Lesson 8

NAME:

Adding and Subtracting Rational Expressions

Start by logging on to the Digital Toolbox and navigating to the Online Lesson for instructions.

Objectives

- Determine the least common denominator (LCD) of a rational expression.
- Add and subtract rational expressions.
- Simplify complex fractions using addition, subtraction, multiplication, and division.
- Apply rational expressions to word problems such as geometric probabilities and efficiency ratios.

Why?

In order to be able to work with rational functions and their graphs in future lessons, you need to be able to simplify expressions containing the four basic operations.

Warm Up

Simplify.

1) $\frac{3}{5} - \frac{5}{3}$

2) $\frac{x}{4} + \frac{4x}{2} - \frac{x}{10}$

3) $\frac{1 + \frac{1}{2}}{10}$

4) $\frac{\frac{5}{8}}{\frac{15}{16}}$

To continue, return to the Online Lesson.

Explore

Adding and Subtracting Rational Expressions

Fill in the guided notes as you watch the video in the Online Lesson.

- Rational expressions are _____ under addition, subtraction, multiplication, and division.
- Mathematically, closed means you _____ a problem with the _____ type of expression.
- Therefore, if you start with a rational expression, you end with _____ _____.
- For example:
  - The _____ of rational expressions is always a rational expression.
  - The _____ between rational expressions is always a rational expression.
  - The _____ of rational expressions is always a rational expression.
- When adding or subtracting _____, determine a least common denominator (LCD) and use it prior to finding the sum or difference.
- Follow these guidelines when simplifying rational expressions by adding or subtracting:

1) Factor the _____ to determine the LCD.

2) _____ each term in the rational expression using the LCD (Identity Property of Multiplication).

3) Simplify and combine the numerators to form _____.

Naming the excluded values (restrictions) is best practice because it gives the most complete answer to a simplified expression.

Example 1

Complete the example as you watch the video in the Online Lesson.

Simplify.

$$\frac{x^2+3x}{x^2+7x+6}-\frac{4x-11}{x^2+7x+6}$$

Plan

Factor denominators to determine the LCD
Name the restrictions for the denominator
Simplify the numerator
Write the answer as a fraction, including restrictions

Implement

$$\frac{x^2+3x}{(x+1)(x+6)}-\frac{4x-11}{(x+1)(x+6)}$$

Explain

- Factor the denominators
- Name the restrictions
- Simplify the numerator
- Write the answer with the factored denominator and restrictions

Example 2

Complete the example as you watch the video in the Online Lesson.

Simplify.

$$\frac{2x^2-5}{x^2-13x+12}+\frac{x-7}{x-12}-\frac{3}{x^2-x}$$

Plan

Factor all denominators to determine the LCD
Write each expression with LCD
Simplify

Implement

$$\frac{2x^2-5}{(x-1)(x-12)}+\frac{x-7}{x-12}-\frac{3}{x(x-1)}$$

$$\frac{x\left(2x^2-5\right)}{x(x-1)(x-12)}+\frac{x(x-1)(x-7)}{x(x-1)(x-12)}-\frac{3(x-12)}{x(x-1)(x-12)}$$

Explain

- Factor the denominators
- Name the LCD and excluded values
- Identity property
- Write each expression with the LCD
- Simplify the numerator
- Write the answer with a factored denominator and excluded values

Example 3

Complete the example as you watch the video in the Online Lesson.

Simplify.

$$\frac{2n-3}{n-3}+\frac{n+2}{(3-n)(2n+3)}$$

Checkpoint: Adding and Subtracting Rational Expressions

Simplify. Determine the restrictions on the denominator.

$$\frac{x}{x+3}-\frac{6}{x-8}$$

To continue, return to the Online Lesson.

Complex Fractions

Fill in the guided notes as you watch the video in the Online Lesson.

- A _____ fraction is a fraction that contains additional fractions, or term(s) with _____ exponents, in the numerator and/or denominator.
- With a complex fraction, it is important to remember that the _____ is a division symbol.
- The problem can be rewritten _____ using the division symbol (÷).
- Before dividing, _____ any addition or subtraction in the numerator and denominator.

