Lesson 21
Piecewise Functions

NAME:

Start by navigating to the Online Lesson for instructions.

Objectives

- Graph piecewise functions, including scenarios, over the given intervals.
- Write piecewise functions with intervals given a graph.
- Find the rate of change over a specified interval.

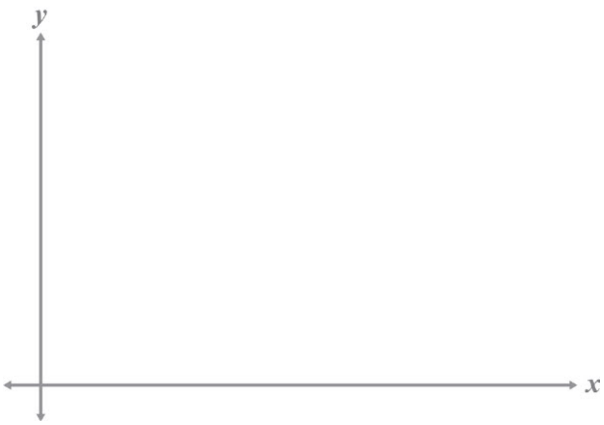
Why?

Piecewise functions are used to model real-life scenarios. For example, you can use a piecewise function to determine the shipping costs of items based on their range of weight. Piecewise functions can also be involved in tax brackets or pricing for bulk items. Many real-life functions are comprised of many smaller pieces that can be linear, quadratic, absolute value, and so on.

Warm Up

A toy car is placed on a ramp at the top of a track. When released, the car rolls down the track, jumps across a gap, lands on the track on the other side of a gap, and rides down a second ramp to come to a stop at a flat part of the track.

Sketch a graph to represent what the track might look like.



To continue, return to the Online Lesson.

Explore

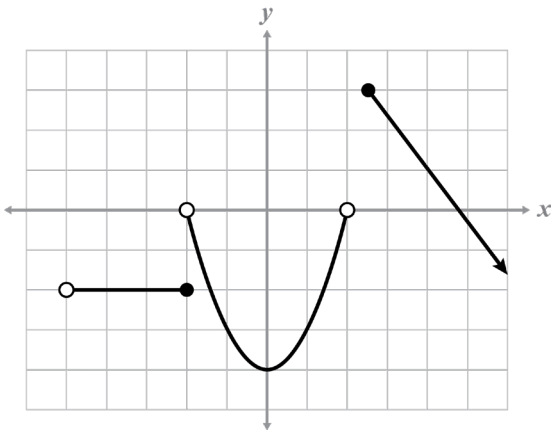
Writing Piecewise Functions

Fill in the notes as you watch the video in the Online Lesson.

- _____ functions are functions that are constructed from two or more expressions over defined intervals of the domain.
- These functions allow you to model _____ situations more realistically because real-life scenarios can rarely be represented by just one function.

- The _____ symbols for the domain intervals determine whether the function will have open points, closed points, or arrows (for continuous functions).

$$f(x)=\begin{cases}-2 & \text{if } -5<x\le -2\\ x^2-4 & \text{if } -2<x<2\\ -1.5x+6 & \text{if } x\ge 2.5\end{cases}$$



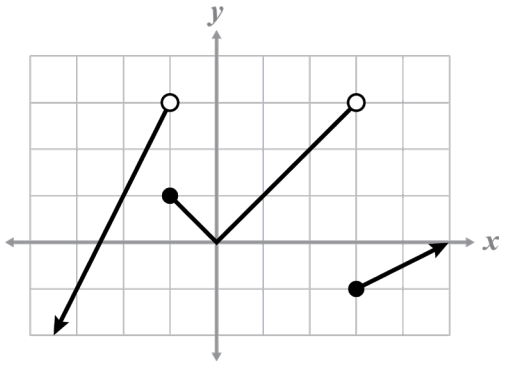
- To determine if an _____ is a solution to a piecewise function:
  - First check what part of the domain inequality interval the _____ is in.
  - Then substitute the x-value in the corresponding expression to find _____.

Example 1

Complete the example as you watch the video in the Online Lesson.

Write the expressions for the piecewise function given the graph and the intervals for the domain.

$$f(x)=\begin{cases} & \text{if } x<-1\\ & \text{if } -1\le x<3\\ & \text{if } x\ge 3\end{cases}$$



Determine $f(x)$ when $x=\{-0.5, 2.99, 3\}$

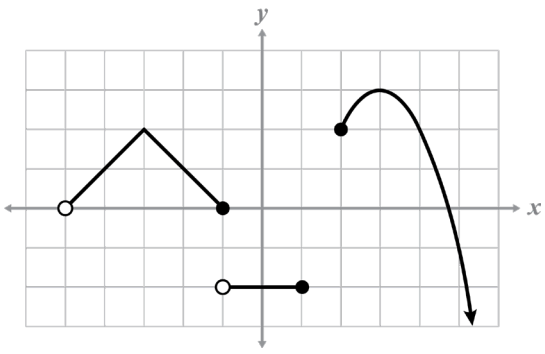
$-1\le -0.5<3$

Example 2

Complete the example as you watch the video in the Online Lesson.

