Lesson 23
NAME:
Solving Polynomials with Factoring

Start by navigating to the Online Lesson for instructions.

Objectives

- Solve polynomial equations by factoring under the set of complex numbers.
- Determine the polynomial equation given the roots.
- Solve equations using the square root property under the set of complex numbers.

Why?

Mathematics is used to solve complex problems in efficient ways. Throughout this unit, you will learn ways to efficiently solve polynomial equations focusing on conics.

Warm Up

Simplify.

1) $\sqrt{-54}$

2) $\sqrt{-121}$

Factor completely.

3) $3x^2 - 27$

4) $-2x^3 - 20x^2 - 50x$

To continue, return to the Online Lesson.

Explore

Writing Equations from Solutions

Fill in the notes as you watch the video in the Online Lesson.

- To write a ____________________ equation from given solutions, work backward.
  - Set each solution ____________ to the variable.
  - Then rewrite the solution as an equation set equal to ___________.
  - Using the ________________________ property, multiply each expression set equal to zero together, resulting in one polynomial equation.
  - The final equation will have __________________ coefficients.
- The ____ says: Polynomial equations with rational coefficients can have complex solutions, which will always come in conjugate pairs.
  - If $a+\sqrt{b}$ is an __________________ root, then $a-\sqrt{b}$ is also a root.
  - If $a+bi$ is a ______________ root, then $a-bi$ is also a root.
- Important: The number of roots can help determine the _______________ of the equation; however, the number of roots can never be greater than the degree.

Example 1

Complete the example as you watch the video in the Online Lesson.

Write a polynomial equation with integer coefficients using the given solutions. Classify the polynomial.

$x = -\frac{3}{2}, 5$

Implement

$x = -\frac{3}{2}$ $\quad x = 5$

$x + \frac{3}{2} = 0$ $\quad x - 5 = 0$

$2x + 3 = 0$

Explain

- Set every solution equal to x
- Rewrite each solution as an equation set equal to zero with integer coefficients
- Multiply the expressions together
- Write the polynomial equation set equal to zero

This is a quadratic ___________________.

Example 2

Complete the example as you watch the video in the Online Lesson.

Write a polynomial equation using the given solutions. Classify the polynomial.

$x = \pm\sqrt{3}, 6$

$x = -\sqrt{3}$ $\quad x = \sqrt{3}$ $\quad x = 6$

Example 3

Complete the example as you watch the video in the Online Lesson.

Given the root $3 - 2i$, determine if there are any missing roots. Then write the quadratic equation with integer coefficients in standard form.

Implement

$3 - 2i,\ 3 + 2i$

$x = 3 - 2i,\ x = 3 + 2i$

$(x - (3 + 2i))(x - (3 - 2i)) = 0$

Explain

- Conjugate Root Theorem
- Set every solution equal to x
- Rewrite each solution as an equation set equal to zero
- Multiply the expressions together until the equation is simplified

Checkpoint: Writing Equations from Solutions

Given the roots $-i$, 4 determine if there are any missing roots. Then write the polynomial equation in standard form.

To continue, return to the Online Lesson.

Comparing Solving Methods

Fill in the notes as you watch the video in the Online Lesson.

- Equations written in the general form $ax^2 + c = 0$ can be solved in two ways:
  - After isolating x^2, find the ______________________. $x^2 = d$, $x = \pm\sqrt{d}$

OR

  - _______________ a difference of two squares. $x^2 - d = (x + \sqrt{d})(x - \sqrt{d})$
- When two identical square roots are ____________________ together, the result is the ______________________.
- Therefore, even if a number is not a _______________ square, it can still be factored as a difference of two squares.
- A difference of two squares reveals another pattern, _________________________.
- Polynomial equations can have solutions that are: ____________________________________.
- To factor equations in $ax^2 + c = 0$ form, rewrite them as: _______________________ and factor using the difference of two squares.

Example 4

Complete the example as you watch the video in the Online Lesson.

Solve and compare methods.

$x^2 + 2 = 0$

Square Root

$$x^2 = -2$$
$$\sqrt{x^2} = \pm\sqrt{-2}$$
$$x = \pm i\sqrt{2}$$

Factoring

$$x^2 - (-2) = 0$$
$$(x + \sqrt{-2})(x - \sqrt{-2}) = 0$$
$$x + i\sqrt{2} = 0, \quad x - i\sqrt{2} = 0$$
$$x = \pm i\sqrt{2}$$

Explain

____________ solving methods result in the ____________ answer because ____________ and the ____________________ of two squares are both ways to represent ____________________ pairs.

