Lesson 5

NAME:

Polynomial Long Division

Start by navigating to the Online Lesson for instructions.

Objectives

- Simplify expressions by dividing a polynomial by a monomial.
- Use long division to divide polynomials.
- Write the remainder of a polynomial expression as a rational expression.

Why?

Polynomial long division is useful when a polynomial needs to be broken down into smaller parts. It is also used in some forms of computer programming, such as error-control coding and cyclic redundancy check (CRC) methods.

Warm Up

Simplify using long division. Write remainders as a fraction.

1) $11\overline{)13672}$

2) $12\overline{)2532}$

Find the GCF of each column and row. Then write the expression as the product of binomial factors and a quadratic trinomial.

3)

	$3x^2$	$12x$
	$2x$	8

4)

	$8x^2$	$40x$
	$-x$	-5

To continue, return to the Online Lesson.

Explore

Dividing by a Monomial

Fill in the notes as you watch the video in the Online Lesson.

- Division can be written in a number of ways.
- For example, "Divide $Ax^2y^2 + Bxy + Cy$ by Axy" can be written symbolically as any of these options seen here:

$(Ax^2y^2 + Bxy + Cy)(Axy)^{-1}$

$(Ax^2y^2 + Bxy + Cy) \div (Axy)$

$\dfrac{Ax^2y^2 + Bxy + Cy}{Axy}$

$Axy\overline{\smash{)}Ax^2y^2 + Bxy + Cy}$

- The ______________________ is the product of the divisor and quotient (plus the remainder, if present).
- A _________________________ is present when the divisor does not go into the dividend evenly.
- Use the _________________________ rules when dividing a polynomial by a monomial.
- Divide every term in the _______________________ (dividend) by the ______________________ (divisor).
- It may be helpful to think of the monomial as the ______________________________________ of each term.
- Simplify the _______________________ like numerical fractions.
- Simplify the _____________________ using the exponent rules.

Example 1

Complete the example as you watch the video in the Online Lesson.

Simplify.

$$\left(9a^4b^2 - 15a^3b^2 + 8a^2b - 6a^2\right)(3ab)^{-1}$$

Implement

$$\frac{9a^4b^2 - 15a^3b^2 + 8a^2b - 6a^2}{3ab}$$

$$\frac{9a^4b^2}{3ab} - \frac{15a^3b^2}{3ab} + \frac{8a^2b}{3ab} - \frac{6a^2}{3ab}$$

Explain

- Rewrite expression as a fraction
- Rewrite the expression using the monomial as the LCD
- Simplify each term (fractional coefficients and exponent rules)

Example 2

Complete the example as you watch the video in the Online Lesson.

Simplify.

$$\frac{5x^3y^2z + 6x^2yz - xyz}{xyz}$$

Implement

Explain

- Rewrite the expression using the monomial as the LCD
- Simplify each term

Checkpoint: Dividing by a Monomial

Simplify.

$$\left(36a^3c^3 + 9a^2c^2 - 16ac\right) \div (8ac)$$

To continue, return to the Online Lesson.

Polynomial Long Division to Factor (No Remainders)

Fill in the notes as you watch the video in the Online Lesson.

- Use polynomial long division when the ________________ is a linear binomial (or higher degree and greater number of terms than a linear binomial).

$$\begin{array}{r} \underline{\qquad\qquad\qquad}, Q(x) + \underline{\qquad\qquad\qquad} \\ \underline{\qquad\qquad\qquad} \overline{\Big) \underline{\qquad\qquad\qquad}, P(x)} \end{array}$$

- ____________________ the term with the ________________ degree out of the expression.
- Write the dividend and divisor in ____________________ form.
- Polynomial expressions in standard form are written with the exponents (degree) in ________________________ order.
- When the quotient has a remainder of zero, this means that the divisor is a ______________ of the polynomial (dividend). Or, (divisor)(quotient) = dividend.

Another use for long division is determining polynomial factors when there is no remainder.

Example 3

Complete the example as you watch the video in the Online Lesson.

Divide $7x^2 - 38x - 24$ by $x - 6$.

Plan

Write the expression with the long division symbol
Simplify from largest to smaller degree terms using the divisor
Write the quotient adhering to place-value by degree

Implement

$$\begin{array}{r|l} & \quad\quad\quad 7x \\ \hline x-6 & 7x^2 - 38x - 24 \\ & -\left(7x^2 - 42x\right) \\ \hline & \quad\quad\quad 4x - 24 \end{array}$$

Explain

- First, find the value that can be used to eliminate $7x^2$ from the dividend
  Place $7x$ over the linear term in the dividend
- Subtract all terms
- Next, eliminate $4x$
  Place $+4$ over the constant in the dividend
- Subtract all terms

This solution has no remainder. The quotient is ____________________ .

This is correct because the product of $(x-6)(7x+4) = 7x^2 - 38x - 24$.

Example 4

Complete the example as you watch the video in the Online Lesson.

Simplify.

