Lesson 11

NAME:

d^{th}-degree Radical Expressions under the set of Reals

Start by logging on to the Digital Toolbox and navigating to the Online Lesson for instructions.

Objectives

- Simplify d^{th}-degree radical expressions.
- Simplify expressions with rational exponents.

Why?

Simplifying d^{th}-degree radicals is the next step in making connections between radicals and exponents. It is important to be able to think about expressions in many different ways. When you can work with an expression in a way that makes the best sense to you, it sets you up for success when you move on to more complex formulas in the future.

Warm Up

Simplify.

1) Write the prime factorization of 48.

2) Simplify $\sqrt{48}$.

3) Simplify $3\sqrt{5} \cdot 4\sqrt{7}$.

4) Simplify $\sqrt{10} \cdot \sqrt{15}$.

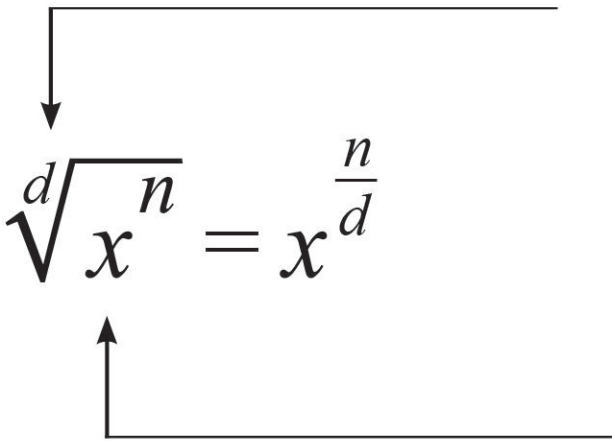
To continue, return to the Online Lesson.

Explore

Simplifying d^{th}-degree Radical Expressions

Fill in the guided notes as you watch the video in the Online Lesson.

- Writing a radical expression with a base raised to a rational exponent allows you to use the _____ to simplify.



- Radical expressions follow the exponent rules because they can be written in the form: $\sqrt[d]{x^n} = x^{\frac{n}{d}}$ where n and d are _____ numbers.
- The rules for simplifying radicands are true for _____.
- To simplify a radical expression, simplify out exact _____ from the radicand using the _____.
- The index d determines the _____ of roots in a simplified expression.
- The most common index is _____, which is the square root.
  - The _____ root is the non-negative square root of a non-negative real number.

$\sqrt{16} = 4$ In this notation, 4 indicates the principal root.

$-\sqrt{16} = -4$ In this notation, –4 indicates the _____ of the principal root.

$\pm\sqrt{16} = \pm 4$ In this notation, ±4 indicates the principal root _____ its opposite.

For this level, if opposite (–) or principal and opposite (±) symbols are **not** shown, then the principal root is implied.

- For an _____ index of $\sqrt[d]{x}$:

If	Then
	one negative root
	one root, 0
	one positive root

- For an _____ index of $\sqrt[d]{x}$:

If	Then
	no real root
	one root, 0
	one positive root, one negative root: $\pm\sqrt[d]{x}$

- Once terms are simplified out of the radicand with an even index, determine if _____ are needed as part of the simplified expression.

- For simplified roots:

If	Then
$\frac{n}{d}$ is an even exponent	no absolute value bars
$\frac{n}{d}$ is an odd exponent	absolute value bars included

Simplifying Examples Chart		
	odd index	**even index**
$\mathbb{W}$ **radicand**	$\sqrt[3]{8}$	$\sqrt{9}$ $-\sqrt{9}$ $\pm\sqrt{9}$
$-\mathbb{Z}$ **radicand**	$\sqrt[3]{-8}$	$\sqrt{-16}$
Variable radicand	$\sqrt[5]{h^{30}m^{15}}$	$\sqrt[4]{a^{24}}$ $\sqrt[4]{n^{12}p^{32}}$

Example 1

Complete the example as you watch the video in the Online Lesson.

Simplify. Write answers in simplified radical form.

$(5^8a^{12}b^{11}c^7)^{\frac{1}{2}}$

$5^{\frac{8}{2}}a^{\frac{12}{2}}b^{\frac{11}{2}}c^{\frac{7}{2}}$

$5^4a^6b^{5\frac{1}{2}}c^{3\frac{1}{2}}$

Explain

- Power of a power rule
- Write exponents as mixed numbers
- Use absolute value bars for terms raised to an odd power when the index is an even number

Example 2

Complete the example as you watch the video in the Online Lesson.

