Lesson 2

NAME:

Systems of Equations with Three Variables

Start by navigating to the Online Lesson for instructions.

Objectives

- Determine if given values represent a solution to a system of equations with three variables.
- Solve a system of linear equations with three variables.
- Write a system of linear equations with three variables.
- Evaluate word problems for linear equations with three variables.

Why?

Should you buy a used car with the best gas mileage, the lowest mileage, or the fewest owners? Should you invest in a backpacking tent that is the lightest, the most durable, or the most waterproof? What about computer models? Three-dimensional computer models require three variables to account for length, width, and height of figures. In the real world, many problems have three or more factors to consider to find the best available option.

Warm Up

Solve the systems of linear equations.

1) $a = \frac{1}{2}b$

$7a - 5b = -105$

2) $5x - 3y = 9$

$3x + 3y = 5$

To continue, return to the Online Lesson.

Explore

Solutions to Systems of Equations with Three Variables

Fill in the notes as you watch the video in the Online Lesson.

- Systems of equations with three variables can have _____ solution, _____ solutions, or _____ solutions.
- For systems with one solution, the answer is often written as an ordered _____, (x, y, z).
- For systems with no solution, there is no common _____ among the three equations.
- For systems with an infinite number of solutions, a solution of _____ occurs for a combination of two or more equations.

Example 1

Complete the example as you watch the video in the Online Lesson.

Determine whether the ordered triple $(22.5, -12, 0.5)$ is a solution to the system of equations.

System A

$x + y - z = 10$

$3x - 2y + z = 0$

$x - y + z = 11$

Implement

$(22.5) + (-12) - (0.5) = 10$ ✓

$3(22.5) - 2(-12) + (0.5) = 92$ ✖

$(22.5) - (-12) + (0.5) = 35$ ✖

Explain

The ordered triple did not make all equations true. Move on to the next system.

System B

$x + 2y + z = -1$

$x + 2y + 5z = 1$

$2x + 3y + 4z = 11$

Implement

Explain

☑ Checkpoint: Solutions to Systems of Equations with Three Variables

Determine whether the ordered triple is a solution to the system of equations. Explain. (1.5, 3, –1.5)

$3x - 2y - z = 0$

$x + y - z = 6$

$10x - y = 12$

To continue, return to the Online Lesson.

Solving Systems of Equations with Three Variables

Fill in the notes as you watch the video in the Online Lesson.

- Solving a system of equations with three variables requires at least _____ equations.
- To solve a system with three variables, you need a plan to work first with _____ variables, and then with _____ variable.
- You will often use _____ and _____ together for most problems, because it is not always possible to find the value of a variable in one step.
- With multiple steps to solve, there are more opportunities for errors, so it is important to remember to persevere and _____.

Remember that it is okay to make mistakes and to learn from them!

Example 2

Complete the example as you watch the video in the Online Lesson.

Find the solution that will satisfy all three variables.

$P\colon 3x + y + 2z = 13$ $Q\colon 2x + 2y + z = 16$ $R\colon x + 3y + 3z = 13$

Plan

Eliminate a variable from any two equations
Eliminate the same variable using a different pair of equations
From the two-variable equations, eliminate another variable
Use substitution to solve for the remaining variables
Check answer in three original equations

Eliminate z from P and Q

$$\begin{aligned} 3x + y + 2z &= 13 \\ (-2)(2x + 2y + z = 16) \Rightarrow \quad & \underline{+\ -4x - 4y - 2z = -32} \\ -x - 3y &= -19 \end{aligned}$$

Eliminate z from Q and R

$$\begin{aligned} (-3)(2x + 2y + z = 16) \Rightarrow \quad & -6x - 6y - 3z = -48 \\ x + 3y + 3z = 13 \quad & \underline{+\ \ x + 3y + 3z = \ \ 13} \\ & -5x - 3y = -35 \end{aligned}$$

Eliminate y from the two new equations

$$\begin{aligned} -x - 3y &= -19 \\ (-5x - 3y = -35)(-1) \quad \Rightarrow \quad & \underline{+\ \ 5x + 3y = \ \ 35} \\ 4x &= 16 \\ x &= 4 \end{aligned}$$

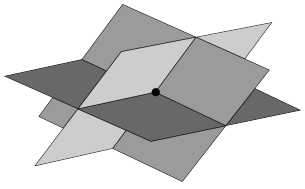
Implement

Explain

- Substitute the x-value into an equation and solve for y
- Use any equation to solve for z
- Substitute all 3 values into each equation. Use a calculator to check that both sides are equal.