$\left(6x^3 + 19x^2 + 7x - 12\right)\left(3x^2 + 5x - 4\right)^{-1}$

$$3x^2 + 5x - 4 \overline{\big)\, 6x^3 + 19x^2 + 7x - 12}$$

Example 5

Complete the example as you watch the video in the Online Lesson.

Simplify.

$$4x-3\overline{\smash{)}20x^2-59x+33}$$

Example 6

Complete the example as you watch the video in the Online Lesson.

Simplify.

$$\left(2x^3-11x^2+13x-\frac{3}{2}\right)\div\left(2x^2-8x+1\right)$$

Checkpoint: Polynomial Long Division to Factor (No Remainders)

Simplify.

$\left(99x^2 - 17x - 40\right) \div (9x + 5)$

To continue, return to the Online Lesson.

Polynomial Long Division with Remainders

Fill in the notes as you watch the video in the Online Lesson.

- When a solution has a remainder, write it as the fraction:
- Add the remainder to the ________________ for the complete solution.

The remainder is a rational expression.

- Some polynomials in standard form have a _______________________ degree.
- For example, $x^2 - 1$, is missing the _______________________________ term.
- It must be rewritten as $x^2 + 0x - 1$ using the _____________________________________.
- Recall that all ________________ terms (numbers) are ____ degree terms since $x^0 = 1$.
- If any degree term is "missing," write it as ________ where n is the missing degree.
- Use the same long division process for finding the quotient once you write all of the terms in _______________________ order.

Example 7

Complete the example as you watch the video in the Online Lesson.

Simplify.

$$\frac{2x^3 - x^2 + x + 1}{x + 1}$$

Plan

Write the expressions with the long division symbol
Simplify from largest to smaller degree terms using the divisor
Write the quotient adhering to place-value by degree
Write remainder as a rational expression

Example 8

Complete the example as you watch the video in the Online Lesson.

Simplify.

$$\left(8x^3 - 4x - \frac{11}{8}\right) \div (4x - 3)$$

Checkpoint: Polynomial Long Division with Remainders

Simplify.

$$\left(3p^3 - 8p + 14\right) \div (p - 6)$$

To continue, return to the Online Lesson.

Practice 1

Complete problems on a separate sheet of paper.

Simplify. Write the polynomial expression with positive exponents.

1) $\dfrac{20a^3b^2 - 15ab^3 + 10a^2b}{5ab}$

2) $\left(13x^4y^5 + 39x^2 - 26\right) \div 13x^2y$

3) $\dfrac{2pq^4 + 12p^2q^2 - 9p^3q + 8pq}{-3p^3q}$

4) $\left(4x^4 - 5x^3 + 8x^2 - 6x + 2\right)(4x)^{-1}$

Simplify. Write the remainder as a fraction if one exists.

5) $\left(3y^3 + 17y^2 + 22y + 8\right) \div (y + 4)$

6) $x^2 - 3x + 1\overline{\smash{\big)}2x^3 - 3x^2 - 7x + 3}$

7) $x - 1\overline{\smash{\big)}3x^2 + 2x + 1}$

8) $\left(x^4 - 3x^2 + x - 5\right)(x + 1)^{-1}$

9) $2y - 1\overline{\smash{\big)}4y^2 - 8y + 3}$

10) $\dfrac{3x^3 + 2x^2 - 8}{x + 2}$

11) $\dfrac{3x^4 + 2x^2 + 16x + 11}{x^2 + 2x + 1}$

12) $\left(2x^3 + 13x^2 - x - 110\right)\left(x - \dfrac{5}{2}\right)^{-1}$

13) $\dfrac{5a^3 - 30a^2 + 70}{5a}$

14) $\left(5y^4 + 3y^3 + 8\right) \div (y + 2)$

15) The volume of a rectangular prism is $2x^3 - 4x^2 - 16x + 42\ \text{cm}^3$. The area of the base is $2x^2 - 10x + 14\ \text{cm}^2$. Find the height.

16) The area of a triangle is $x^2 + 8x + 7\ \ m^2$. The height is $x + 1$ meters. Determine the length of the base.

To continue, return to the Online Lesson.

Mastery Check

Show What You Know

A) A student completed the following problem. They know that the solution contains an error because the check does not result in the original problem. Find the correct solution. Prove your solution is correct by checking your work.

$$\begin{array}{rl}
 & 3x^2 + 11x + 22 + \dfrac{51}{x+2} \\
x+2 & \overline{\big)\ 3x^3 + 5x^2 + 0x + 7} \\
 & \underline{-\left(3x^3 + 6x^2\right)} \\
 & \qquad 11x^2 + 0x \\
 & \qquad \underline{-\left(11x + 22x\right)} \\
 & \qquad\qquad 22x + 7 \\
 & \qquad\qquad \underline{-\left(22x + 44\right)} \\
 & \qquad\qquad\qquad 51
\end{array}$$

B) Explain the error in the given work from part A.

C) Another student found when $12x^4y^3 - 6x^3y^2 - 5xy + 3$ is divided by $3xy$ the quotient is $4x^3y^2 - 2x^2y$. Find the correct solution. Show your work.