Simplify. Write answers in simplified radical form.

$\sqrt[3]{54p^5q^9}$

$54 = 2 \cdot 3^3$

$2^{\frac{1}{3}} \; 3^{\frac{3}{3}} \; p^{\frac{5}{3}} \; q^{\frac{9}{3}}$

Example 3

Complete the example as you watch the video in the Online Lesson.

Simplify. Write answers in simplified radical form.

$\sqrt[4]{81a^{15}b^{21}}$

Checkpoint: Simplifying d^{th}-degree Radical Expressions

Simplify. Write answers in simplified radical form.

$\sqrt{18xy^5z^{18}}$

To continue, return to the Online Lesson.

Multiplying Radicals

Fill in the guided notes as you watch the video in the Online Lesson.

- When radicals are multiplied:
- The _____ are multiplied and remain outside of the radical symbol.
- The _____ are multiplied and then simplified further if possible.

Example 4

Complete the example as you watch the video in the Online Lesson.

Simplify.

$7\sqrt[3]{6p^5q^3} \cdot 4\sqrt[3]{12p^{11}q^5}$

$28\sqrt[3]{72p^{16}q^8}$

$72 = 2^3 \cdot 3^2$

Explain

- Multiply coefficients
- Product Rule (Add exponents with like bases)
- Prime factor of the numerical radicand
- Radicand with rational exponents
- Simplify

Example 5

Complete the example as you watch the video in the Online Lesson.

Simplify.

$$-2x\sqrt{10xyz}\cdot\sqrt{10x^3yz}$$

Checkpoint: Multiplying Radicals

Simplify.

$$\sqrt[4]{12a^3b^2c}\cdot\sqrt[4]{27ab^3c^2}$$

To continue, return to the Online Lesson.

Practice 1

Complete problems on a separate sheet of paper.

Simplify. Write answers in simplified radical form.

1) $\pm\sqrt{12a^{10}b^6}$

2) $\left(11^3q^4c^{10}\right)^{\frac{1}{2}}$

3) $\sqrt[5]{32n^{15}}$

4) $-\left(8^5v^{13}w^{33}\right)^{\frac{1}{3}}$

5) $\sqrt[4]{81k^{44}u^{13}r^{51}}$

6) $\left(7^6g^7h^{16}\right)^{\frac{1}{4}}$

7) $\sqrt[3]{125x^{33}y^{14}}$

8) $\sqrt[5]{16b^{19}c^{13}d^{34}}$

9) $3\sqrt{12xy^3}\cdot 6\sqrt{3x^5y^3}$

10) $-4x\sqrt{7g^8h^2}\cdot 5\sqrt{21g^5h^{11}}$

11) $7v^2\sqrt[3]{4v}\cdot 2\sqrt[3]{2v}$

12) $\left(3^4a^3b^5\right)^{\frac{1}{5}}\cdot\left(3^2a^4b^{10}\right)^{\frac{1}{5}}$

13) $\sqrt[4]{27a^{10}b^9c}\cdot\sqrt[4]{18a^{26}b^{21}c^3}$

14) $-\sqrt{11p^4q^6}\cdot 21\sqrt{33q^{16}}$

15) Find the area of a rectangle with the dimensions $\left(5x^3y^6z\right)^{\frac{1}{2}}$ and $\sqrt{5xy}$ units. Write your answer in simplified radical form.

16) A triangle has a height of $\left(21a^4b^7\right)^{\frac{1}{3}}$ centimeters and base of $\sqrt[3]{49ab^2}$ centimeters. Find the area of the triangle in simplified radical form.

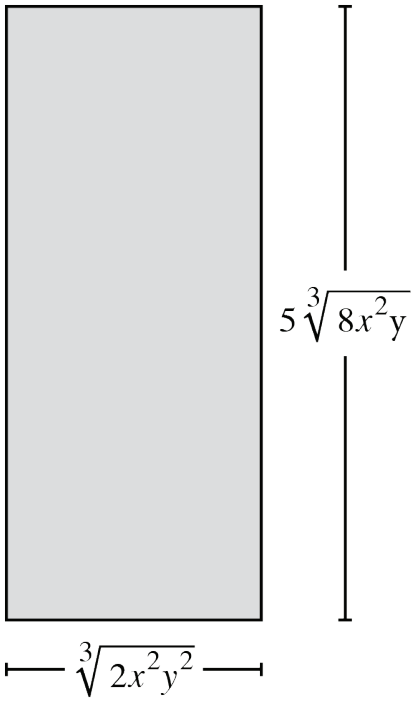
To continue, return to the Online Lesson.

