

Lesson 7

NAME:

Multiplying and Dividing Rational Expressions

 Start by logging on to the Digital Toolbox and navigating to the Online Lesson for instructions.

Objectives

- ✓ Name restrictions for a denominator.
- ✓ Simplify the product or quotient of rational expressions.

Why?

Working with rational expressions is a foundational step to developing skills and understanding rules for later lessons and levels. Rational expressions extend what you have learned about fractions to include fractions with variables in the numerator and denominator.

Warm Up

Factor to solve.

1) $2x^2 - 10x = 0$

2) $4x^2 + 12x + 9 = 0$

Simplify.

3) $\frac{5a^3b}{15ab^2}$

4) $\frac{4x^2}{9y^2} \cdot \frac{y}{24x}$



To continue, return to the Online Lesson.

 Explore Restrictions for Rational Expressions

 Fill in the guided notes as you watch the video in the Online Lesson.

- A _____ is a ratio between two polynomial expressions in the form $\frac{p}{q}$ where p and q are polynomials and _____.
- Notice that q cannot equal zero. A denominator of zero makes the expression undefined. This is called a _____ or excluded value.
- Therefore, any values that result in a rational expression with a denominator of zero are _____ or restricted from the expression.
- To find the restrictions for a rational expression:
 - 1) _____ the numerator and denominator.
 - 2) Set each expression in the denominator equal to _____.
 - 3) _____. The resulting values are the excluded values, or restrictions.
- Use the _____ symbol to name restrictions on the denominator.

Example 1

▶ Complete the example as you watch the video in the Online Lesson.

Determine the restrictions for the denominator.

$$\frac{20x^2 + 13}{3x^2 - 12}$$

Plan

Factor the denominator

Set each expression in the denominator equal to zero

Solve for the restrictions of the denominator

Implement

$$3x^2 - 12$$

$$3(x^2 - 4)$$

$$3(x - 2)(x + 2)$$

Explain

- ▶ Factor
- ▶ Solve
- ▶ Write the restrictions (or excluded values) for x .

Example 2

▶ Complete the example as you watch the video in the Online Lesson.

State all numbers that make the given expression undefined.

$$\frac{11}{3x^3 - 19x^2 + 6x}$$

Implement**Explain**

- ▶ Factor the denominator
- ▶ Solve each expression in the denominator
- ▶ Write the values that make the expression undefined (in other words, the restrictions or excluded values).

☒ **Checkpoint: Restrictions for Rational Expressions**

Determine the restrictions for the denominator.

$$\frac{5x - 8}{40x^3 + 11x^2 - 2x}$$



To continue, return to the Online Lesson.

Simplifying Rational Expressions with Multiplication and Division

 Fill in the guided notes as you watch the video in the Online Lesson.

- A _____ rational expression has _____ between the numerator and denominator other than 1 or -1.

- To simplify a rational expression:

- 1) _____ the numerator and denominator completely.
- 2) Determine if there are any _____ on the domain.
- 3) _____ common factors out of the expression.

- Rational expressions can have _____ that are monomials, binomials, trinomials, etc.

- To simplify monomials:

- Use the rules you have already learned to simplify _____.

- In this example, the _____ were simplified, and the _____

$$\frac{\cancel{18}x^9}{\cancel{3}\cancel{4}x^5} = \frac{1x^{9-5}}{3} = \frac{x^4}{3}$$

were subtracted.

- To simplify polynomials:

- The numerator and denominator must be _____ in order to divide out a polynomial from the rational expression.

- In this example, once factored, only the _____ can be simplified out of the expression.

$$\frac{(x-3)\cancel{(x+7)}}{(2x-3)\cancel{(x+7)}} = \frac{x-3}{2x-3}$$

- It may be helpful to classify expressions by _____ and _____ verbally to help determine if they are identical.

Example 3

▶ Complete the example as you watch the video in the Online Lesson.

Simplify. State the restrictions on the denominator.

$$\frac{2x(x+3)(3x+11)}{8x^2(2x-5)(x+3)}$$

Plan

Solve for the restrictions on the denominator

Simplify the rational expression

Implement

$$\frac{\cancel{2x}(x+3)(3x+11)}{\cancel{2x}(4x)(2x-5)(x+3)}$$

$$\frac{(\cancel{x+3})(3x+11)}{4x(2x-5)(\cancel{x+3})}$$

$$\frac{3x+11}{4x(2x-5)},$$

Explain

- ▶ Solve for the x -values in the denominator to name the restrictions
- ▶ Simplify the monomial $\frac{2x}{8x^2}$
- ▶ Simplify *identical* binomials
- ▶ Write the answer with the restrictions



To continue, return to the Online Lesson.