Example 4

Complete the example as you watch the video in the Online Lesson.

Simplify.

$$\frac{2+\frac{5}{x}}{\frac{2}{x}-7}$$

Plan

Simplify the numerator (dividend)
Simplify the denominator (divisor)
Write as a horizontal fraction using ÷
Take the reciprocal of the second fraction
Simplify

numerator:

$2+\frac{5}{x}$ $\quad \text{LCD}(1, x) = x$

$\frac{2x}{x}+\frac{5}{x}$

$\frac{2x+5}{x}, x \neq 0$

Example 5

Complete the example as you watch the video in the Online Lesson.

Simplify.

$$\frac{1-\frac{2}{x}-\frac{3}{x^2}}{\frac{x}{4}-\frac{3}{x+1}}$$

Example 6

Complete the example as you watch the video in the Online Lesson.

Simplify.

$$\left(\frac{3}{x-5}+1\right)\left(4-\frac{x-1}{x+3}\right)^{-1}$$

$$\left(\frac{3}{x-5}+1\right)\div\left(4-\frac{x-1}{x+3}\right)$$

Implement

Explain

- Simplify the first group of terms
- Simplify the second group of terms
- Combine rational expressions
- Take the reciprocal of expression after division symbol
- Write as one rational expression

Checkpoint: Complex Fractions

Simplify.

$$\frac{\dfrac{6}{x^2+7x-18}-\dfrac{1}{x-2}}{\dfrac{1}{x+9}-1}$$

To continue, return to the Online Lesson.

Applications for Rational Expressions

Fill in the guided notes as you watch the video in the Online Lesson.

- Complex fractions are commonly used with ratios that:
  - find a _____.
  - use formulas for _____.
- A geometric probability is written as:
- An efficiency ratio is comparing two options to find the best one.
  - First _____ the rational expression, then _____ ratios to find the best choice.
  - A _____ efficiency rating is considered best because it means there is less _____.

Example 7

Complete the example as you watch the video in the Online Lesson.

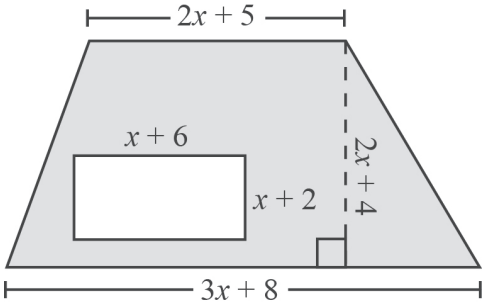
Benji finished wiping the kitchen island countertop (trapezoid) and tosses the rag toward the sink (rectangle).

A) Determine the probability of the rag landing in the sink instead of the counter.

B) Determine the probability of the rag landing anywhere on the counter *except* in the sink.

Plan

Find the area of the rectangle
Find the area of the trapezoid
Find the area of the (trapezoid – rectangle)
Find P(rectangle)
Find P(not rectangle)



Rectangle

$A = bh$

$A = (x + 6)(x + 2)$

$A = x^2 + 8x + 12$

Trapezoid

$A = \frac{1}{2}h(b_1 + b_2);\ b_1 = 2x + 5,\ b_2 = 3x + 8$

$A = \frac{1}{2}(2x + 4)(2x + 5 + 3x + 8)$

$A = (x + 2)(5x + 13)$

$A = 5x^2 + 23x + 26$

Trapezoid — Rectangle

$(5x^2 + 23x + 26) - (x^2 + 8x + 12)$

$5x^2 + 23x + 26 - x^2 - 8x - 12$

$4x^2 + 15x + 14$

$(4x + 7)(x + 2)$

A) $P(\text{rectangle}) = \frac{\text{rectangle area}}{\text{total area}}$

$= \frac{x^2 + 8x + 12}{5x^2 + 23x + 26}$

B) $P(\text{not rectangle}) = \frac{\text{trapezoid} - \text{rectangle}}{\text{total area}}$

Example 8

Complete the example as you watch the video in the Online Lesson.

