

Lesson 3

Operations with Polynomials

NAME:



Start by navigating to the Online Lesson for instructions.

Objectives

- ✓ Determine if an expression represents a polynomial.
- ✓ Add, subtract, and multiply to simplify polynomial expressions.
- ✓ Factor polynomial expressions.
- ✓ Factor the sum and difference of cubes.

Why?

Factoring polynomials is integral to many new Algebra 2 concepts. Factoring is a key skill for finding solutions/roots to polynomials. It is important to practice and master this skill so that you can build new concepts on this solid foundation.



Warm Up

- 1) Define like terms in your own words.
- 2) Explain the difference between whole numbers and integers.



To continue, return to the Online Lesson.

Explore

Polynomials

▶ Fill in the notes as you watch the video in the Online Lesson.

- A _____ expression is a group of one or more terms that follow these rules:
 - The variables must have _____ that are _____ numbers, and:

- The variables cannot have _____ exponents.
- The variables cannot have _____ exponents.
- The variables cannot be in the _____.
- The variables must have no _____ or _____ symbols.

Example 1

🎥 Complete the example as you watch the video in the Online Lesson.

Determine if the expression is a polynomial. Explain.

A) $\frac{1}{3}x^3 - xy + 4$

B) $4x^{-5} + \frac{1}{x}$

C) $5\sqrt{x} + x^2y$

A) _____ a polynomial because all the variables have exponents that are whole numbers.

B) _____ a polynomial because the first term has a negative exponent and the second term has a variable in the denominator.

C) No, this is NOT a polynomial because _____
_____.

Polynomials have _____.

Another way to write a square root is to the _____.



To continue, return to the Online Lesson.

 Polynomials (cont.)

🎥 Fill in the notes as you watch the video in the Online Lesson.

- If an expression, written in _____ form, is a polynomial, then it can be classified by the _____ and number of _____.
- The degree of a polynomial is the largest _____ when there is a _____ variable, or the largest _____ of the exponents of the variables in a _____ polynomial.
- A multivariate polynomial is a polynomial with _____ variables.
- With multivariate polynomials, _____ the exponents to find the largest sum even though the bases are not the same.

Degree	Classification
0	
1	
2	
3	
4	
5	
n	

Number of Terms	Classification
	monomial
	binomial
	trinomial
	polynomial with n -terms

Example 2

▶ Complete the example as you watch the video in the Online Lesson.

Classify the polynomial expression by the degree and number of terms.

A) $x^4 + 7x^3 - 4x + 15$

Expression A is a 4th degree, or quartic, polynomial with 4 terms.

B) $2x^3y^2 - 3x^2y - 10xy + 2xy$

$$2x^3y^2 - 3x^2y - 8xy$$

$$\text{1st term exp: } 3 + 2 = 5$$

$$\text{2nd term: } 2 + 1 = 3$$

$$\text{3rd term: } 1 + 1 = 2$$

C) $8a^2b^5 + 3a^5b^4 + a^3b^3$

Example 3

▶ Complete the example as you watch the video in the Online Lesson.

Determine whether each expression is a polynomial. If it is a polynomial, state the degree of the polynomial.

A) $|3x^2| - 2xy$

B) $\frac{2}{3}a^4bc^4$

C) $\sqrt{4}x^8y^3z - 3yz^3 + \frac{1}{2}xy$

☒ **Checkpoint: Polynomials**

Explain if the given expressions are polynomials. If it is a polynomial, name the degree.

A) $14x^{-3}y + 2xy - 9$

B) $7p^2q^2 - 4p^4q^2 - 6p^5q^2$



To continue, return to the Online Lesson.

Operations with Polynomial Expressions

🎥 Fill in the notes as you watch the video in the Online Lesson.

- When working with polynomial expressions in Algebra 2, the directions will be to _____, but this can mean different things depending on the given problem.

Example 4

🎥 Complete the example as you watch the video in the Online Lesson.