Exactly one solution

The planes intersect in a single point.



Example 3

Complete the example as you watch the video in the Online Lesson.

Find the solution that will satisfy all three variables.

$P\colon a-3b+c=4$ $Q\colon 3a-6b+9c=5$ $R\colon 4a-9b+10c=9$

$-3P+Q$

$$(a-3b+c=4)(-3) \Rightarrow \begin{array}{r} -3a+9b-3c=-12 \\ +\ \ 3a-6b+9c=\ \ 5 \\ \hline 3b+6c=-7 \end{array}$$

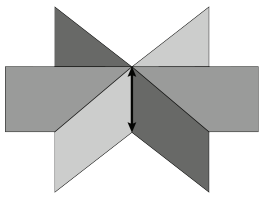
$-4P+R$

$$(a-3b+c=4)(-4) \Rightarrow \begin{array}{r} -4a+12b-\ \ 4c=-16 \\ +\ \ 4a-\ \ 9b+10c=\ \ 9 \\ \hline 3b+6c=-7 \end{array}$$

Since $(-3P+Q)$ and $(-4P+R)$ result in the same equation, there are infinite solutions.

Infinitely many solutions

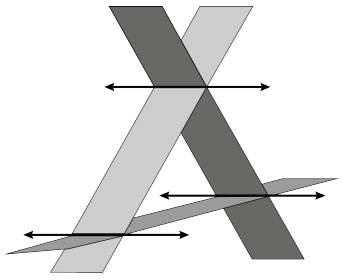
The planes intersect in a line or are the same plane.



Suppose Equation R is replaced with S: $a-3b+c=-9$. How will this change the solution?

$-P+S$

Using equation S, the system has **no solution**.



☑ Checkpoint: Solving Systems of Equations with Three Variables

Find the solution that will satisfy all three variables.

$A: x + y = -1$ $B: x + 2y = 1$ $C: 2x + 3y + 4z = 11$

To continue, return to the Online Lesson.

Word Problems with Three Variables

Fill in the notes as you watch the video in the Online Lesson.

- When working with word problems, as always, first _____ the variables.
- Then _____ a system of equations for a word problem.
- Finally, _____ the problem.

Example 4

Complete the example as you watch the video in the Online Lesson.

Define your variables. Then write and solve the system of equations with three variables.

A catering company purchased shipments of fruits, grains, and vegetables for their next three events. Event A requires one crate of each and the total shipment weighed 41 pounds. Event B requires two crates of grains and two crates of vegetables for a total weight of 56 pounds. Event C requires two crates of vegetables, one crate of grains, and two crates of fruit, weighing 71 pounds. What are the individual weights of each crate?

Plan

Define the variables
Write a system of equations
Solve
Check

Implement

$$(2g + 2v = 56)\left(\frac{1}{2}\right) \Rightarrow g + v = 28$$

$$f + g + v = 41$$
$$f + (28) = 41$$
$$f = 13$$
$$2f + g + 2v = 71$$
$$2(13) + g + v + v = 71$$

Explain

- Multiply equation B by $\frac{1}{2}$
- Solve for f in equation A
- Substitute 28 for $(g + v)$
- Substitute in the value of f and the value of $(g + v)$ in equation C
- Solve for v
- Solve for g using equation A
- Substitute all values into each equation
- Check with a calculator

The fruit crates weigh _____ pounds each, the grain crates weigh _____ pounds each, and the vegetable crates weigh _____ pounds each.