Simplifying Rational Expressions with Multiplication and Division (cont.)

▶ Fill in the guided notes as you watch the video in the Online Lesson.

- When a division symbol is present in a rational expression:
 - Find the _____ of the fraction after the symbol (the divisor).
 - And change the operation to _____.

- Therefore, when rational expressions are _____, state the restrictions for the _____.
- Which means, you must determine all the values that are excluded from the numerator and the denominator of the _____.

Example 4

▶ Complete the example as you watch the video in the Online Lesson.

Simplify the expression. State the restrictions on the denominator.

$$\frac{2x+10}{12x-28} \div \frac{x^2-25}{6x-14}$$

Implement

$$\frac{2(x+5)}{4(3x-7)} \div \frac{(x-5)(x+5)}{2(3x-7)}$$

divisor (numerator and denominator)

$$3x-7=0 \quad x-5=0 \quad x+5=0 \quad 3x-7=0$$

Explain

- ▶ Factor all parts of the expression
- ▶ State the restrictions for first denominator and the *entire* divisor
- ▶ Write the fraction after the division symbol as its reciprocal.
- ▶ Write as one large expression
Simplify like terms and expressions
- ▶ Write answer with the restrictions

$$\frac{2(x+5)}{4(3x-7)}$$

Example 5

▶ Complete the example as you watch the video in the Online Lesson.

Simplify. State when the given expression is undefined.

$$\frac{x^2 + 6x - 16}{2x^2 + 5x - 3} \div \frac{3x^2 - 5x - 2}{4x^2 - 36} \cdot \frac{x^3 + 3x^2 - 10x}{6x^2 + 12x - 90}$$

Implement**Explain**

- ▶ Factor all parts of the expression
- ▶ Factor completely
- ▶ State the restrictions
First and last denominator,
middle expression: numerator
and denominator
- ▶ Write the fraction after
the division symbol as its
reciprocal
- ▶ Write as one large expression
Simplify like terms
and expressions

☒ **Checkpoint: Simplifying Rational Expressions with Multiplication and Division**

Simplify. State when the given expression is undefined.

$$\frac{x^2 - 1}{7x^2 + 5x - 2} \cdot \frac{21x^2 - 6x}{10x^2 - 7x - 3}$$



To continue, return to the Online Lesson.

 Practice 1*Complete problems on a separate sheet of paper.***State the restrictions on the denominator.**

1) $\frac{8x^2}{x^3 + 2x^2 + x}$

2) $\frac{r + 1}{2r^3 + 3r^2 - 2r - 3}$

3) $\frac{4p + 5}{4p^2 - 16}$

4) $\frac{x + 3}{x^4 - 81}$

Simplify. State the restrictions on the denominator.

5) $\frac{6x^2 + x - 1}{2x^2 - 13x + 6} \cdot \frac{16x^2 + 40x + 25}{12x^2 + 11x - 5}$

6) $\frac{x^2 + 7x + 12}{x^2 - x - 12} \div \frac{x^2 + 3x + 2}{x^2 - 9}$

Simplify.

7) $\frac{2z^2 + 13z + 15}{z^2 + 4z + 4} \cdot \frac{2z^2 + z - 6}{z^2 - 25}$

8) $\frac{b^2 - 9b + 20}{4b^2 + 6b} \div \frac{2b^2 - 32}{4b^2 + 12b + 9} \cdot \frac{8b + 8}{b^2 - 3b - 4}$

9) $\frac{g^4 - 13g^2 + 36}{g^2 - 10g + 25} \div \frac{g^2 - 5g + 6}{g^2 - g - 30} \cdot \frac{g^2 - 25}{g^2 + 7g + 10}$

10) $\frac{7x}{x^2 - 4x - 21} \div \frac{x^3 - 49x}{x^2 + 9x + 18}$

11) $\frac{14c^4 - c^3 - 3c^2}{9c^3 - 25c} \cdot \frac{9c^2 - 9c - 10}{49c^2 + 42c + 9}$

12) $\frac{4y^2 + 4y - 35}{3y^2 - 20y - 7} \cdot \frac{y^2 - 11y + 28}{6y^2 + 19y - 7} \div \frac{y^2 - 16}{9y^2 - 1}$

Find the missing side.