Say What You Know

In your own words, talk about what you have learned using the objectives for this part of the lesson and your work on this page.

To continue, return to the Online Lesson.

Practice 2

Complete problems on a separate sheet of paper.

Simplify. Write the polynomial expression with positive exponents. Write the remainder as a fraction if one exists.

1) $(x^4 - 8x^3 + 15x^2 - 23x - 9)(x+3)^{-1}$

2) $\dfrac{36x^2y^3 + 12xy^2 - 24xy}{-9xy}$

3) $\dfrac{3x^3 + 5x^2 - 2x + 1}{x - 1}$

4) $(2y^3 - 6y - 4) \div (y + 1)$

5) $\dfrac{y^4 - 3y^3 + 3y^2 + 72y - 22}{y^2 + 3y - 1}$

6) $(14a^4b^3 - 28a^2b^2 + 35ab)(7a^2b)^{-1}$

7) $(5r^3w + 9r^2w^2 - 3rw^3) \div 45r$

8) $5x - 1\overline{)5x^3 + 14x^2 - 53x + 14}$

9) $\dfrac{3x^4 + 17x^3 - 12x^2 - 19x + 7}{3x^2 + 2x - 1}$

10) $(2x^4 - 3x^3 + x - 2) \div (x - 2)$

11) $(27y^3 - 8)(3y - 2)^{-1}$

12) $(10g^5h^4 - 12g^4h^3 + 15g^3h + 5gh^2) \div (-15g^2h)$

13) Determine the height of the pyramid when the volume is $6x^3 + 19x^2 - 11x - 14$ cubic inches and an area of the base is $2x^2 + 5x - 7$ square inches.

14) The area of a rectangle is $5x^2 + 13x + 6$ square feet. Find the width of the rectangle if the length is $5x + 3$ feet.

To continue, return to the Online Lesson.

Targeted Review

Complete problems on a separate sheet of paper.

1) What is an objective function when working with an optimization problem?

2) Determine the least common multiple (LCM) for $7x^2$, $14xy$, $2z$.

Simplify.

3) $\dfrac{\left(\dfrac{5}{3}-\dfrac{1}{2}\right)}{2}$

4) $\left(2\dfrac{3}{5}\right)\div\left(3\dfrac{1}{2}\right)$

5) Factor completely: $4\left(x^2-1\right)+z^2\left(1-x^2\right)$

6) Simplify: $(x-3)(x-5)-\left(x^2-2x-3\right)$

7) Bixby's Bead Shop placed three orders for black, white, and purple beads. In September, 30 black, 50 white, and 80 purple beads were purchased for \$460. In October, 80 black and 20 white beads were purchased for \$260. In November, \$166 was spent on 22 white and 36 purple beads. Write a system with three variables. Do not solve.

8) Determine the value of Q that will make the equation a polynomial identity.
$(Qx-3)^2=(2x-1)^2-8(x-1)$

Multiple Choice

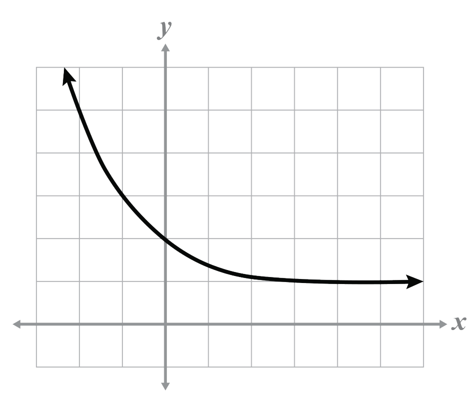
_____ **9)** Determine the *range* of the function when the domain is all real numbers.

A) all real numbers

B) $y \le 1$

C) $y \ge 0$

D) $y \ge 1$



_____ **10)** Determine the value of $(y + z)$ for the system: $\begin{aligned} 2x - 3y - 3z &= 22 \\ 2x + y + z &= 14 \end{aligned}$

A) 16

B) 8

C) 2

D) -2

_____ **11)** Determine the expression that when set equal to $(ax)^3 - (by)^3$ would form a polynomial identity.

A) $(ax - by)\left((ax)^2 - abxy - (by)^2\right)$

B) $(ax - by)\left((ax)^2 + abxy + (by)^2\right)$

C) $(ax - by)\left(ax^2 + abxy + by^2\right)$

D) $(ax - by)\left((ax)^2 + 2abxy + (by)^2\right)$

_____ **12)** Select the word that best represents the polynomial.
An expression with three terms with 2 as the highest degree

A) linear binomial

B) linear trinomial

C) quadratic trinomial

D) binomial trinomial

Problem	1	2	3	4	5	6	7	8	9	10	11	12
Origin	L01	FD	FD	FD	L03	L03	L02	L04	A1	L02	L04	A1

L = Lesson in this level, A1 = Algebra 1: Principles of Secondary Mathematics, FD = Foundational Knowledge

To continue, return to the Online Lesson.