Example 5

Complete the example as you watch the video in the Online Lesson.

Define your variables. Then write and solve the system of equations with three variables.

The average of three numbers is zero. The range for the set of numbers is eight. Three times the middle number, plus the smallest number, minus four times the largest number is –29. What are the three numbers in the data set?

x: smallest

average: $\dfrac{x+y+z}{3}=0$

y: middle

range: $z-x=8$

z: largest

$3y+x-4z=-29$

Implement

$$\frac{x+y+z}{3}=0$$

$$x+y+z=0$$

$$z-x=8$$

$$z=x+8$$

Explain

- Multiplication Property of Equality
- Solve for z in the range equation
- Substitute into average equation
- Substitute into third equation
- Combine equations from the previous two steps
- Solve for x
- Solve for z
- Solve for y

☑ Checkpoint: Word Problems with Three Variables

Define your variables. Then write a system with three variables. Do not solve.

After three tests, Charlie had a test average of 91. The difference between the highest and lowest test score is eight. The sum of the highest and median test minus the lowest is 99.

To continue, return to the Online Lesson.

Practice 1

Complete problems on a separate sheet of paper.

Solve the system of equations.

1) $3x + 6y - 4z = 17$
$-x + 5y + 4z = 11$
$x + y - 5z = 0$

2) $-3x - y - 2z = -13$
$2x + 2y + z = 16$
$x + 3y + 3z = 13$

3) $x + 2y + 3z = 32$
$x - 3y + z = 7$
$-2x + 6y - 2z = 4$

4) $x - 8y + z = 6$
$2x + 7y - z = 11$
$2x - 10y - 3z = -22$

5) Chari has a total of 17 coins in her pocket that are pennies, nickels, and dimes. The value is \$0.87. There are two more dimes than nickels. How many of each type of coin does Chari have?

6) The average of three numbers is 2. The second is three more than the third. The sum of the first and third is −1. Find the three numbers.

To continue, return to the Online Lesson.

Mastery Check

Show What You Know

Clothing Limited has a budget of $\$12{,}000$ to restock shirts, pants, and shoes in the store. There is space for a total of 300 items. Shirts cost $\$35$ to purchase, pants are $\$25$, and shoes are $\$50$. Based on the current inventory, Clothing Limited needs the combined number of pants and shoes to equal the total number of shirts purchased.

A) Define your variables and write a system of equations.

B) Solve the system you created in part A.

C) What if the equations were written as these inequalities:

$$x + y + z \leq 300 \qquad 35x + 25y + 50z \leq 12000 \qquad x \leq y + z$$

Would 150 shirts, 110 pairs of pants, and 40 pairs of shoes be a solution? How would this impact the budget? Show your work and explain your thinking.

Say What You Know

In your own words, talk about what you have learned using the objectives for this part of the lesson and your work on this page.

To continue, return to the Online Lesson.

Practice 2

Complete problems on a separate sheet of paper.

Solve the system of equations.

1) $-2x + 3y + 5z = -7$
$-6x - 2y - z = -15$
$-4x + 4y + 5z = -15$

2) $2x - 5y + 2z = -5$
$-4x + 10y - 4z = 10$
$5x + 6y - z = 18$

3) $6x + 3y - 5z = 5$
$-2x - 3y - z = -1$
$2x + y - 3z = -1$

4) $x - 2y + 4z = -4$
$3x + 4y - 5z = 25$
$5x - 3y + 2z = 12$

5) The perimeter of a right triangle is 30 cm. The length of the hypotenuse is the sum of twice the length of the shorter leg plus three. The length of the longer leg is one less than the hypotenuse. Find the measurement of each side.

6) A local library needs to buy 44 children's, juvenile, and young adult books for its inventory. Each children's book costs the library $2, the juvenile books cost $5 each, while the young adult books cost $6 each. The library has a budget of $184. Twice the amount of young adult novels minus one is equal to the total of children's and juvenile books. Determine the number of each type of book.

To continue, return to the Online Lesson.