Simplify.

$$(12x^3 - 8x^2 + 5x) - (-7x^2 + x + 4)$$

Implement

Explain

- ▶ Distribute -1 across the second expression to change subtraction to adding the opposite
- ▶ Combine like terms

Example 5

🎥 Complete the example as you watch the video in the Online Lesson.

Simplify.

$$(5x - 1)(2x^2 + 3x + 6)$$

Implement

Explain

- ▶ Distribute $5x$ across all terms in the 2nd expression
- ▶ Distribute -1 across all terms in the 2nd expression
- ▶ Combine like terms

Example 6

▶ Complete the example as you watch the video in the Online Lesson.

Simplify.

$$(10x - 3)(2x + 5) - 4x^2 + 16x$$

Implement**Explain**

- ▶ Distribute
- ▶ Combine like terms in standard form

☒ **Checkpoint: Operations with Polynomial Expressions****Simplify.**

$$(9x + 5)(x - 1) + (x^2 + 2)(x - 2)$$

**To continue, return to the Online Lesson.** **Factoring Polynomials Completely**

▶ Fill in the notes as you watch the video in the Online Lesson.

- To factor _____ means that you cannot factor the expression any further.
- To factor completely, _____ multiple methods of factoring in the following order:
 - 1) Find the _____ (other than 1).
 - 2) Factor by _____ (when given 4 terms).
 - 3) Analyze the _____ patterns.

- 4) Factor _____ products (ex. difference of two squares, perfect square trinomials).
- 5) Factor using your _____ factoring method (ex. ac-grouping, modeling, or mental math).
- Some expressions cannot be _____. When this occurs, answer “cannot be _____.”
 - To check if you have factored an expression correctly, _____ the product of terms back together to see if the given expression results.

Example 7

▶ Complete the example as you watch the video in the Online Lesson.

Factor completely.

$$8x^2 - 6x - 44$$

Implement

$$2(4x^2 - 3x - 22)$$

$$2(\quad + \quad)(\quad - \quad)$$

Explain

- ▶ Factor out the GCF
- ▶ Use sign patterns to factor

Example 8

▶ Complete the example as you watch the video in the Online Lesson.

Factor completely.

$$4x^4 - 36x^2 - x^2y^2 + 9y^2$$

Implement

$$(4x^4 - 36x^2) + (-x^2y^2 + 9y^2)$$

$$4x^2(x^2 - 9) - y^2(x^2 - 9)$$

Explain

- ▶ Group terms
- ▶ Factor out the GCF from each group of terms
- ▶ Regroup terms
- ▶ Factor difference of two squares

☒ **Checkpoint: Factoring Polynomials Completely**

Factor completely.

$$45x^2 - 33x - 36$$



To continue, return to the Online Lesson.

 The Sum and Difference of Cubes

 Fill in the notes as you watch the video in the Online Lesson.

- Factoring _____ occur for the sum and difference of cubes.
- Perfect _____ occur when the base is multiplied by itself three times.

$$4^3 = 64 \quad (2x)^3 = 2^3 x^3 = 8x^3$$

- Continue to look for the _____ as the first step of factoring the sum and difference of cubes.

Sum of Cubes

$$a^3 + b^3 = (a + b)(a^2 - ab + b^2)$$

Difference of Cubes

$$a^3 - b^3 = (a - b)(a^2 + ab + b^2)$$

Example 9

▶ Complete the example as you watch the video in the Online Lesson.

Factor completely.

$$8x^3 + 125y^3$$

Implement

$$(2x)^3 = 8x^3, (5y)^3 = 125y^3$$

$$(2x + 5y)((2x)^2 - (2x \cdot 5y) + (5y)^2)$$

Explore

- ▶ Find the cubed root of the terms
- ▶ Substitute values into the sum of cubes rule
- ▶ Simplify

Example 10

▶ Complete the example as you watch the video in the Online Lesson.

Factor completely.

$$10m^3 - 640$$

Implement

Explore

- ▶ Factor out the GCF
- ▶ Find the cubed root of 64
- ▶ Find the difference of cubes

☒ **Checkpoint: The Sum and Difference of Cubes**

Factor completely.

$$81a^3 + 24b^3$$



To continue, return to the Online Lesson.