- 13) The area of a triangle is $\frac{2x + 1}{5x + 2} \text{ m}^2$ and the area of a rectangle is $\frac{4x^2 + 2x}{5} \text{ m}^2$. Find the probability of the triangle. $P(\text{triangle}) = \frac{\text{area of triangle}}{\text{area of rectangle}}$ What is the probability of the triangle if $x = 1$?

- 14) Find the height of a trapezoid when the sum of the bases is $\frac{4x + 5}{x^2 + 9x - 10}$ inches and the area is $\frac{12x^2 + 19x + 5}{x^2 - 100}$ inches². Then explain if $x = 1$ is a possible solution.

- 15) All numbers that make the denominator of a rational expression equal to _____ must be excluded as possible values.
- 16) A rational expression can also be described as the _____ of two polynomials, since the fraction bar represents a division problem.

**To continue, return to the Online Lesson.**

 Mastery Check **Show What You Know**

The surface area and volume of a cylinder are used in the efficiency ratio $\frac{SA}{V}$.

$$SA = 2\pi rh + 2\pi r^2 \quad V = \pi r^2 h$$

A) Simplify the efficiency ratio for a cylinder.

B) The restriction for the radius r is $x \neq \frac{1}{2}$.

The restriction for the height h of the cylinder is $x = 3$.

Write a binomial expression for the radius and height of the cylinder.

C) Use part A and B to write a simplified expression for the efficiency ratio $\frac{SA}{V}$.

 Say What You Know

In your own words, talk about what you have learned using the objectives for this part of the lesson and your work on this page.



To continue, return to the Online Lesson.

 Practice 2*Complete problems on a separate sheet of paper.***Simplify. State when the given expression is undefined.**

1) $\frac{3x}{x^2 + 7x + 10} \cdot \frac{x + 2}{x^2 + 5x}$

2) $\frac{b^2 - 9b + 14}{3b + 18} \div \frac{3b^2 - 15b - 72}{6b + 36} \cdot \frac{b^2 + 6b + 9}{b^2 - b - 2}$

3) $\frac{2n - 3}{n^3 + 4n^2 + n + 4} \div \frac{4n^2 - 9}{4n^2 + 22n + 24}$

4) $\frac{k^2 - j^2}{k - j} \div \frac{k^4 - j^4}{k^3 - j^3}$

Simplify.

5) $\frac{\frac{x^2 + 4x - 5}{x^2 - 3x - 18}}{\frac{x^2 + 6x + 5}{x^2 - 4x - 12}}$

6) $\frac{3x^2 + x - 14}{2x^2 + 13x + 20} \cdot \frac{3x^2 - 48}{10x^2 - 24x + 8} \div \frac{18x^2 - 98}{10x^2 + 21x - 10}$

7) $\frac{8h^3 - 1}{6h^2 + 6h} \cdot \frac{6h^4 + 12h^3 + 6h^2}{4h^2 - 1}$

8) $\frac{r^2 + 3r - 10}{r^2 + 6r + 9} \div \frac{r^2 + r - 20}{20r^2 - 11r - 3} \div \frac{5r^2 - 9r - 2}{r^2 - r - 12}$

9) $\frac{4w^2}{w^2 + 8w + 16} \div \frac{16}{5w^2 - 80}$

10) $\frac{\frac{y^2 - 5y + 6}{y^2 + 3y + 2}}{\frac{y^2 - 2y - 3}{y^2 - 4}}$

- 11) Find the length of a rectangle when the width is
- $\frac{x^2 + 2x + 1}{x^2 + 8x + 15}$
- meters and the area is
- $\frac{x^2 - 1}{x^2 + 10x + 25}$
- meters
- ²
- . Explain whether
- $x = 1$
- is a possible solution.

- 12) Find the area of a triangle with base
- $\frac{2x^2 + 5x + 3}{x - 3}$
- feet and the height of
- $\frac{x^2 - x - 6}{x^2 - 1}$
- feet.
-
- Then determine if a length of
- $x = 3$
- is possible. Explain your reasoning.