Write the intervals for the domain given the graph and the expressions for the piecewise function.

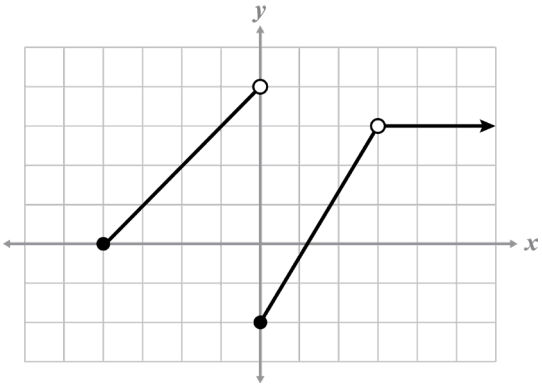
$$p(x) = \begin{cases} -|x+3|+2 \\ -1 \\ -(x-3)^2+3 \end{cases}$$



Example 3

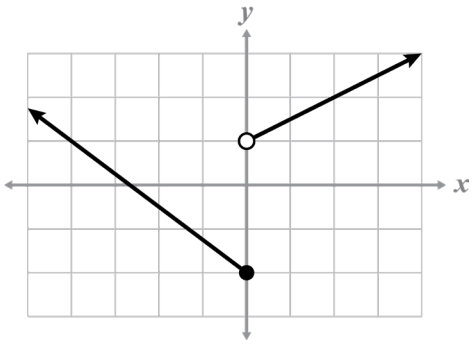
Complete the example as you watch the video in the Online Lesson.

Write the piecewise function using the given graph.



☑ Checkpoint: Writing Piecewise Functions

Write the piecewise function using the given graph.



To continue, return to the Online Lesson.

Graphing Piecewise Functions

Fill in the notes as you watch the video in the Online Lesson.

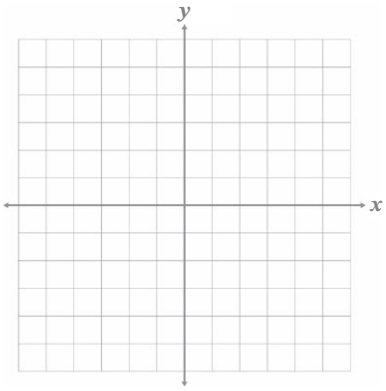
- When graphing a piecewise function, use what you know about transforming parent functions to mark open and closed points that note _____.
- The specific domain intervals beside each expression in the function determine:
  - the _____ on the graph.
  - what _____ is graphed over that domain interval.

Example 4

Complete the example as you watch the video in the Online Lesson.

Graph.

$$y = \begin{cases} (x+1)^2 + 2 & \text{if } -2 \le x < 1 \\ -\frac{1}{3}(x-1) + 2 & \text{if } x \ge 1 \\ 3x+7 & \text{if } x < -2 \end{cases}$$

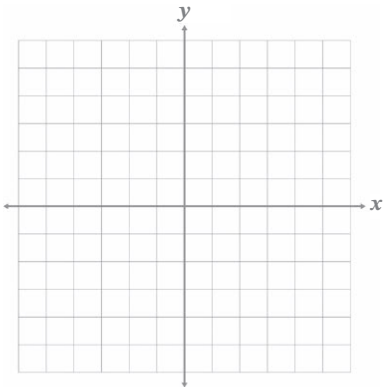


Example 5

Complete the example as you watch the video in the Online Lesson.

Graph.

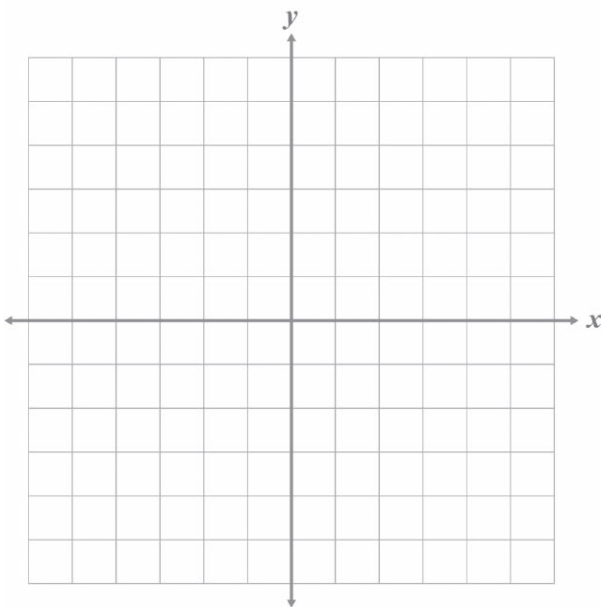
$$h(x) = \begin{cases} x+3 & \text{if } x < 2 \\ 5 & \text{if } x = 2 \\ -x+7 & \text{if } x > 2 \end{cases}$$



Checkpoint: Graphing Piecewise Functions

Graph.

$$w(x) = \begin{cases} -2x + 5 & \text{if } x > 1 \\ 1 & \text{if } -2 < x \leq 1 \\ -\frac{2}{3}x + 1 & \text{if } -6 < x \leq -2 \end{cases}$$



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Special Piecewise Functions

Fill in the notes as you watch the video in the Online Lesson.