Mastery Check

Show What You Know

A) The area of a figure is determine to be $\left(144x^7y^{12}z\right)^{\frac{1}{4}}$ square units. Write in simplified radical form.

B) Determine the simplified area of the given rectangle.



Say What You Know

In your own words, talk about what you have learned using the objectives for this part of the lesson and your work on this page.

To continue, return to the Online Lesson.

Practice 2

Complete problems on a separate sheet of paper.

Simplify. Write answers in simplified radical form.

1) $\sqrt[3]{24x^6y^{10}z^{12}}$

2) $-\sqrt{120gh^8j^{19}}$

3) $\sqrt[4]{625f^{28}g^{48}}$

4) $\left(3^8x^3y^{13}z^{15}\right)^{\frac{1}{5}}$

5) $\pm\left(6^4a^7b^3c^{10}\right)^{\frac{1}{2}}$

6) $\sqrt[3]{54x^4y^3}$

7) $\sqrt[5]{243q^5v^9}$

8) $\left(48x^{10}y^6z^2\right)^{\frac{1}{4}}$

9) $-5x^2\sqrt{xy}\cdot 2x\sqrt{x^3y}$

10) $\sqrt[3]{7c^3n^4}\cdot\sqrt[3]{42c^{17}n^{13}}$

11) $\sqrt[4]{32b^{47}d^{29}}\cdot\sqrt[4]{13b^{50}d^8}$

12) $\left(12m^7n^{14}p^7\right)^{\frac{1}{4}}\cdot\left(4m^5n^2p^3\right)^{\frac{1}{4}}$

13) $11x^2\sqrt{24x^2y^2}\cdot 7x^8\sqrt{2y^8}$

14) $5\sqrt[3]{7^6n^4u^{18}}\cdot 13nu\sqrt[3]{7^3nu}$

15) Determine the area of a triangle with a base of $\left(12a^3b^4\right)^{\frac{1}{2}}$ units and a height of $6a^2\sqrt{3ab}$ units. Write your answer in simplified radical form.

16) Determine the area of the rectangle with the dimensions $\left(8xy^3\right)^{\frac{1}{3}}$ and $\left(216x^8\right)^{\frac{1}{3}}$ units.

To continue, return to the Online Lesson.

◎ Targeted Review

Complete problems on a separate sheet of paper.

Solve.

1) $m + \frac{m}{m-5} = \frac{5}{m-5}$

2) $2(x-3) = \frac{1}{2}(4x+5)$

3) $5p - 3 > \frac{1}{3}(6p+27)$

4) Graph the solution to problem 3 on a number line.

5) Graph the linear function on a coordinate plane.

$$y - 3 = \frac{2}{3}(x-1)$$

6) Name the domain and range of a linear function in set-builder notation.

7) Sketch a graph of the rational function on the coordinate plane.

$$f(x) = \frac{1}{x+4} - 3$$

8) Name the domain and range for problem 7 in set-builder notation.

Multiple Choice

____ **9)** Ron and Will volunteer to pick up trash along the road. If Ron worked alone, it would take him 5 hours. Working together, it would take them 3 hours. How long would it take Will if he worked alone?

A) 1.875 hours

B) 7.5 hours

C) 8 hours

D) 15 hours

____ 10) Determine the expression that will form a polynomial identity with $(2x+1)^2$.

A) $(5x^2-6x+10)-(x^2+10x-9)$

B) $(5x^2-6x+10)-(x^2+6x+9)$

C) $(5x^2-6x+10)-(x^2-10x+9)$

D) no identity given

____ 11) Determine the restrictions for the denominator of the rational function.

$$y=\frac{4(x+5)}{x^2+3x-10}$$

A) $x \neq -5$

B) $x \neq 2$

C) $x \neq -5, 2$

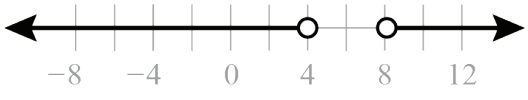
D) $x \neq -2, 5$

____ 12) Select the number line that best represents the solution to $|x-6|<2$.

A)

B)

C)



D)

Problem	1	2	3	4	5	6	7	8	9	10	11	12
Origin	L9	A1	A1	A1	A1	A1	L10	L10	L9	L4	L9	A1

L = Lesson in this level, A1 = Algebra 1: Principles of Secondary Mathematics, FD = Foundational Knowledge

To continue, return to the Online Lesson.