Example 5

Complete the example as you watch the video in the Online Lesson.

Solve by finding the square root and factoring.

$x^2 - 2 = 0$

Checkpoint: Comparing Solving Methods

Solve using the method that is indicated.

Square Root

$x^2 = d$

Factoring

$x^2 - d = 0$

To continue, return to the Online Lesson.

Factoring to Solve

Fill in the notes as you watch the video in the Online Lesson.

- One method that can be used to solve polynomial expressions is ________________. It is not possible to solve every polynomial by factoring, but it is a method that can be used in many cases.
- To factor:
  - **1)** Find the greatest common factor, ___________, (other than one).
  - **2)** Factor by ____________________ when given four terms.
  - **3)** Factor __________________________________ (difference of two squares, sum/difference of cubes, perfect square trinomials).
  - **4)** Solve using your _____________________ factoring method (or another more efficient solving method).
- Once the expression is factored, solve using the ________________________________.
- The Zero-Product Property:
  - Says: If ________________, then $a = 0$, or $b = 0$.
  - Allows you to solve an equation by setting each expression _________________________ and solving for the value(s) of the variable.
- Remember that the ________________ of the equation tells you the number of solutions, or ________________.
- You will also use the ____________________________________ Theorem to factor equations and then solve.

Example 6

Complete the example as you watch the video in the Online Lesson.

Solve.

$x^2+2x\sqrt{3}+3=0$

Implement

$x^2+2x\sqrt{3}+3=0$

$(x+\square)(x+\square)=0$

Explain

- Special product pattern: Perfect Square Trinomial

The Conjugate Root Theorem does not apply to this problem because the middle coefficient is not a rational number. This is an example of a repeated root.

Example 7

Complete the example as you watch the video in the Online Lesson.

Solve.

$4x^3+8x^2-5x-10=0$

Implement

$(4x^3+8x^2)+(-5x-10)=0$

$(2x+\sqrt{5})(2x-\sqrt{5})(x+2)=0$

Explain

- Factor by grouping
- Recall: $x^2-d=(x+\sqrt{d})(x-\sqrt{d})$
- Zero-Product Property

The term with the ± symbol will be written first so as not to confuse this with a single expression for the solution.

Example 8

Complete the example as you watch the video in the Online Lesson.

Solve.

$x^2 = -5x$

Implement

Explain

- Set equal to zero
- GCF

Watch out! When problems are written in $ax^2 + bx = 0$ form, factor out the GCF to solve.

Checkpoint: Factoring to Solve

Solve.

$x^3 + 7x = 0$

To continue, return to the Online Lesson.

The Square Root Property to Solve

Fill in the notes as you watch the video in the Online Lesson.

- The ______________________________ property can be a more efficient method to solve equations when in the form $ax^2 + c = 0$ or $a(x - d)^2 + c = 0$.
- The _______________________________ or expression will be isolated and then you solve for the variable.

Example 9

Complete the example as you watch the video in the Online Lesson.

Solve.

$$(x+2)^2 = -\frac{2}{25}$$

Implement

Explain

- Square root property

Example 10

Complete the example as you watch the video in the Online Lesson.

Solve using the square root property.

$$4x^2 + 80 = 0$$

Implement

Explain

- Isolate the squared expression
- Solve for x

Example 11

Complete the example as you watch the video in the Online Lesson.

Solve.

$(3x+5)^2-144=0$

Implement

Explain

- Isolate the squared expression
- Square root property
- Solve for x

Checkpoint: The Square Root Property to Solve

Solve.

$(3x-1)^2=-12$

To continue, return to the Online Lesson.

Practice 1

Complete problems on a separate sheet of paper.

Be sure to carefully read the directions. Some problems have all solutions given. Others require including the conjugate before writing the equation.

Given the root(s), determine if there are any missing roots and write an equation of the polynomial in standard form with integer coefficients.

1) $x = 3, -2$

2) $x = 5i$

3) $x = 1 - \sqrt{5}$

4) $x = -i, \sqrt{6}$

5) $x = \frac{1}{2}, \frac{2}{3}$

6) $x = \pm\sqrt{11}, \frac{3}{4}$

7) $x = 0, 9$

Solve.

