Lesson 1
Linear Programming

NAME:

Start by navigating to the Online Lesson for instructions.

Objectives

- Determine the optimization, minimum or maximum, for a linear programming graph using the objective function.
- Given a system of linear inequalities, create a graph and use the objective function to optimize.
- Apply linear programming to word problems.

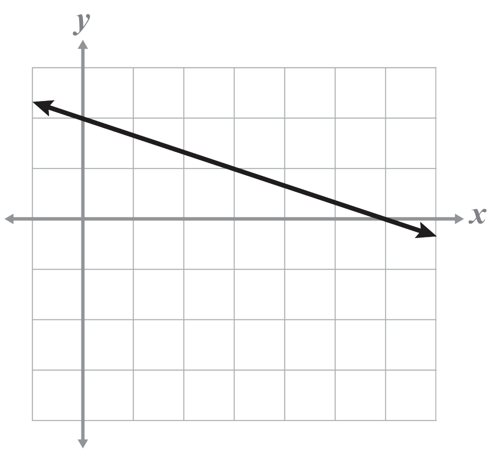
Why?

Have you ever wondered how a bakery knows how many cookies to make to maximize profits? Or a retailer knows how many sneakers to stock? How does a lumber manufacturer minimize fuel usage while maximizing the amount of product that is shipped? Determining the answer to these questions uses linear programming.

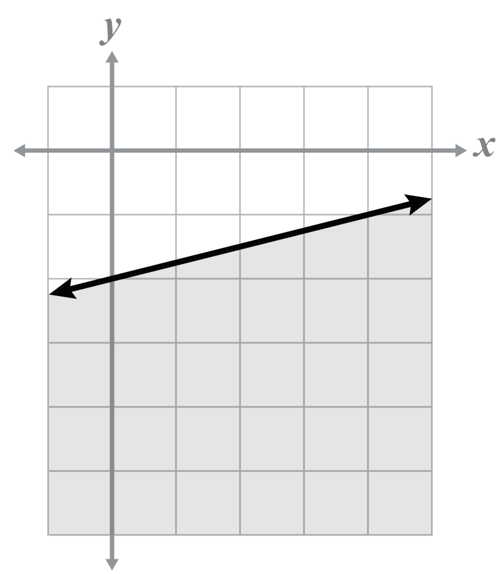
Warm Up

Write the equation or inequality of the line in slope-intercept form.

1)



2)



3) On the coordinate plane for problem 1, add the graph of the equation: $y = \frac{4}{3}x - 3$

4) Name the solution to the system created from problems 1 and 3.

To continue, return to the Online Lesson.

Explore

Optimization

Fill in the notes as you watch the video in the Online Lesson.

- ______________________________ is the mathematical approach that optimizes a function.
- ____________________ is finding the minimum and maximum values of a function, which allows you to ______________ problems and scenarios.
- Specifically, the __________________________ $f(x, y) = x + y$ is used to determine the minimum and/or maximum values within the feasibility region.
- The ___________________________ is another way to reference the region where all possible solutions exist for a system of linear inequalities.
- The x- and y-values that are substituted into the objective function come from the ______________________ that form the __________ of the feasibility region on the graph.
- A ____________ is a point where two or more lines intersect.

Reading graphs and creating graphs accurately on the coordinate plane is critical for linear programming problems.

Example 1

Complete the example as you watch the video in the Online Lesson.

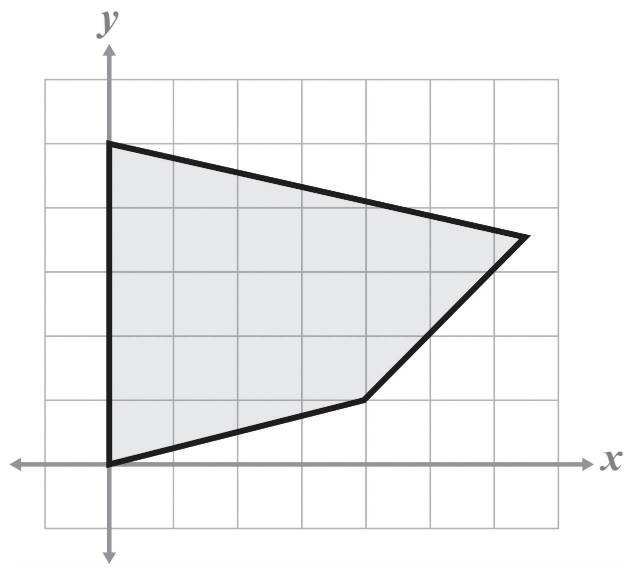
Lucy is given the following graph and asked to find the minimum and maximum values using the objective function $f(x, y) = 3x - y$.