- The following special piecewise functions do not require that you note the domain intervals:
  - _____ functions
  - _____ functions
- Because the domain and range are not listed for these functions, you must remember:
  - what the _____ for the function means,
  - as well as the _____.

Absolute Value Function

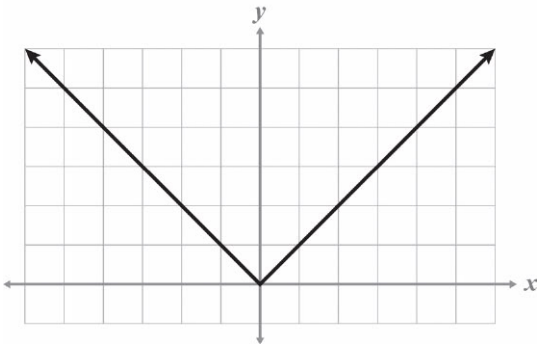
$g(x) = |x|$

OR

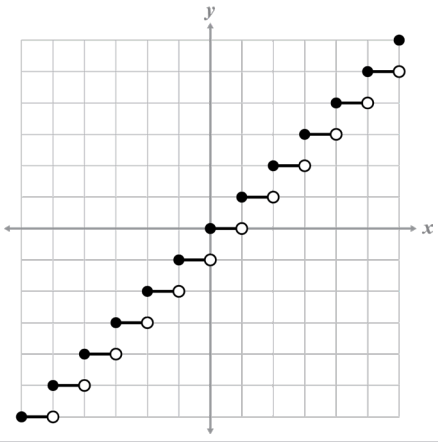
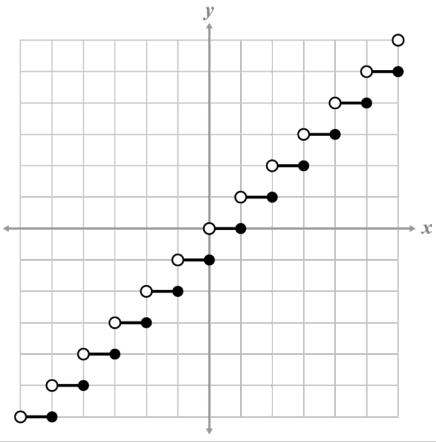
$$g(x) = \begin{cases} x & \text{if } x > 0 \\ -x & \text{if } x < 0 \\ 0 & \text{if } x = 0 \end{cases}$$

domain:

range:



Step Functions

Function	Greatest Integer	Least Integer
Also Known As	_____ function	_____ function
***y*-value**	rounded _____ to the _____ of the two integers that it is between	rounded _____ to the _____ of the two integers that it is between
Closed Point Value	the _____ value is an included point	the _____ value is an included point
Equation	$f(x) = \lfloor x \rfloor$	$h(x) = \lceil x \rceil$
Parent Graph		
Domain and Range	domain: range:	domain: range:

Example 6

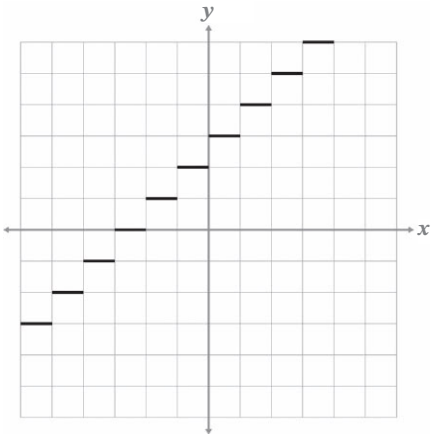
Complete the example as you watch the video in the Online Lesson.

Finish the graph of the step function. Name the domain and range for the function.

$f(x) = \lfloor x \rfloor + 3$

domain:

range:



Describe how the graph is translated as compared to the parent graph.

The y-intercept is _____ rather than $(0, 0)$ which

means the function is _____.

Example 7

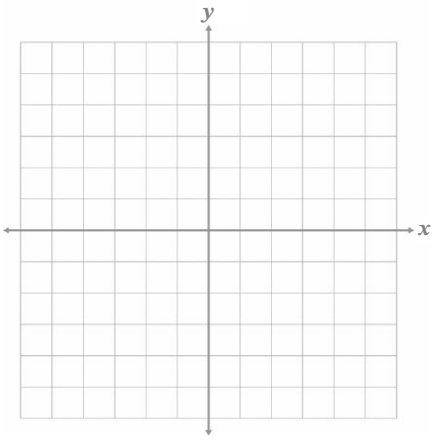
Complete the example as you watch the video in the Online Lesson.

Graph.

$y = -\lceil x + 2 \rceil$

domain:

range:



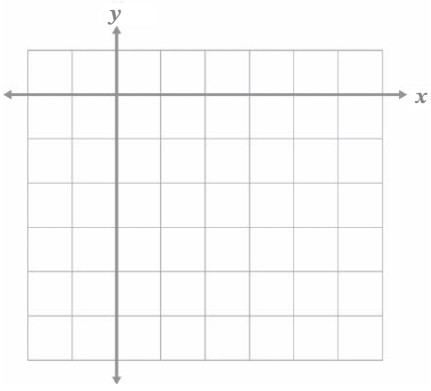
Checkpoint: Special Piecewise Functions

Graph. Name the domain and range.