8) $x^3 - 7x^2 + 9x - 63 = 0$

9) $9x^2 - 6x\sqrt{2} + 2 = 0$

10) $x^2 - \frac{7}{25} = 0$

11) $3x^2 + 54 = 0$

12) $5x^3 - 15x^2 - 10x + 30 = 0$

13) $x^3 - 5x^2 - 6x + 30 = 0$

14) $x^2 + 4 = 0$

15) $x^4 + 13x^2 + 36 = 0$

16) $(5x - 1)^2 = 4$

17) $2(x - 5)^2 = -48$

18) $x^3 - 7x^2 + 8x - 56 = 0$

19) $x^2 - 10 = 0$

20) $(x + 8)^2 = -9$

21) $x^4 + 6x^2 - 7 = 0$

22) $x^2 - x\sqrt{13} = 0$

To continue, return to the Online Lesson.

Mastery Check

Show What You Know

A) The given equation and solutions are incomplete. Determine any additional solutions that are not shown and show all work. Then explain your reasoning.

Incomplete reasoning: If $(4x+1)^2 = 81$, then $x = 2$

B) The equation is incorrect for the given solutions. Determine a correct equation. Explain where you think the error occurred to get the incorrect equation.

Given $x = i, 3$, determine a possible polynomial equation with integer coefficients.

Incorrect equation: $x^2 - 3x - ix + 3i = 0$

Say What You Know

In your own words, talk about what you have learned using the objectives for this lesson and your work on this page.

To continue, return to the Online Lesson.

Practice 2

Complete problems on a separate sheet of paper.

Given the root(s), determine if there are any missing roots and write an equation of the polynomial in standard form with integer coefficients.

1) $x=-5, \frac{6}{7}$

2) $x=2-i$

3) $x=1, \sqrt{13}$

4) $x=\pm i$

5) $x=\frac{1}{2}, 7i$

6) $x=2i, \sqrt{3}$

Solve.

7) $4x^2-4x\sqrt{5}+5=0$

8) $(2x+5)^2=3$

9) $x^4-3x^2-70=0$

10) $x^3-x^2+16x-16=0$

11) $(7x-5)^2+10=0$

12) $x^2+2x\sqrt{7}+7=0$

13) $4x^3-8x^2+100x-200=0$

14) $x^3-10x^2=0$

15) $4(x-3)^2+8=0$

16) $(4x+9)^2-16=0$

17) $x^3-13x^2+4x-52=0$

18) $x^4+20x^2+64=0$

19) $x^2-\frac{8}{9}=0$

20) $3x^3-12x^2-12x+48=0$

21) $5x^2-25=0$

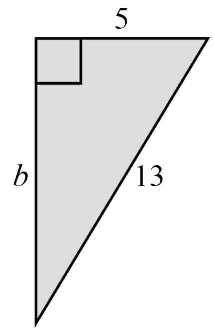
22) $(x-11)^2+8=0$

To continue, return to the Online Lesson.

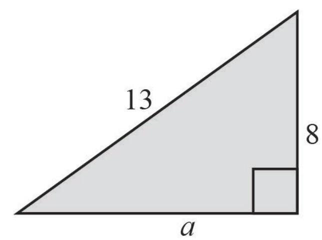
Targeted Review

Complete items on a separate sheet of paper.

1) Determine the perimeter of the right triangle.



2) Determine the area of the triangle. Round to the nearest hundredth.



3) Determine the value of Q and R.
$(7x+1)^2 = 49x^2 + Qx + R$

4) Determine the value of n.
$(2x-n)^2 = 4x^2 - 36x + 81$

5) Simplify.
$\sqrt{-6}\left(i+\sqrt{3}\right)$

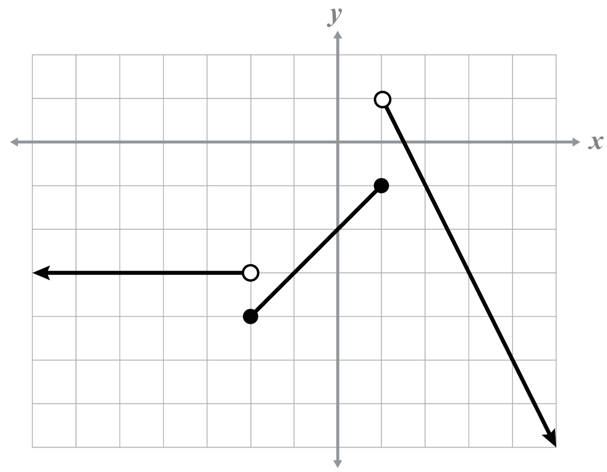
6) Simplify.
$\sqrt{-10}\left(\sqrt{-10}+5\sqrt{2}\right)-3i\sqrt{5}$

7) A parabola is reflected over the x-axis and translated 4 spaces up and left 2 spaces as compared to the parent graph. Write the equation of the new parabola in vertex form.

8) Graph the equation you wrote in the previous problem.