Plan

Name the vertices

Substitute each vertex into the objective function and evaluate

Determine the minimum and maximum values



Implement

There are 4 vertices, so there will be 4 equations to evaluate using $f(x, y) = 3x - y$.

$f(0, 0) = 3(0) - (0)$
$f(0, 0) =$

$f(0, 5) = 3(0) - (5)$
$f(0, 5) =$

$f(6.5, 3.5) = 3(6.5) - (3.5)$
$f(6.5, 3.5) =$

$f(4, 1) = 3(4) - (1)$
$f(4, 1) =$

The vertices are named in a clockwise order in this level when possible.

Explain

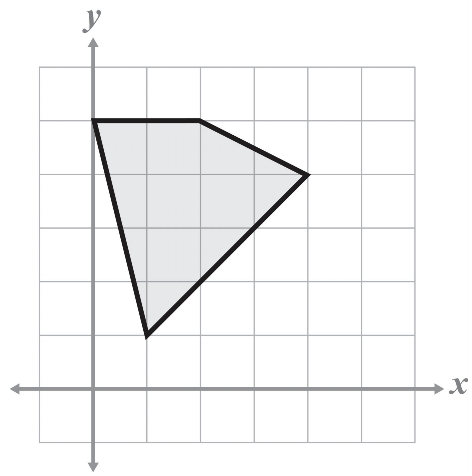
The vertex with the minimum value is ______________. The vertex with a maximum value is _______________.

Example 2

Complete the example as you watch the video in the Online Lesson.

Write a system of inequalities given the graph. Then use the objective function to find the minimum and maximum values.

Objective function: $f(x, y) = y - x$



Plan

Mark and name all of the vertices

Find the slope and y-intercepts

Determine if the shading is above(↑) or below(↓) the line

Write an inequality for each of the 4 sides of the figure

Implement

Vertices: __________ __________

__________ __________

Lines:

$\overline{AB}$	$\overline{BC}$	$\overline{CD}$	$\overline{AD}$
$m = -4, b = 5, \uparrow$	$m = 0, b = 5, \downarrow$		
$y \geq -4x + 5$	$y \leq 5$		

Function: $f(x, y) = y - x$

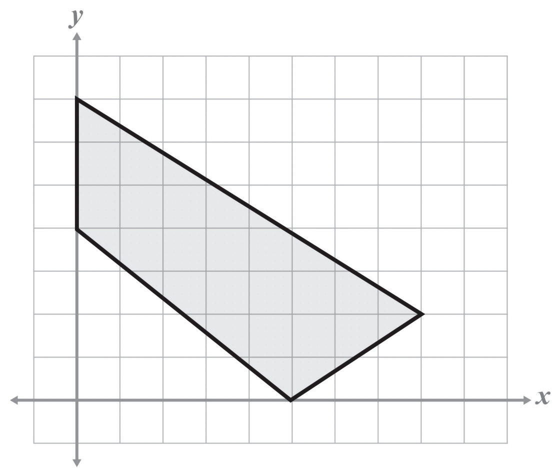
$f(1, 1) = 1 - 1$

$f(1, 1) = 0$

Explain

Checkpoint: Optimization

Label all of the vertices on the graph. Then find the minimum and maximum values using the objective function: $f(x, y) = 2x + y$.