$f(x) = \lfloor x \rfloor - 4$

domain:

range:



To continue, return to the Online Lesson.

Piecewise Scenarios

Fill in the notes as you watch the video in the Online Lesson.

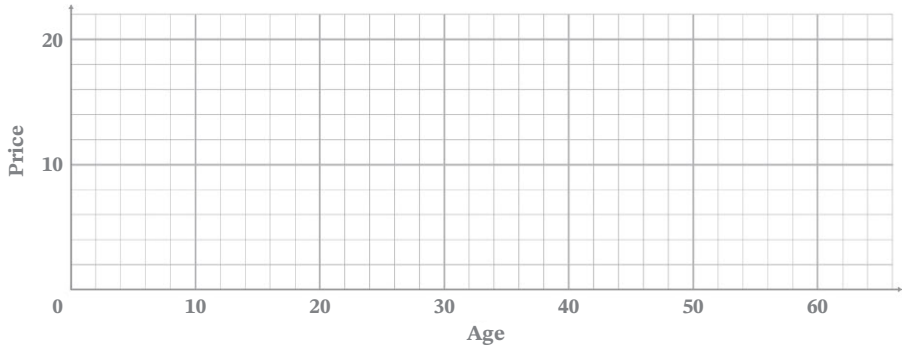
- When working with application problems, determine the independent and dependent _____ in order to write the piecewise function, create a graph with labels, or both.
- The _____ variable (x) is the variable that does not depend on the other variable for its value.
- The _____ variable ($f(x)$) is the variable that is dependent on the other variable for its value.
- Writing an ordered pair in words (independent, dependent) helps remind you how the two variables _____ to one another.
- Some piecewise functions will have _____ in the graph or domain intervals.
- As long as there are no _____ sections that align vertically, the graph will be a function.

Example 8

Complete the example as you watch the video in the Online Lesson.

The community center is selling tickets for their upcoming concert. The table shows the ticket prices based on the ages of the concert-goers. Create a graph that represents the ticket prices for the concert. Write a piecewise function to represent the table and graph.

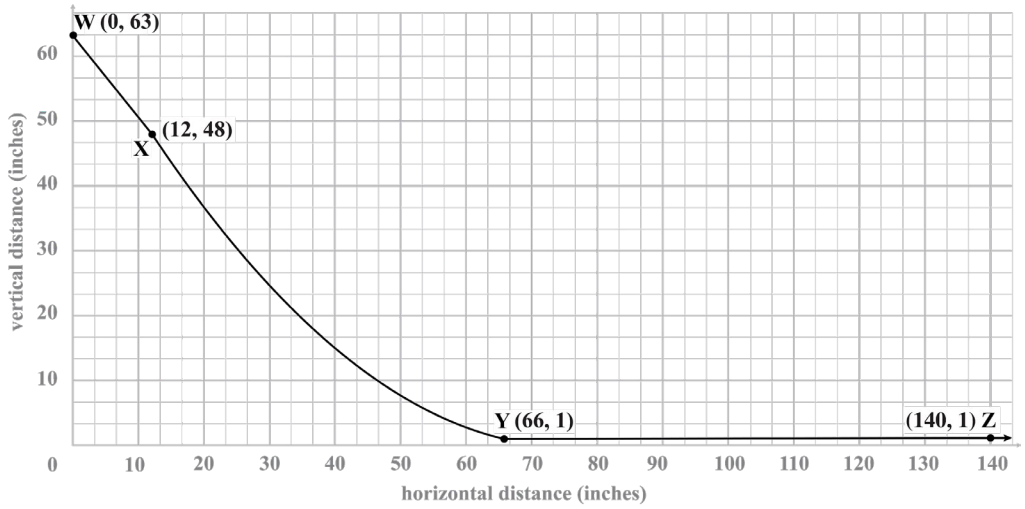
age	price
0–5	free
6–15	$10
16–60	$15
61+	$12



Example 9

Complete the example as you watch the video in the Online Lesson.

A physics class was using a toy car track to learn about the relationship between kinetic and potential energy. The graph of the ramp shows the relationship between the vertical and horizontal distances in inches.



A) Find the rate of change between points W and X.
$(0, 63), (12, 48)$

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

B) Estimate the rate of change between points X and Y.

The graph between points X and Y is a curve, but the rate of change can still give an estimate of what is happening over time.

C) Find the rate of change between points Y and Z. Explain why this slope does not mean that the toy car is stopped.

D) What is the steepest part of the ramp? Explain.

☑ Checkpoint: Piecewise Scenarios

The art club designed t-shirts and plans on placing an order with a screen-printer company. They charge $\$28$ per shirt for orders of 20 or fewer shirts, $\$23$ per shirt for orders of 21 to 40 shirts, $\$20$ per shirt for 41 to 65 shirts, and $\$15$ per shirt for orders greater than 65 shirts. Write a piecewise function to represent t-shirt prices, where x represents whole numbers of t-shirts.

What is the cost if 25 t-shirts are ordered?

What is the cost if 100 t-shirts are ordered?

To continue, return to the Online Lesson.