Multiple Choice

_____ **9)** Select the equation that best matches the graph.



A) $y = \begin{cases} -2x+3 & x > 1 \\ x-2 & -2 \le x \le 1 \\ -3 & x < -2 \end{cases}$

B) $y = \begin{cases} -2x+3 & x > -2 \\ x-2 & -2 \le x \le 1 \\ -3 & x < 1 \end{cases}$

C) $y = \begin{cases} -2x+1 & x > 1 \\ x-2 & -2 \le x \le 1 \\ -3 & x < -2 \end{cases}$

D) $y = \begin{cases} 2x+3 & x \ge 1 \\ x-2 & -2 \le x < 1 \\ -3 & x < -2 \end{cases}$

_____ **10)** Determine the missing value, M.
$(8i-5)+(2i+7)^2 = 4(M+10)$

A) 9

B) $2i$

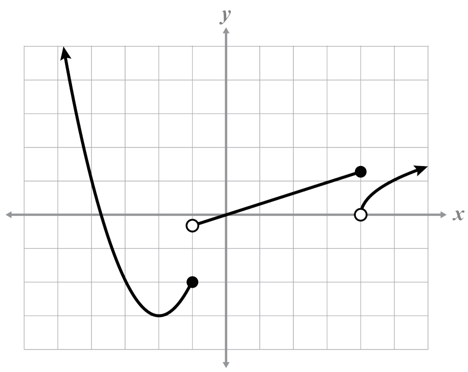
C) $9i$

D) $36i$

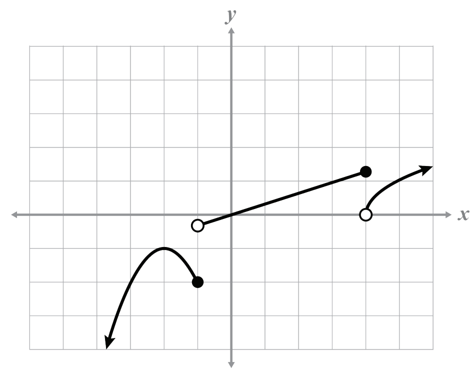
_____ **11)** Determine the graph that best represents the piecewise function.

$$y = \begin{cases} -(x+2)^2 - 1 & x < -1 \\ \dfrac{1}{3}x & -1 < x \le 4 \\ \sqrt{x-4} & x > 4 \end{cases}$$

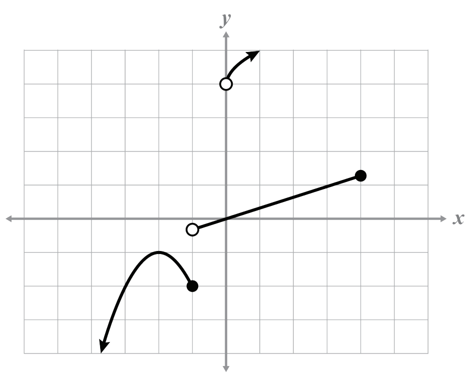
A)



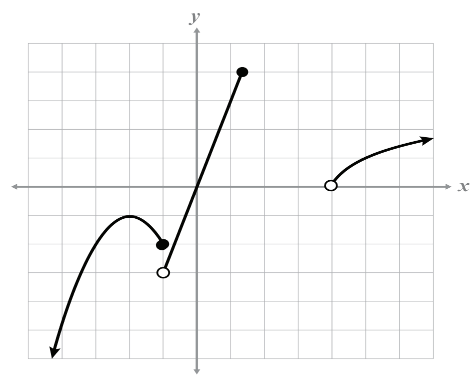
B)



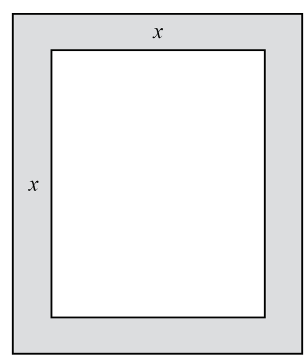
C)



D)



_____ **12)** Determine the expression that represents the total area, including the x inch wide frame, when the picture is a 5 inch by 7 inch photo.



A) $x^2 + 12x + 35$ square units

B) $4x^2 + 28x + 35$ square units

C) $x^2 - 12x + 35$ square units

D) 35 square units

Problem	1	2	3	4	5	6	7	8	9	10	11	12
Origin	–	–	L04	L04	L12	L12	L18	L18	L21	L16	L21	A1

L = Lesson in this level, A1 = Algebra 1: Principles of Secondary Mathematics

To continue, return to the Online Lesson.