Three companies, X, Y, and Z, are being compared based on efficiency ratios. The operating costs are determined using the formulas:

$$C = \frac{m^2}{2}(3m-1)(4c+5) \qquad T = \frac{5m}{2}(3m-1)^2(6c+7)$$

A) Determine each company's efficiency rating by comparing operating cost to total income, $\frac{C}{T}$.

B) Using the table, find the best efficiency rating for the three companies.

	Company X	Company Y	Company Z
m	7.8	9.2	4.7
c	5.2	1.8	2.3

A) $$\frac{C}{T} = \frac{\frac{m^2}{2}(3m-1)(4c+5)}{\frac{5m}{2}(3m-1)(3m-1)(6c+7)} = \frac{\frac{m^2}{2}}{\frac{5m}{2}} \cdot \frac{(3m-1)(4c+5)}{(3m-1)(3m-1)(6c+7)}$$

B) $$\frac{C}{T} = \frac{m(4c+5)}{5(3m-1)(6c+7)}$$

Company X

$$\frac{(\square)(4(\square)+5)}{5(3(\square)-1)(6(\square)+7)} =$$

Company X rating: _____

Company Y

Company Y rating: _____

Company Z

Company Z rating: _____

Explain

☑ Checkpoint: Applications for Rational Expressions

Find the efficiency ratio $\frac{SA}{V}$ that compares the surface area and volume of a sphere.

$$SA = 4\pi r^2 \qquad V = \frac{4}{3}\pi r^3$$

To continue, return to the Online Lesson.

Practice 1

Complete problems on a separate sheet of paper.

Complete each sentence with the word that best describes rational expressions (always, sometimes, never).

1) The sum of rational expressions is _____ a rational expression.

2) The denominator of a rational expression should _____ be undefined.

3) Rational expressions are _____ closed under addition, subtraction, multiplication, and division.

Simplify.

4) $\dfrac{3x}{x+3} - \dfrac{x+1}{x+2}$

5) $\dfrac{y}{y-1} + \dfrac{5}{y+2} - \dfrac{3}{y^2+y-2}$

6) $\dfrac{7}{m+5} + \dfrac{4}{5-m} + \dfrac{2m-1}{m^2-25}$

7) $\dfrac{2x+3}{4x^2+6x} + \dfrac{2x}{2x+3} + \dfrac{3}{2x}$

8) $\dfrac{r^2-6r-7}{r^2-3r-28} + \dfrac{r^2+4r+3}{r^2+3r+2}$

9) $\dfrac{2g}{9g^2-25} - \dfrac{3g+5}{5-3g} + \dfrac{g}{3g+5}$

10) $\dfrac{\dfrac{x}{y}+3}{\dfrac{x}{y}-4y}$

11) $\dfrac{\dfrac{2}{a}+\dfrac{3}{b}}{\dfrac{6b+9a}{ab}}$

12) $\dfrac{\dfrac{1}{x}+\dfrac{2}{x+1}}{\dfrac{2}{x+2}-\dfrac{1}{x+1}+\dfrac{3}{x}}$

13) $\dfrac{\dfrac{1}{x-3}+\dfrac{5}{x+3}}{\dfrac{3}{x}-\dfrac{2x}{x-3}}$

14) An electrical circuit with three resistors connected in parallel is shown using the equation: $R_T = \dfrac{1}{\dfrac{1}{R_1}+\dfrac{1}{R_2}+\dfrac{1}{R_3}}$. Simplify the expression.

15) Find the total resistance of the electrical circuit if three light bulbs in parallel have the resistance of: $R_1 = \dfrac{2}{5}$ ohms, $R_2 = \dfrac{5}{3}$ ohms, and $R_3 = 4$ ohms.

To continue, return to the Online Lesson.

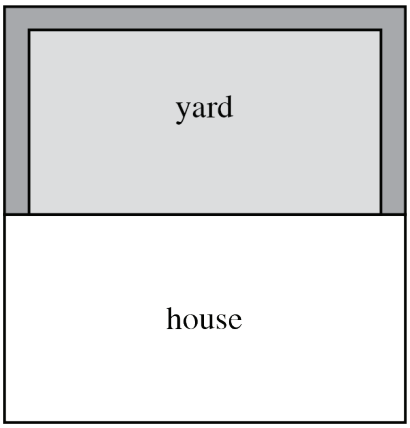
Mastery Check

Show What You Know

The backyard of the Klein house is fenced on three sides.