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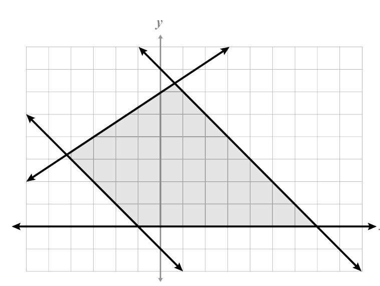
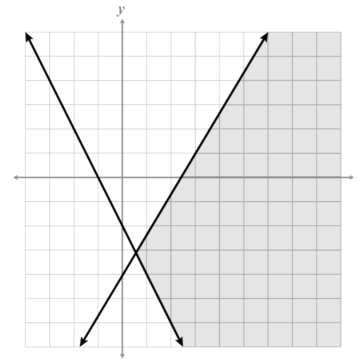
Graphing for Linear Programming

Fill in the notes as you watch the video in the Online Lesson.

- If a linear programming problem does not provide a graph, you must ___________ ___________________________________ using the provided system of inequalities.
- Once the graph is created, ___________________________________ so you can find the minimum and/or maximum using the objective function.
- Use ______________________ to confirm that the vertices are correct. To confirm, replace the variables with the point (or vertex) where the two equations or inequalities intersect.
- Graphs of systems of inequalities can have feasibility regions that are _______________ or ___________________.
  - A bounded region is an _________________, shaded region with vertices that provide _________________one minimum _______ one maximum using the objective function.



- An unbounded region continues infinitely in at least one direction with vertices that provide a minimum _______ a maximum, but not both. The minimum or maximum depends on the _________________ of the shaded region.



Example 3

Complete the example as you watch the video in the Online Lesson.

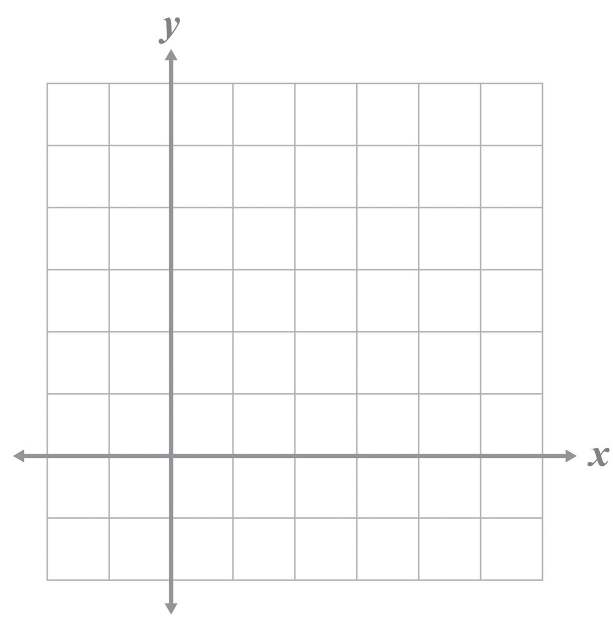
Find the minimum and maximum values using the objective function: $f(x, y) = 5y - 2x$. Explain if the system is bounded or unbounded.

$y \le -2x + 6$

$x + y \le 4$

$x \ge 0$

$y \ge 0$



Plan

Graph system of inequalities

Name all the vertices

Evaluate the objective function using the vertices

Name the minimum and maximum values

Implement

Explain

This is a ________________ system. The maximum is ________________ and the minimum is ________________.

Example 4

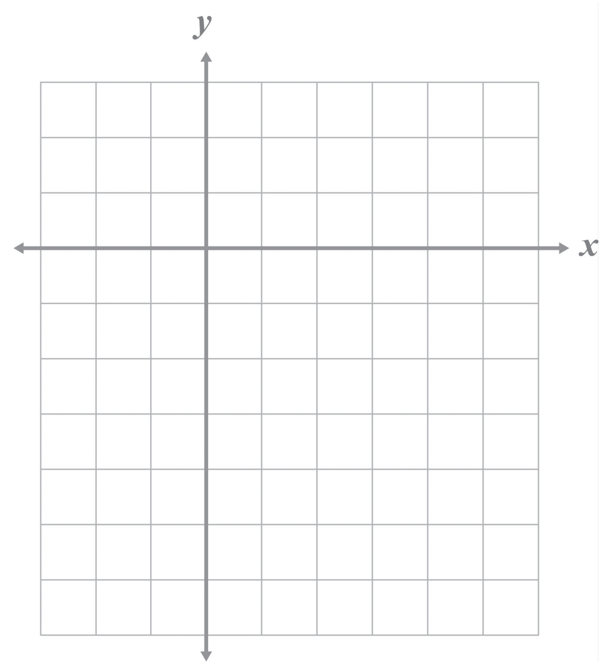
Complete the example as you watch the video in the Online Lesson.