**Practice 1***Complete problems on a separate sheet of paper.***Determine if the following are polynomials. If so, classify by degree and number of terms.**

1) $3x^2y + 2xy - 4$

2) $x^4y^{-2} + 5x + 1$

3) $2x^2 + 4x - \sqrt{xy}$

4) $x^5 + 5x^4 + 10x^3 + 10x^2 + 5x + 1$

Simplify. Write answers in standard form.

5) $x^2(xy + 7) - (x + 1)(x - 1)$

6) $(ab + 2)(ab - 3) - ab + 4$

7) $(g + 5)(g^2 - 3g + 2)$

8) $2xy(7x^2y - 2) - 3x(x^2y^2 + 5)$

9) $(k + 2)(k - 2) + (k + 3)(k - 3)$

10) $(2r + 1)(r^2 + 4r - 2)$

Factor completely.

11) $3x^2y^2 - 5xy - 12$

12) $4a^2b^2 - 4a^2 - 9b^2 + 9$

13) $5m^2 + 70m + 245$

14) $4x^2 - 26x + 30$

15) $35q^4 - 8q^3 - 3q^2$

16) $2xy^2 + 8x - 9y^2 - 36$

Factor the sum or difference of cubes completely.

17) $8y^3 - 1$

18) $27c^3 + 64$

19) $250v^3 + 16$

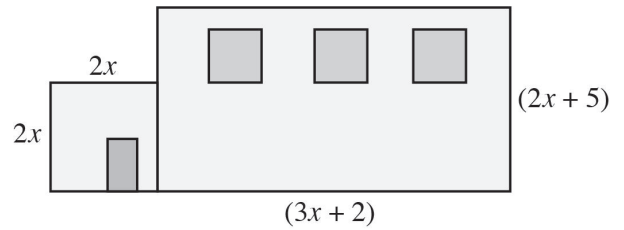
20) $x^3y^3 - a^3b^3$

**To continue, return to the Online Lesson.**

Mastery Check

Show What You Know

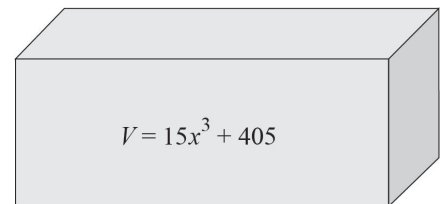
Wallies Warehouse needs to refinish the front of their building. The total surface area is needed for the project. The surface area of just the door is $2x$ square meters and the area of each window is x^2 square meters.



All lengths are in meters

- A)** What is the total area of the front of the building, *excluding* the door and windows?

The volume of a standard-size box in Wallies Warehouse is given in cubic centimeters.



- B)** Factor to find the dimensions of the box.

Say What You Know

In your own words, talk about what you have learned using the objectives for this part of the lesson and your work on this page.



To continue, return to the Online Lesson.

**Practice 2***Complete problems on a separate sheet of paper.***Determine if the following are polynomials. If so, classify by degree and number of terms.**

1) $|6x| - 7x^3 + 10$

2) $5xy + \sqrt{2x^2 + x^2y^2}$

3) $\frac{1}{3}x^5 - 4x^2$

4) $\frac{3}{4x} - \frac{1}{2}y$

Simplify. Write answers in standard form.

5) $(x^2 - y^2)(x^2 + y^2)$

6) $5x(x + 7) - 3x^2(x - 2)$

7) $(a + 2b)(a^2 - 2ab + 4b^2)$

8) $3(5r^2 + 7rv - 2v^2) - 2(2r^2 - rv)$

9) $(n - 4)(n - 3) - (2n + 1)$

10) $(w + 1)^3$

Factor completely.

11) $9nv + 24n + 12v + 32$

12) $x^4 - 16y^4$

13) $8x^2 - 22x - 21$

14) $12x^3y^2 = 48x^2y^2 - 3x + 12$

15) $200b^2 + 120b + 18$

16) $36x^2y^2 - 15xy - 6$

Factor the sum or difference of cubes completely.

