

# Lesson 4

## Polynomial Identities

NAME:



Start by navigating to the Online Lesson for instructions.

### Objectives

- ✓ Determine if a polynomial identity exists.
- ✓ Determine the value of an unknown to make a polynomial expression or equation true.

### Why?

Being able to prove that both sides of an equation are equal strengthens mathematical understanding and use of the algebraic properties.

### Warm Up

1) Multiply.  
 $(2x - 3)^2$

2) Multiply.  
 $(4x - 3)(x - 3)$

3) Solve.

$$7x - \frac{1}{2}(2x - 6) = 6x + 3$$



To continue, return to the Online Lesson.

### Explore

#### Polynomial Identities

Fill in the notes as you watch the video in the Online Lesson.

- An \_\_\_\_\_ is an equation that will be true for any value of variable(s) in the equation.
- This means that the \_\_\_\_\_ will be true for \_\_\_\_\_ that replaces the variable.

- Recognizing a polynomial identity can help simplify \_\_\_\_\_ of an equation.
- This is because when the identity exists, the sides of the equation are \_\_\_\_\_ because they are equal.
- Some polynomial identities that you are already familiar with are:
  - Difference of two squares: \_\_\_\_\_
  - Perfect square trinomials:
    - \_\_\_\_\_
    - \_\_\_\_\_
  - Sum of cubes: \_\_\_\_\_
  - Difference of cubes: \_\_\_\_\_

**Example 1**

▶ Complete the example as you watch the video in the Online Lesson.

**Determine if a polynomial identity exists. Explain.**

$$(x^2 + y^2)^2 = (x^2 - y^2)^2 + (2xy)^2$$

**Implement**

**Explain**

Left side

Right side

$$(x^2 + y^2)^2$$

$$(x^2 - y^2)^2 + (2xy)^2$$

$$(x^2 + y^2)(x^2 + y^2)$$

$$x^4 + x^2y^2 + x^2y^2 + y^4$$

$$x^4 + 2x^2y^2 + y^4$$

- ▶ Write each side of the problem
- ▶ Expand expressions
- ▶ Distribute
- ▶ Combine like terms

When written in the same form, the left and right sides of the equation have identical terms, which means \_\_\_\_\_.

**Example 2**

▶ Complete the example as you watch the video in the Online Lesson.

**Determine if a polynomial identity exists. Explain.**

$$(x-5)^2 = (x-4)(x+4) + (x-3)(x+3)$$

Left side

$$(x-5)^2$$

Right side

$$(x-4)(x+4) + (x-3)(x+3)$$

**Example 3**

▶ Complete the example as you watch the video in the Online Lesson.

**Determine if a polynomial identity exists. Explain.**

$$(a-b)^3 = a^3 - b^3 - 3ab(a-b)$$

Left side

$$(a-b)^3$$

Right side

$$a^3 - b^3 - 3ab(a-b)$$

**Checkpoint: Polynomial Identities**


Determine if a polynomial identity exists. Explain.

$$(4x)^2 + 24x + 9 = (4x + 3)^2$$



To continue, return to the Online Lesson.

### Finding Unknowns in Polynomial Equations

 *Fill in the notes as you watch the video in the Online Lesson.*

- Finding an \_\_\_\_\_ coefficient or constant in a polynomial equation combines the operations with polynomials and solving equations.
- When comparing like terms in an equation, it is important to look at \_\_\_\_\_ of the equation.
- To check your answer, \_\_\_\_\_ the values back into the given equation. Then make sure both sides of the equation are \_\_\_\_\_.

**Example 4**

▶ Complete the example as you watch the video in the Online Lesson.

**Find the missing values,  $M$  and  $N$ .**

$$(2x + M)(Nx - 1) = 8x^2 + 10x - 3$$

**Implement**

$$2Nx^2 - 2x + MNx - M = 8x^2 + 10x - 3$$

**Explain**

- ▶ Distribute on the left side of the equation
- ▶ Solve for  $N$  using 2nd degree terms
- ▶ Solve for  $M$  using constant terms
- ▶ Substitute  $M$  and  $N$  into the given problem
- ▶ Check using mental math

**Example 5**

▶ Complete the example as you watch the video in the Online Lesson.

**Find the missing value,  $P$ .**

$$(x^2 - Px + 7) - (6x^2 - 5x + 3) = -5x^2 - 3x + 4$$

$$x^2 - Px + 7 - 6x^2 + 5x - 3 = -5x^2 - 3x + 4$$

**Checkpoint: Finding Unknowns in Polynomial Equations**

Find the missing value when  $P$  is a whole number.

$$(Px - 2)(8x + P) = 24x^2 - 7x - 6$$



To continue, return to the Online Lesson.

