) $y = a(x-h)^2 + k$	C) $y = \frac{a}{x-h} + k; x \neq h$
D) domain: $\{x x \in \mathbb{R}\}$ range: $\{y y \in \mathbb{R}, y > 0\}$	E) domain: $\{x x \in \mathbb{R}, x \neq 0\}$ range: $\{y y \in \mathbb{R}, y \neq 0\}$	F) domain: $\{x x \in \mathbb{R}\}$ range: $\{y y \in \mathbb{R}\}$

_____ **6)** Quadratic
 $y = x^2$

_____ **7)** Cubic
 $y = x^3$

_____ **8)** Rational
 $y = \frac{1}{x}$

Multiple Choice

- _____ **9)** Determine the perimeter of an isosceles right triangle with a hypotenuse that measures $4\sqrt{2}$ meters.
- ± 4
 - 4 meters
 - $8 + 4\sqrt{2}$ meters
 - cannot be determined

____ 10) Solve: $x^4 - 2x^2 - 9x^2 + 18 = 0$

A) $x = \pm\sqrt{2}$

B) $x = \pm 5$

C) $x = \pm 2, \pm 3$

D) $x = \pm\sqrt{2}, \pm 3$

____ 11) The work provided for solving a quadratic equation contains an error. Select the *correct* roots.

A) $x = 1 \pm 3i\sqrt{2}$

B) $x = -3, 5$

C) $x = \pm 4i$

D) $x = 1 \pm 4i$

Step 1 $x^2 - 2x + 17 = 0$

Step 2 $x^2 - 2x = -17$

Step 3 $x^2 - 2x + 1 = -16$

Step 4 $(x - 1)^2 = -16$

Step 5 $\sqrt{(x - 1)^2} = \pm\sqrt{-16}$

Step 6 $x - 1 = \pm 4$

12) Classify the expression: $-4 \pm \sqrt{6}$

rational

irrational

real

imaginary

complex

Problem	1	2	3	4	5	6	7	8	9	10	11	12
Origin	L23	L24	L21	L18	L24	L17	L17	L17	–	L23	L24	L15

L = Lesson in this level, A1 = Algebra 1: Principles of Secondary Mathematics



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