17) $27x^3 - 343$

18) $8v^3 - 125m^3$

19) $2b^3 + 16c^3$

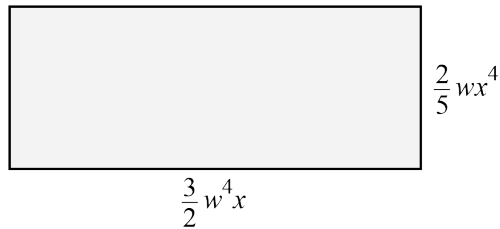
20) $5z^3 + 625$

**To continue, return to the Online Lesson.**

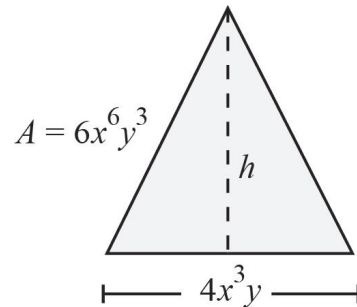
Targeted Review

Complete items on a separate sheet of paper.

- 1) Find the area of the figure.



- 2) Determine the missing side length.



Divide. Show ALL work. Write any remainders as a simplified fraction.

3) $25 \overline{)13572}$

4) $14 \overline{)3128}$

- 5) Graph the system of inequalities. Label the intersection points.

$$y \geq 0$$

$$x \geq 0$$

$$y \leq -\frac{1}{10}x + 30$$

$$y \geq \frac{1}{20}x + 15$$

- 6) Use the objective function to find the minimum and maximum values.

$$f(x, y) = -x + 3y$$

- 7) Braun's Bakery uses three types of flour for their bakery. Three times the number of bags of triple-zero flour purchased is equal to the sum of all-purpose and wheat flour. Sixty-four total bags of flour are purchased weekly. Five times more all-purpose flour is purchased than wheat flour. Write a system of equations.

- 8) Solve the system you wrote in problem 7.

Multiple Choice

_____ 9) Determine the ordered triple that is the solution to the system of equations.

$$4x + 5y - 6z = 2$$

$$3x + 2y - 7z = 15$$

$$-x + 4y + 2z = -13$$

A) $(3, 2, 4)$

B) $(-3, -2, -4)$

C) $(-49, 18, -18)$

D) No solution

_____ 10) The answer when one term is divided by another.

A) dividend

B) divisor

C) quotient

D) remainder

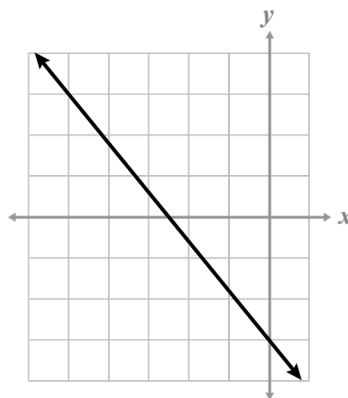
_____ 11) Use the graph to determine the equation in slope-intercept form.

A) $y = -\frac{6}{5}x - 3$

B) $y = -\frac{6}{5}x - 2.5$

C) $y = \frac{6}{5}x - 2.5$

D) $y = \frac{6}{5}x - 3$



_____ 12) Solve the quadratic equation: $(2x - 5)(3x + 2) = 0$

A) $x = -2, 5$

B) $x = -\frac{3}{2}, \frac{2}{5}$

C) $x = -\frac{2}{3}, \frac{5}{2}$

D) $x = -\frac{5}{2}, \frac{2}{3}$

Problem	1	2	3	4	5	6	7	8	9	10	11	12
Origin	A1	A1	D	D	L1	L1	L2	L2	L2	FD	A1	A1

L = Lesson in this level, A1 = Algebra 1: Principles of Secondary Mathematics, D = Delta, FD = Foundational Knowledge



To continue, return to the Online Lesson.