 Practice 1

Complete problems on a separate sheet of paper.

Determine if the given expressions will form an identity.

1)  $(a + b)^3$  and  $a(a^2 + ab) + 2ab(a + b) + b^2(a + b)$

Left side:  $(a + b)^3$

Right side:  $a(a^2 + ab) + 2ab(a + b) + b^2(a + b)$

2)  $-6x^2 - 2(25x + 28)$  and  $3x(-2x - 14) - 8(x + 7)$

3)  $\left(\frac{1}{2}n - 7\right)(n + 8)$  and  $\frac{1}{2}n(n + 26) - 56(n + 1)$

4)  $(3g)^2 + 10(3g + 2) + 1$  and  $(3g)^2 + 10(3g + 3) - 9$

5)  $x(5x + 8)(5x - 8)$  and  $x(5x)^2 - 8^2$

6)  $(2ab)^2 + 4(5ab) + (5ab)^2$  and  $4ab(ab + 4) + ab(25ab + 4)$

Find the missing value(s) in the given equation.

7)  $(Rx + 4)(2x - R) = 6x^2 - x - 12$

8)  $(Bx^2 - Px + 8) + (4x^2 + 1x - 15) = 10x^2 - 14x - 7$

9)  $3x(Wx + Q) + 8(Wx + Q) = 12x^2 + 35x + 8$

10)  $(Ax + 3)^2 = 25x^2 + 30x + 9$

11)  $(5x^2 - Gx + 1) - (9x^2 - Mx + 2) = -Gx^2 + 3x - 1$

12)  $(Rx + K)(Rx - K) = 36x^2 - 81$ ; Assume  $R$  and  $K$  are whole numbers.



To continue, return to the Online Lesson.

 **Mastery Check**
 **Show What You Know**

- A)** A student was asked to show their work to prove the equation represented a polynomial identity. Find their error and correct it to prove the identity exists. Indicate the line(s) in which the error occurs.

$$x^4 - y^4 = (x^2 + y^2)(x + y)(x - y)$$

Simplifying the right side

$$(x^2 + y^2)(x + y)(x - y)$$

$$x^3 + x^2y + x^3 - x^2y$$

$$\underline{\hspace{2cm}} + xy^2 + y^3 + xy^2 - y^3$$

$$2x^3 + 2xy^2$$

$$x^4 - y^4 \neq 2x^3 + 2xy^2$$

Student Response

I distributed the 1st binomial across the 2nd and 3rd binomial.

This is NOT an identity because the left and right sides of the equation are NOT equal.

- B)** Determine the non-zero value of  $Q$  that will form a polynomial identity. Then rewrite the polynomial using the value you found for  $Q$ .

$$(Qx + 1)^2 = Qx^2 + (Qx + 2)(5x + 1) - 4x - 1$$

 **Say What You Know**

In your own words, talk about what you have learned using the objectives for this part of the lesson and your work on this page.



**To continue, return to the Online Lesson.**

 **Practice 2**

Complete problems on a separate sheet of paper.

**Determine if the given expressions will form an identity.**

- 1)  $(2z + 1)^2 - 2(z - 1)$  and  $2z(2z + 1) - 1$
- 2)  $(x + 2)(x + 5) - (x - 7)(x - 3)$  and  $(x - 1)((x + 7) - (x - 10)) + 6$
- 3)  $2x^2y^2 - 4x^2 - 9y^2$  and  $x^2(y + 2)(y - 2) + y^2(x + 3)(x - 3)$
- 4)  $a(b + 3) - b(b + 3)$  and  $b(a - b) + 3(a - b)$
- 5)  $(a + 2)^3$  and  $(a + 2)(a^2 - 2a + 4) + 6a(a + 2)$
- 6)  $(x^2 - 4)(x^2 - 9) = (x + 2)(x - 3)(x^2 - x - 6)$

**Find the missing value(s) of the given equation.**

- 7)  $(Bx - 5)(2x - C) = 6x^2 + 11x - 35$
- 8)  $((Ax)^3 + B^3) = (5x + 4)(25x^2 - 20x + 16)$
- 9)  $(Qx^2 + 3x + Y) - (4x^2 - 5x - 1) = 9x^2 + 8x + 7$
- 10)  $(Gx - 4)(x + 5) - B = 3x^2 + 11x + 8$
- 11)  $(Wx + 1)(2x - 5) + (3x - R)(x + 1) = 7x^2 - 7x - 7$
- 12)  $5x^2 + Hx - 2 - (Vx^2 - Hx - 3) = 14x + 1$



To continue, return to the Online Lesson.