Find the minimum and maximum values using the objective function: $h(x, y) = x + y - 1$.

$y \le 2x - 3$

$y \ge -3$

$y \ge \frac{1}{3}x - 4$



Checkpoint: Graphing for Linear Programming

Graph the system of inequalities. Name all of the vertices and evaluate using the objective function for the minimum and maximum values.

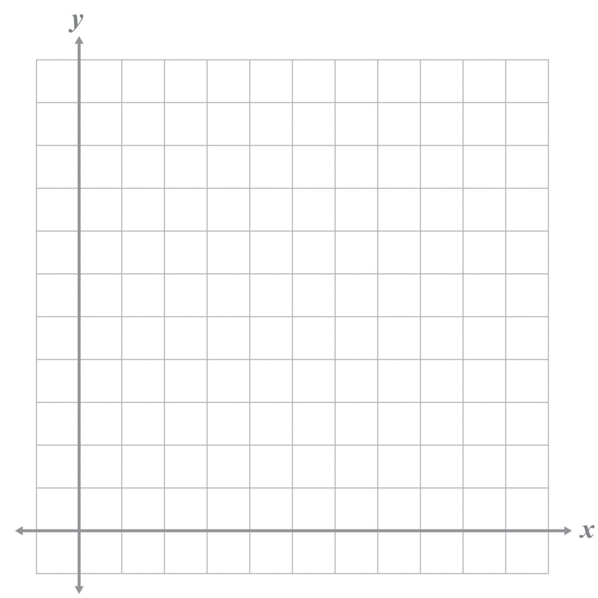
$f(x, y) = -0.5x + 1.5y$

$y \le \frac{1}{4}x + 5$

$y \le -x + 10$

$y \ge -\frac{3}{5}x + 5$

$y \ge \frac{1}{2}x - \frac{1}{2}$



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Applications of Linear Programming

Fill in the notes as you watch the video in the Online Lesson.

- Linear optimization has many ____________________ applications.
- Linear programming word problems typically provide the information needed to write a system of inequalities so that you can graph and then ____________________ ____________________.
- When writing a system of inequalities for a scenario, consider what ______________ make the most sense for the solutions.

Example 5

Complete the example as you watch the video in the Online Lesson.

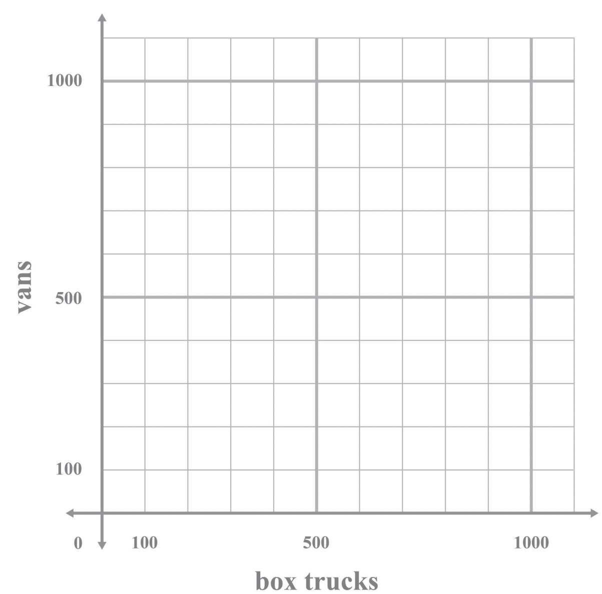
Speedy Delivery has two types of delivery vehicles, trucks (x) and vans (y). The total daily mileage of the trucks and vans cannot exceed 1000 miles. If 3 trucks are on the road, then 2 less vans can be used to maintain a total mileage that is less than or equal to 1000 miles. Each day the vans will drive at least 100 miles and the box trucks will drive at least 100 miles. Write a system of inequalities to maximize deliveries using the equation $f(x, y) = 10x + 15y$.