The length of the fence is $\frac{f-5}{f^2-3f-10}$ feet.

The other two sides of the fence are each $\frac{f}{f^2+f-2}$ feet.



A) Find the total length of the fence.

The Klein family hired a surveyor to determine if their yard is large enough for a pool. The ratio of the width to the length of the yard will be used by the surveyor.

B) Determine the simplified ratio, $\frac{w}{l}$.

Three members of the Klein family will paint the fence in their backyard.

Joe's time in minutes	Jack's time in minutes	Judy's time in minutes
$\frac{1}{r}$	$\frac{1}{r+3}$	$\frac{1}{r-3}$

C) Find the total time when Joe, Jack, and Judy work together.

Say What You Know

In your own words, talk about what you have learned using the objectives for this part of the lesson and your work on this page.

To continue, return to the Online Lesson.

Practice 2

Complete problems on a separate sheet of paper.

Complete each sentence with the word that best describes rational expressions (always, sometimes, never).

1) The restrictions for the expression _____include the numerator and denominator of a rational expression.

2) Closed means that you will _____start and end a problem within the same set of terms.

3) When the product or quotient of rational expressions is found, the result is _____a rational expression.

Simplify.

4) $\frac{1}{x+3}+\frac{1}{x-2}-\frac{1}{x-3}$

5) $\frac{3x}{2x+5}-\frac{x}{x-6}$

6) $\frac{y+3}{y-2}-\frac{6y-7}{y^2-3y+2}$

7) $\frac{4}{x-5}+\frac{1}{x-1}-\frac{3x}{x^2-6x+5}$

8) $\frac{2}{x^2+3x+2}+\frac{2}{2x^2+3x+1}$

9) $\frac{5}{a}+\frac{2a-5}{a^2-10a}+\frac{1}{a^2-100}$

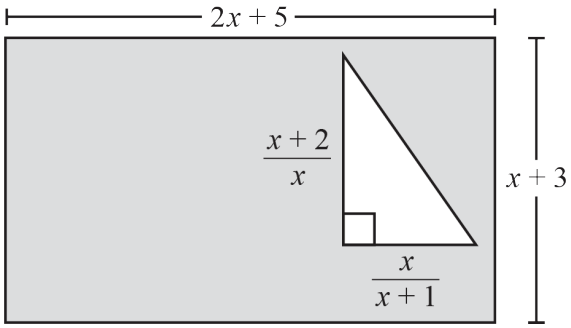
10) $\frac{\frac{4}{y}}{2-\frac{y}{y+6}}$

11) $\frac{\frac{a}{b}+\frac{a}{b+1}}{\frac{b}{a}+\frac{b}{a+1}}$

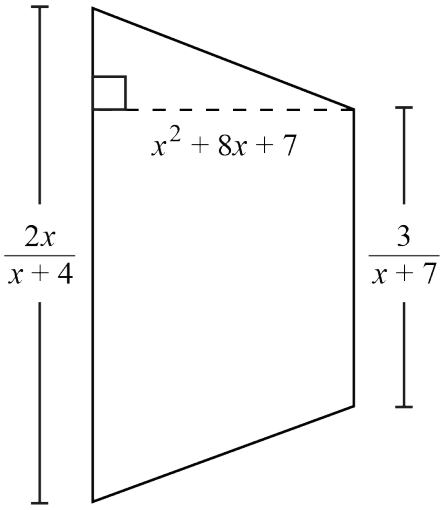
12) $\frac{\frac{x}{y}-\frac{y}{x}}{\frac{y}{x}-\frac{x}{y}}$

13) $\frac{\frac{2}{x-4}+\frac{3}{x+1}}{\frac{5}{x+1}-\frac{1}{x-2}}$

14) Find the area of the shaded region.



15) Find the area of the trapezoid.



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