Practice 1

Complete problems on a separate sheet of paper.

Match each graph below (numbered) with its correct piecewise function and intervals (letters). Problems 1–3 have two matches.

A) $f(x) = \begin{cases} -x \\ x^2 \\ x \end{cases}$

B) $g(x) = \begin{cases} 2 \\ |x+3| \\ (x-2)^3 \end{cases}$

C) $\begin{cases} \text{if } -6 < x \le -4 \\ \text{if } -4 < x < 1 \\ \text{if } 1 \le x \le 3 \end{cases}$

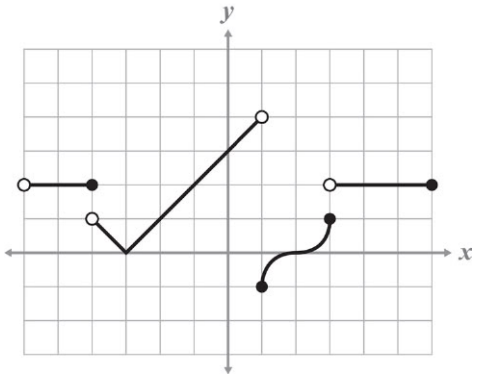
D) $\begin{cases} \text{if } -4 \le x < -3 \\ \text{if } -3 < x < -1 \\ \text{if } -1 \le x \le 2 \end{cases}$

E) $\begin{cases} \text{if } -6 \le x \le -2 \\ \text{if } -2 < x < 2 \\ \text{if } 2 \le x \le 6 \end{cases}$

F) $h(x) = \begin{cases} -x-3 \\ x+3 \\ x^2 \end{cases}$

G) $\begin{cases} \text{if } -6 < x \le -4 \\ \text{if } -4 < x < 1 \\ \text{if } 1 \le x \le 3 \\ \text{if } 3 < x \le 6 \end{cases}$

____ 1)



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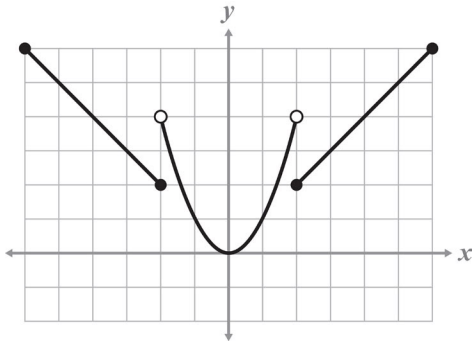
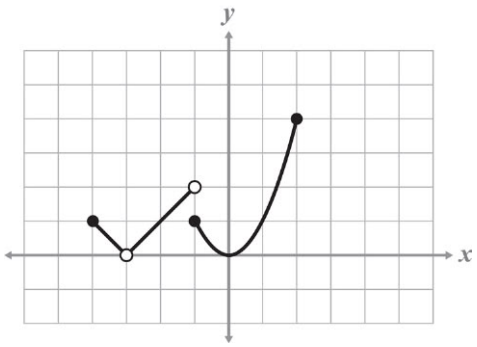


____ 3)



Graph the piecewise function.

4) $h(x) = \begin{cases} -\frac{1}{3}x & \text{if } x < -3 \\ x & \text{if } -3 \le x \le 3 \\ -2x + 9 & \text{if } x > 4 \end{cases}$

5) $b(x) = \begin{cases} (x+4)^2 + 1 & \text{if } x \le -2 \\ -|x+1| + 6 & \text{if } x > -2 \end{cases}$

6) Find the domain and range of problem 5.

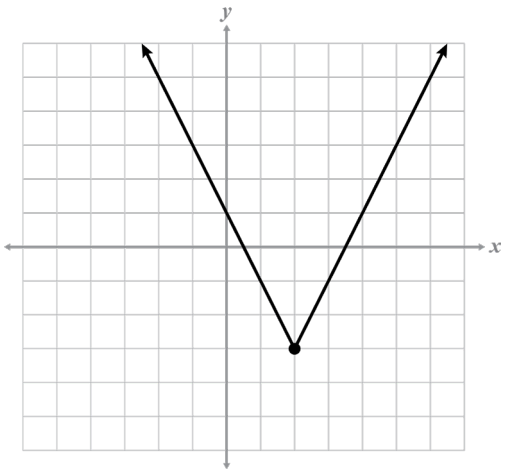
7) $f(x) = \lfloor x + 3 \rfloor + 1$

8) $g(x) = \lceil -x \rceil$

9) $h(x) = \begin{cases} \sqrt{-x} - 2 & \\ x - 2 & \text{if } x > 0 \end{cases}$

10) Find the domain and range of problem 9.

11) Write the absolute value function as a piecewise function.



Use the information below to complete problems 12–15.

The STEM club will host a catered dinner to celebrate their accomplishments. Tickets will be sold over a period of 45 days and are more expensive the closer it is to the event. Use the table to complete the following problems.

Dates tickets will be sold	Price of tickets
Day 0–14	$30
Day 15–30	$35
Day 31–45	$40

12) Write a piecewise function and draw a graph to represent the scenario.

13) Determine the cost for 5 students on the 16th day.

14) Calculate the cost when 3 tickets were purchased on day 6, and then 2 additional tickets were purchased on the 32nd day.