Plan

Write and graph a system of inequalities

Find the vertices

Determine the optimization



Implement

Explain

Checkpoint: Applications of Linear Programming

Use the graph from Example 5 to complete the Checkpoint.

Suppose the company updated their objective function to $f(x, y) = 5x - y$. Explain the maximum number of miles that each type of vehicle can drive now.

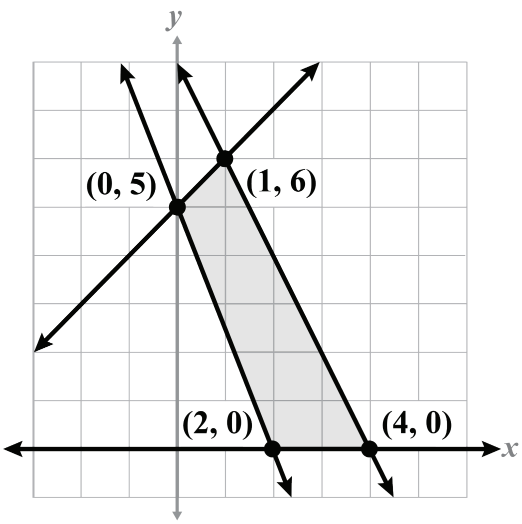
To continue, return to the Online Lesson.

Practice 1

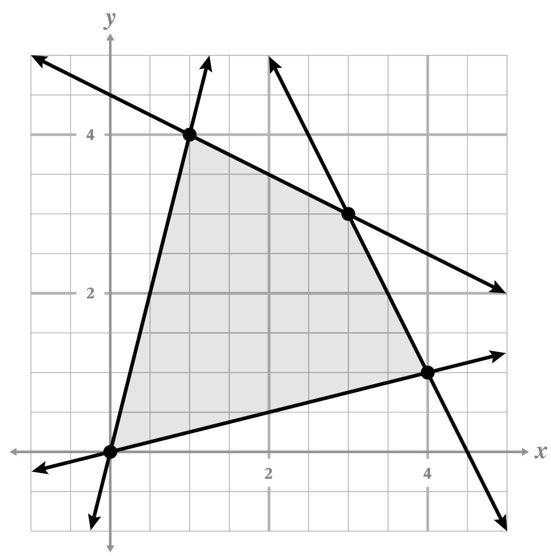
Complete problems on a separate sheet of paper.

Given the graph, find the minimum and maximum values using the objective function.

1) $f(x, y) = \frac{1}{2}x + 4y$

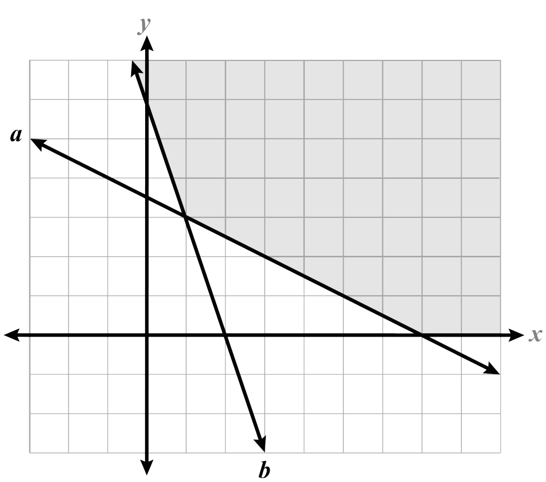


2) $f(x, y) = 5y - x$

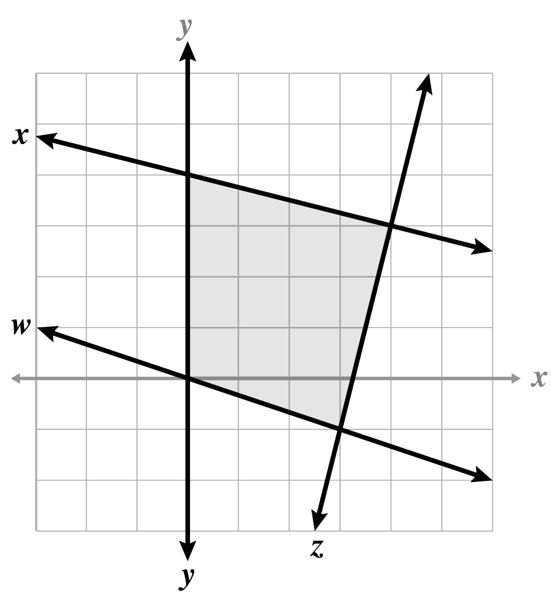


Write a system of inequalities given the graph. Then use the objective function to find the minimum and maximum.