- 13) When rational expressions are divided, the _____ of the term _____ the division symbol must be written.

- 14) When a rational expression is _____, the denominator has a value of zero.

**To continue, return to the Online Lesson.**

Targeted Review

Complete items on a separate sheet of paper.

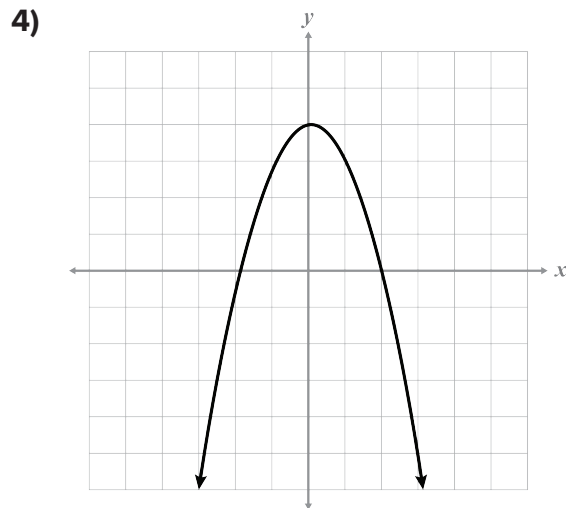
1) Solve: $\frac{4x+1}{6} = \frac{3x-4}{8}$

2) Solve: $\frac{3}{4}\left(3x + \frac{1}{2}\right) = -1$

Name the domain and range.

3)

x	y
-2	1
-1	2
0	4
1	8
2	16



5) Solve: $\frac{x-8}{5} = \frac{2x+5}{10}$

6) Factor: $x^3 + 64$

Divide.

7) $(3x^3 + x^2 + 2x + 10)(x^2 - 2x + 1)^{-1}$

8) $(4x^3 - 14x^2 - 2) \div (2x - 1)$

Multiple Choice

_____ 9) Describe the transformation of the parabola $y = (x - 2)^2 + 5$ as compared to the parent function.

- A) The parabola shifts 2 spaces right and 5 spaces up.
- B) The parabola shifts 2 spaces left and 5 spaces up.
- C) The parabola shifts 2 spaces right and 5 spaces down.
- D) The parabola shifts 2 spaces left and 5 spaces down.

_____ 10) Solve: $\frac{2}{3}n - 5 = \frac{5}{4}$

A) $\frac{8}{75}$

B) $\frac{75}{8}$

C) $-\frac{45}{8}$

D) $\frac{55}{8}$

Use the graph to answer problems 11–12.

11) Select all vertices of the system:

$$y \leq \frac{4}{5}x + 4$$

$$y \geq 4x - 12$$

$$y \geq -\frac{4}{5}x + 4$$

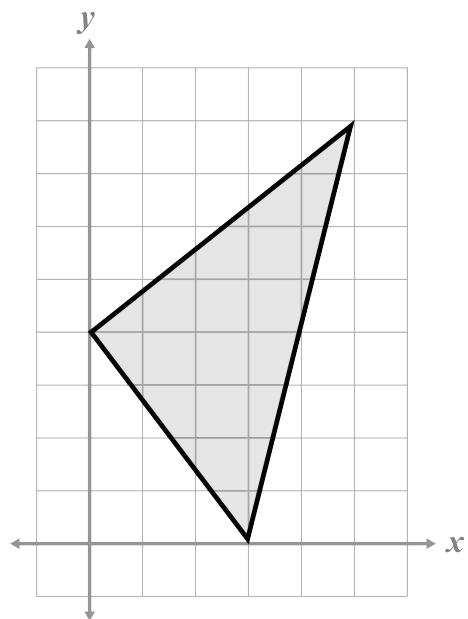
☐ (4, 0)

☐ (3, 0)

☐ (0, 4)

☐ (5, 8)

☐ (8, 5)



12) Using the graph from problem 11, select the minimum and maximum values using the objective function $f(x, y) = x - 2y$.

☐ 4

☐ 3

☐ -8

☐ -11

☐ -2

Problem	1	2	3	4	5	6	7	8	9	10	11	12
Origin	A1	A1	A1	A1	A1	L03	L05	L06	A1	A1	L01	L01

L = Lesson in this level, A1 = Algebra 1: Principles of Secondary Mathematics, FD = Foundational Knowledge



To continue, return to the Online Lesson.