15) What is the cost for a student on the 46th day?

Use the information below to complete problems 16–21.

Natalee left her house for a drive at 8:06 a.m. At 8:12 a.m., she was three miles from the house. At 8:20 a.m. she was 10 miles from the house. She then turned around. It took 10 minutes to drive home.

16) Graph the scenario.

17) Write the rate of change as a ratio in words.

18) What is the rate of change from the start of the trip at 8:06 until 8:12 a.m.?

19) Calculate the rate of change between 8:20 and 8:30 a.m.. Explain your reasoning for the sign of the slope in your solution.

20) How far did Natalee travel round trip?

21) What would Natalee's speed be in miles per hour from 8:12 to 8:20 a.m.? (Hint: You need to convert miles per minute to miles per hour.)

To continue, return to the Online Lesson.

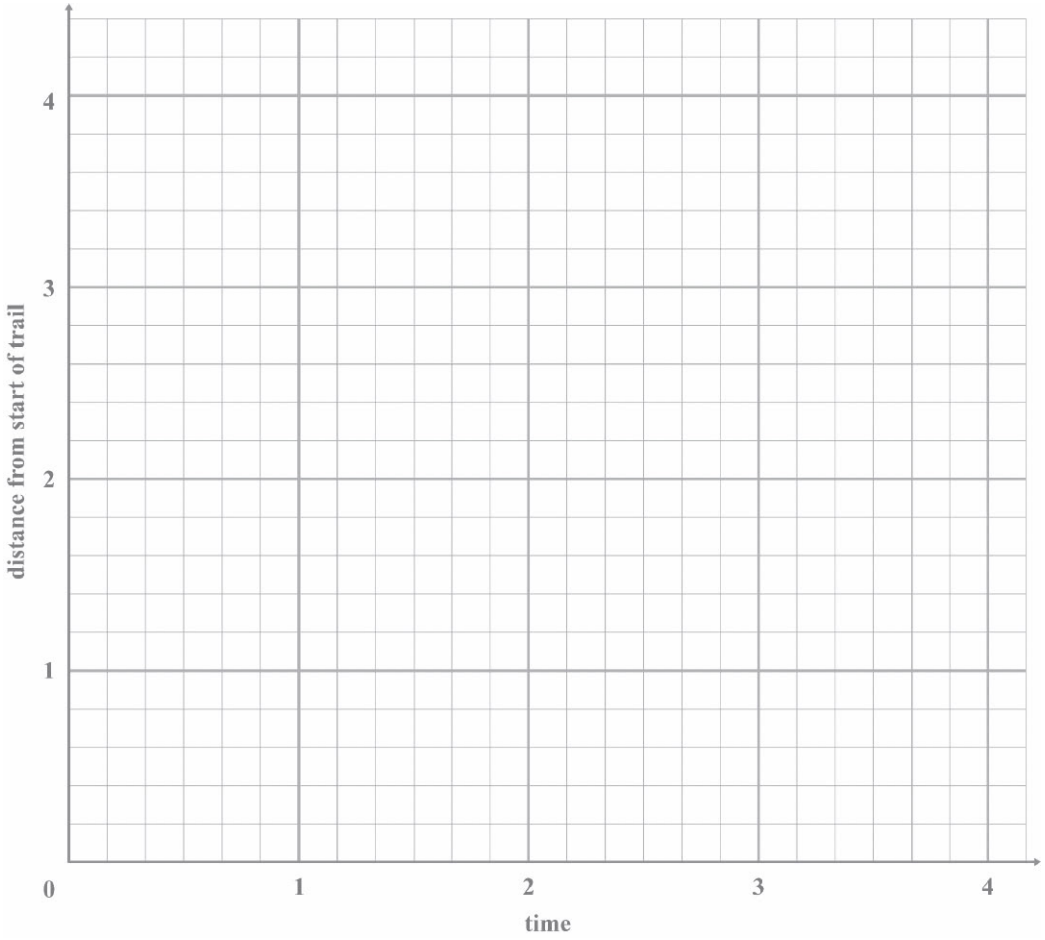
Mastery Check

Show What You Know

A novice hiker's loop-trail hike was recorded using a GPS device. Because the trail is a loop, the start and end is at the same location where the distance is zero.

- The first 3 miles of the hike took one hour.
- Then the hiker took a 10 minute break.
- The next mile took one hour, as this part of the trail was very steep.
- The hiker's next break was for 20 minutes.
- The last part of the loop-trail took 100 minutes.

A) Use the information provided to graph the scenario.



An expert hiker took the same loop-trail. Their GPS tracked them in the same way as the novice hiker.

$$e(x) = \begin{cases} 6x & \text{if } 0 \le x \le 0.5 \\ 2(x-0.5)+3 & \text{if } 0.5 < x \le 1 \\ -5(x-1)+4 & \text{if } 1 < x \le 1.8 \end{cases}$$

B) Graph the piecewise function for the expert hiker on the same graph as part A.

C) What is the total distance that both hikers completed?

D) What is the time difference between the hikes of the novice and expert?

E) What does a negative rate of change mean for both hikers?