3) $f(x, y) = 6x + 5y$



4) $f(x, y) = x - \frac{3}{4}y$



Graph the system of inequalities. Name all of the vertices and evaluate using the objective function for the minimum and maximum values.

5)
$x + 2y \le 12$
$x + y \ge 5$
$x + 3y \le 15$
$x \ge 0$
$y \ge 0$

$f(x, y) = 4.5y - 3x$

6)
$y \le -x + 4$
$y \le -\frac{1}{3}x + 2$
$x \ge 0$
$y \ge 0$
$f(x, y) = 3x + 2y$

7)
$y \le \frac{1}{2}x + 2$
$y \le -x + 8$
$x \ge 2$
$y \ge 1$
$f(x, y) = 2x - 3y$

8)
$y \ge -3x + 7$
$x + 2y \ge 9$
$x \ge 0$
$y \ge 0$
$f(x, y) = 4x - y$

9) Computer Concepts plans to build new computer chips with different materials, Type A and Type B. The Type A material weighs x grams. The Type B material weighs y grams. The maximum weight for the computer chips is 250 grams. Type A costs $150 per gram, while Type B costs $200 per gram. Local investors will invest no more than $45,000 in materials.

A) Write and graph the system of inequalities to find the vertices.

Type A yields 600 mb of storage per gram, and Type B yields 750 mb of storage per gram.

B) Determine the optimization equation for the amount of Type A and Type B material that Computer Concepts should build to maximize storage capacity.

C) Find the optimized number of Type A and Type B material to provide the highest storage capacity for Computer Concepts.

10) Friendship INC is creating a new set of rings for a special event. It takes x grams of silver and y grams of gold to make a ring. All rings have a combination of gold and silver. The "traditional" ring has twice as much gold as silver and weighs no more than 4 grams. The "contemporary" ring has twice as much silver as gold and weighs no more than 5 grams. The rings need a minimum of 0.5 grams of silver and a minimum of 0.5 grams of gold. Write and graph the system of inequalities to find the vertices.

The profit earned per gram of silver is $6 per gram and $8 per gram of gold.

How much of each type of metal should they use in their rings to optimize profit?

To continue, return to the Online Lesson.

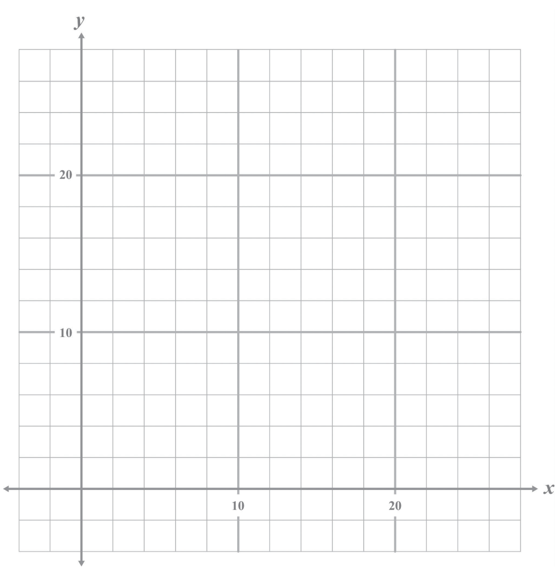
Mastery Check

Show What You Know

Pups Nutrition Company is creating new dog food supplements with two main nutritional components, x and y. The Product A supplement contains 2 times as much x nutrition as y nutrition and must have at least 20 grams of nutrition per serving. The Product B supplement contains 2 times as much y nutrition as x nutrition and has at least 25 grams of nutrition per serving.

A) Write a system of inequalities to represent the given scenario.

B) Graph your system from part A.



C) Evaluate the objective function, $g(x, y) = 3x + 5y$ using part B.

D) The company wants to minimize the cost of creating each new product. Component x costs \$3 per gram, while y costs \$5 per gram. How much of each should they order to *minimize* costs?

Say What You Know

In your own words, talk about what you have learned using the objectives for this part of the lesson and your work on this page.

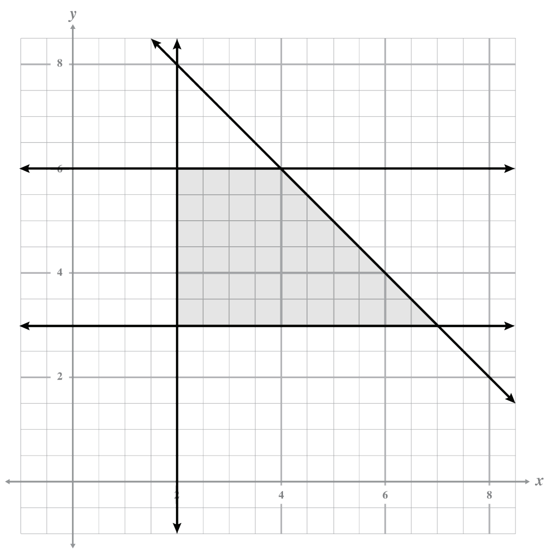
To continue, return to the Online Lesson.

Practice 2

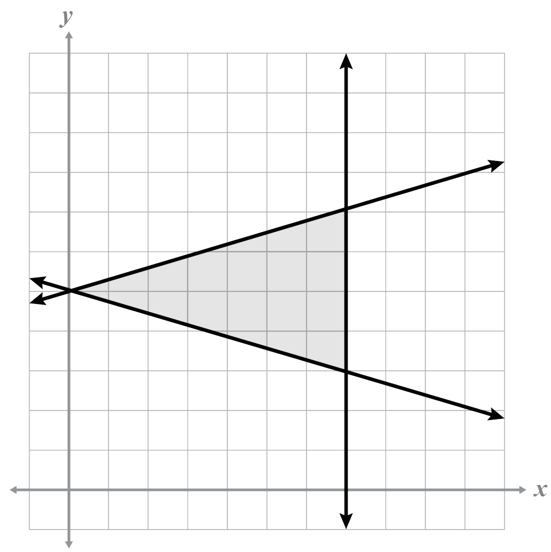
Complete problems on a separate sheet of paper.

Given the graph, find the minimum and maximum values using the objective functions.

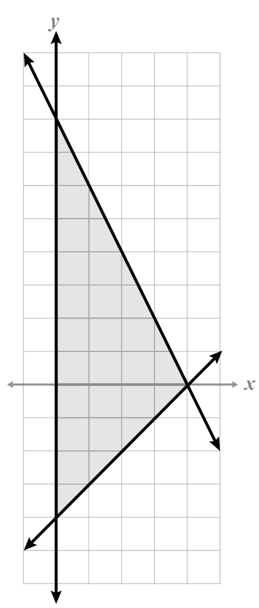
1) $f(x, y) = x + \frac{2}{3}y$



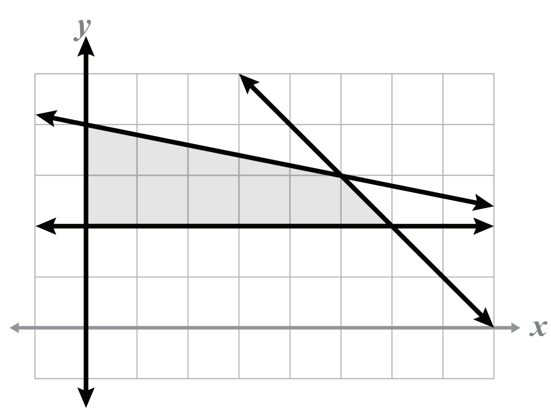
2) $f(x, y) = 3y - x$



3) $f(x, y) = 8x - 5y$



4) $f(x, y) = 3x + 2y$



Graph the system of inequalities. Name all of the vertices and evaluate using the objective function for the minimum and maximum values.