Say What You Know

In your own words, talk about what you have learned using the objectives for this part of the lesson and your work on this page.

To continue, return to the Online Lesson.

Practice 2

Complete problems on a separate sheet of paper.

Match the piecewise functions (numbered) with the correct intervals (letters).

Intervals

A) if $x < -2$
if $-2 \le x \le 2$
if $x > 2$

B) if $x \le -1$
if $1 < x \le 3$
if $3 < x \le 5$

C) if $-6 \le x < -2$
if $-2 \le x \le 2$
if $2 < x < 6$

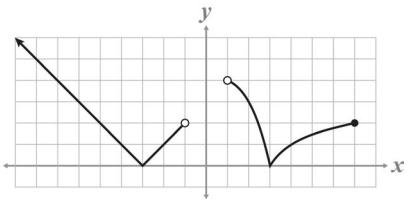
D) if $-\infty < x < \infty$

E) if $-7 \le x \le -6$
if $-6 < x \le 3$
if $3 < x \le 5$

F) if $x < -1$
if $1 < x < 3$
if $3 \le x \le 7$

G) if $x < -1$
if $-1 \le x \le 1$
if $x \le 1$

____ 1)



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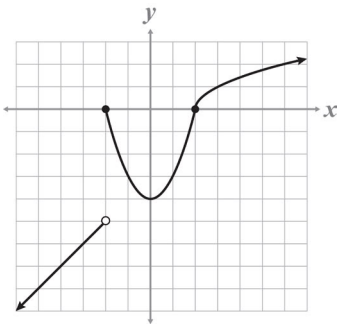

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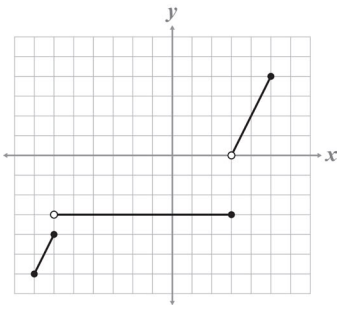
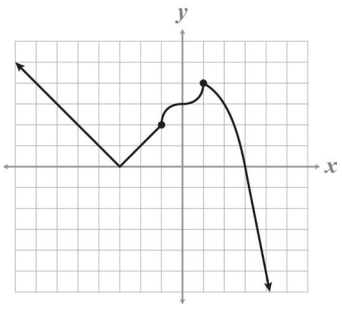


____ 4)



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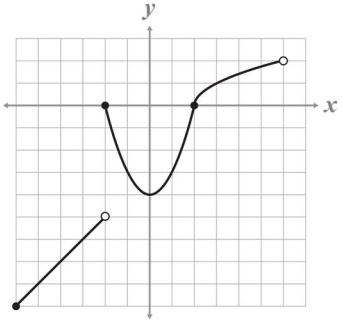
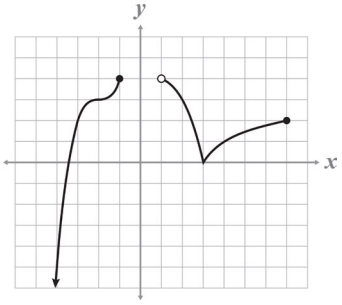


____ 6)



Graph the piecewise function.

7) $f(x) = \begin{cases} -\frac{1}{2}(x+3)^2 & \text{if } -5 \le x < -3 \\ \sqrt[3]{x+2} - 1 & \text{if } -3 \le x < 6 \end{cases}$

8) Find the domain and range of problem 7 using interval notation.

9) $m(x) = \begin{cases} -2(x+5)+3 & \text{if } -6 \le x \le -2 \\ 1 & \text{if } -1 \le x \le 1 \\ 2x-7 & \text{if } 2 \le x \le 6 \end{cases}$

10) $f(x) = \begin{cases} -\frac{1}{2}(x-4)^2+6 & \text{if } 0 \le x \le 6 \\ \frac{1}{2}(x+2)^2-4 & \text{if } -6 \le x < 0 \end{cases}$

11) Find the domain and range of problem 10 in set notation.

12) $a(x) = \begin{cases} -x-1 & \text{if } x < 1 \\ -2 & \text{if } x = 1 \\ x-3 & \text{if } x > 1 \end{cases}$

13) Create a graph that represents a child's age from 0 to 7 years old.

14) Describe and explain the type of function you graphed in the previous problem.

Use the information below to complete problems 15–20.

Gracie's Babysitting charges different amounts depending on the number of children. She babysits for one hour. If there are 1–3 children, she charges $8 per child. If there are 4–5 children she charges $6 per child. If there are 6 children, she charges $5 per child.

15) Create a piecewise function.

16) Graph the piecewise function.

17) How much will Gracie make to babysit 5 children for an hour?

18) How much will Gracie make to babysit 2 children for an hour?

19) Based on the information, does Gracie babysit 8 children?

20) The information was for one hour based on the number of children. How much would Gracie make if she babysat 3 children for 3 hours?

To continue, return to the Online Lesson.

Targeted Review

Complete items on a separate sheet of paper.

1) Write answers in simplified radical form. Use absolute value bars when necessary.

$\sqrt[4]{162a^7b^{12}c^4}$

2) Rationalize the denominator.

$$\frac{2x-5}{x+\sqrt{3}}$$

3) Solve. Check your work.

$\sqrt{x+8}=\sqrt{2x-1}$

4) Solve.

$\sqrt{4x+5}>\sqrt{4x+1}+2$

5) Simplify. Then classify by all sets to which it belongs: real, pure imaginary, complex.

$i\left(\sqrt{-12}+3i\right)-2\sqrt{3}+i\sqrt{-49}$

6) Simplify.

i^{67}

7) Write the equation for each parent function in vertex form.

A) cubic

B) absolute value

C) square root

D) quadratic

E) rational

8) Two students were asked to describe the transformation from the parent function using a, h, and k. Explain the errors and if either student is correct. State the correct answer if both students are incorrect.

$$y=-\frac{1}{4}\sqrt{x+14}+33$$

Student A's answer

$a=-\frac{1}{4}$, $h=14$, $k=33$

Student B's answer

$a=\frac{1}{4}$, $h=-14$, $k=-33$

____ 9) Name domain restrictions to represent the function.

A) $\{x \mid x \in \mathbb{R}, x > 3\}$

B) $\{x \mid x \in \mathbb{R}, x \geq 3\}$

C) $\{x \mid x \in \mathbb{R}, x \geq 1\}$

D) $\{x \mid x \in \mathbb{R}, x \geq 6\}$

____ 10) Solve.

$\sqrt{2x+2} = 3\sqrt{2x-4}$

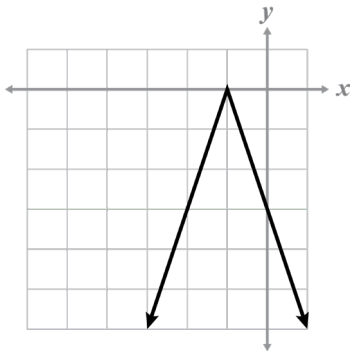
A) $\frac{19}{8}$

B) $\frac{7}{2}$

C) $\frac{3}{8}$

D) no solution

____ 11) Name the equation that represents the graph of the transformed parent function.



A) $y = -|x+3|$

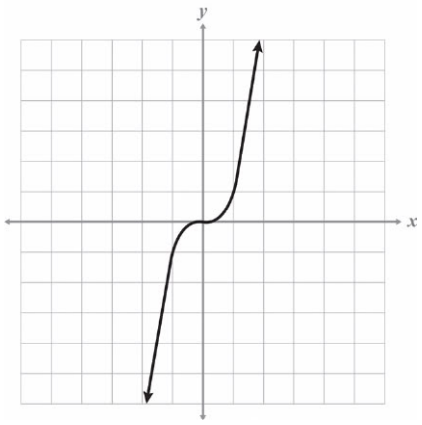
B) $y = 3|x+1|$

C) $y = -3|x+1|$

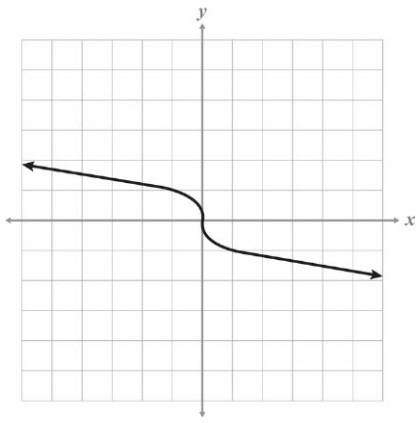
D) $y = -3|x-1|$

_____ **12)** Select the graph that matches the description of the end behavior.
As $x \to +\infty, f(x) \to +\infty$, and as $x \to -\infty, f(x) \to +\infty$

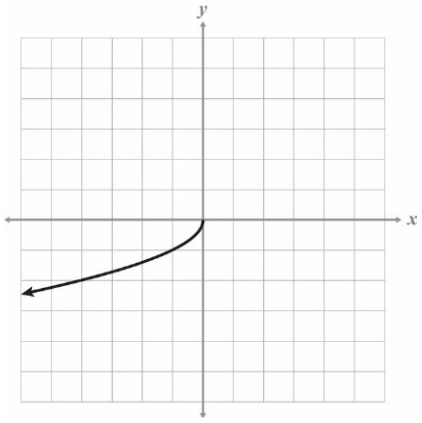
A)



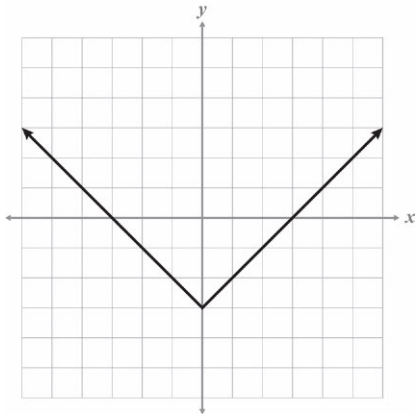
B)



C)



D)



Problem	1	2	3	4	5	6	7	8	9	10	11	12
Origin	L11	L12	L13	L14	L16	L15	L17	L18	L18	L13	L18	L18

L = Lesson in this level, A1 = Algebra 1: Principles of Secondary Mathematics, FD = Foundational Knowledge

To continue, return to the Online Lesson.