5) $y \le -x + 12$

$y \ge \frac{1}{2}x$

$x \ge 2$

$y \ge 2$

$f(x, y) = 5x - y$

6) $3x + 5y \le 45$

$y \ge -2x + 6$

$x \ge 0$

$y \ge 0$

$f(x, y) = 3x + y$

7) $y \ge -3x + 9$

$y \ge \frac{2}{3}x - 2$

$x \ge 2$

$f(x, y) = 2x + 4y$

8) $x + 5y \le 25$

$x + y \le 9$

$y \ge 1$

$x \ge 0$

$f(x, y) = 4x + y$

9) A food supplier has options for selling almonds and cashews in bulk as a way to save on restaurant food costs. Option A allows restaurants to buy 5-pound bags of almonds and 9-pound bags of cashews with a minimum order of at least $420. Option B allows restaurants to purchase 6-pound bags of almonds and 10-pound bags of cashews with a maximum order of $730. At least 30 pounds of almonds and at least 25 pounds of cashews must be ordered from the food supplier. Determine the minimum purchase when the cost of almonds is $6 per pound and the cost of cashews $8 per pound.

10) Sci Labs is producing two new robotic arms. Sci Labs will use at least 20 grams of aluminum and at least 15 grams of iron. Model A uses 30 grams of aluminum and 40 grams of iron and can weigh no more than 2700 grams. Model B uses 20 grams of aluminum and 30 grams of iron and can weigh no more than 1900 grams. Find the maximum number of grams of aluminum and iron Sci Labs should use with the optimization equation $f(x, y) = 25x + 38y$.

To continue, return to the Online Lesson.

Targeted Review

Complete items on a separate sheet of paper.

Simplify. Answers should contain positive exponents only.

1) $\dfrac{x^3y^{-2}}{y^2x^4}$

2) $(3p^2q^{-5})^2$

Multiply.

3) $(x-3)(5x+1)$

4) $(2x+3)(x^2-4x-5)$

Factor.

5) $25x^2y^2+15xy^2+30xy$

6) $8x(x+11)-6(x+11)$

Solve.

7) $\dfrac{2}{3}x-5=\dfrac{1}{6}$

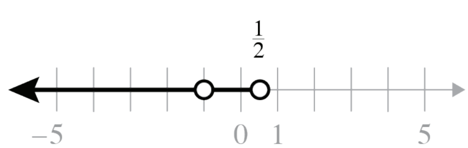
8) $9\left(x-\dfrac{2}{3}\right)+2x=6(2x+1)$

Multiple Choice

9) Select all terms that are *like terms*.

- ☐ $3x^3$
- ☐ $3x$
- ☐ x^3
- ☐ 3

_____ **10)** Solve: $|4x+3|>5$

A)


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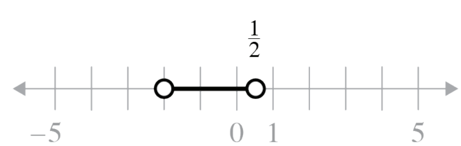

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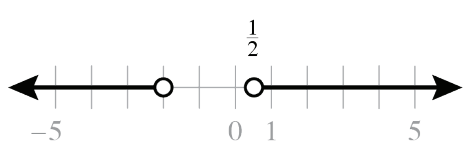

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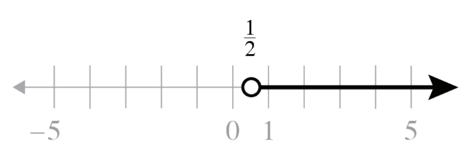

11) Select all solutions to the equation $4x^2 = 64$.

- ☐ -4
- ☐ 4
- ☐ ± 4
- ☐ ± 8

_____ **12)** Bob did chores for his neighbor. He already has $\$8$ saved. Then he earned $\$10$ per day for two weeks. If he took off one day each week, how much did he save at the end of two weeks?

A) $\$18$

B) $\$68$

C) $\$128$

D) $\$148$

Problem	1	2	3	4	5	6	7	8	9	10	11	12
Origin	A1	A1	A1	A1	A1	A1	A1	A1	A1	A1	A1	A1

L = Lesson in this level, A1 = Algebra 1: Principles of Secondary Mathematics, FD = Foundational Knowledge

To continue, return to the Online Lesson.